No office hours on Monday

• (I’ll be giving the CS colloquium talk)

• Final project meets MS plan C project requirements
Topics

- Rigid body dynamics
- Collisions and contact
- Fluid simulation
- Elastic bodies
- Cloth and thin shells

Based on time and interest:
- Hair
- Plasticity and fracture
- Fire and explosions
- Snow, foams, and goop
- Artistic control
- Real-time techniques
Survey results

• Prior knowledge:
  • Graphics & animation: 13/17
  • Numerical methods: 11/17
Topics of interest

- Particles
- Rigid bodies
- Collisions/contact
- Fluids
- Deformation
- Cloth
- Hair
- Fracture
- Real-time
- Crowds
Expected difficulties

- Math
- Physics
- Theory
- Reading papers
- Implementation
- Quiz
Differential equations and particle dynamics
Quick recap

• A particle system is a collection of *particles*, acted upon by *forces* (gravity, wind, springs, collisions, …)

• Particles 1, 2, 3, … have positions \( x_1, x_2, x_3, \ldots \) and velocities \( v_1, v_2, v_3, \ldots \)

• The acceleration \( a_i \) of particle \( i \) is determined by the total force \( f_i \) acting on it,

\[
    f_i = m_i a_i
\]
Quick recap

\[ f_i = m_i a_i \]

- Acceleration is rate of change of velocity. Velocity is rate of change of position.

- So now we have a system of differential equations:

\[
\begin{align*}
    x_1' &= v_1 \\
    v_1' &= f_1/m_1 \\
    x_2' &= v_2 \\
    v_2' &= f_2/m_2 \\
    &\vdots
\end{align*}
\]
Quick recap

• Given $x' = g(x, t)$ and $x(t_0) = x_0$, how to compute $x(t_1)$?

$$x(t_1) = x(t_0) + x'(t_0) \Delta t + \frac{1}{2} x''(t_0) \Delta t^2 + \cdots$$

- $x(t_1) \approx x(t_0) + x'(t_0) \Delta t$

• Applies equally well when $x$ is a vector, or a collection of vectors (e.g. positions and velocities of all particles).
More ways of looking at it

- **Fundamental theorem of calculus:**

  \[ x(t_1) - x(t_0) = \int x'(t) \, dt \]

- Assume \( x'(t) = x'(t_0) \) over the whole interval.
More ways of looking at it

- **Mean value theorem**: There exists some $t^*$ between $t_0$ and $t_1$ such that

  $$x(t_1) - x(t_0) = x'(t^*) \Delta t.$$ 

- Assume $t^* = t_0$. 

![Diagram showing secant and tangent lines at points $t_0$, $t^*$, and $t_1$.](image-url)
Mass-spring systems
[World of Goo]
[Choi and Ko, “Stable but Responsive Cloth”, 2002]
[Selle et al., “A Mass Spring Model for Hair Simulation”, 2008]
A two-dimensional sheet made of masses and springs.

What could go wrong?
What could go wrong

- No shear resistance? Add more springs
- No bending resistance? Add more springs
- Resolution-dependent behavior? Um…
Homework

• Implement a simple mass-spring system

• Starter code and detailed information will be provided soon (sorry…)}
Next class

- *Time integration*

- **Reading**: “Implicit Methods” from Witkin and Baraff