Problem 1. (10 points)
Show if network has an anti-parallel edge (directed edges going both ways), there exists a maximum flow that doesn’t use both.

Problem 2. (15 points)
Given an arbitrary graph $G = (V, E)$, design an algorithm to determine if $G$ is bipartite. Your algorithm should run in time linear in the number of vertices and edges (in other words the performance should be $O(V + E)$).

Your answer should consist of the following parts:
- A brief description of your idea and a justification of why it works,
- Pseudocode,
- Analysis of the time complexity.

Problem 3. (10 points)
Suppose we are given N points in the plane and we want to find the shortest set of line segments connecting all the points. This problem is called Euclidean minimum spanning tree problem. Describe an algorithm that can find the Euclidean minimum spanning tree in time proportional to $N^2$.

Problem 4. (15 points extra credit)
If you are given the convex hull $H$ of n points, describe and algorithm to find if a single point $(x,y)$ lies inside or outside the convex hull.

If you have the convex hull $(H)$ of a simple polygon $P$ (the polygon may not be convex) made of n points, design an algorithm to determine if some point $(x,y)$ is inside of $P$. Your algorithm should run linear time compared to the number of points on the polygon/convex hull, namely $O(n)$.

Problem 5. (10 points)
Reweight the following directed graph to contain no negative weights, yet maintain the same shortest path properties (as described in the first part of the Johnson's algorithm). Show the reweighted graph (you do not need to run anymore of the Johnson's algorithm). You do not need to show how you find the shortest paths when showing work.

Adjacency matrix:

$\begin{array}{cccccc}
0 & \infty & \infty & -3 & 5 \\
-2 & 0 & 2 & 4 & \infty \\
-3 & \infty & 0 & -2 & \infty \\
\infty & 7 & \infty & 0 & 10 \\
\infty & \infty & 6 & \infty & 0 \\
\end{array}$