1 Let $Q = [q_1, q_2, \cdots, q_n]$ an $m \times n$ matrix with orthonormal columns (so $m \geq n$). Show that any vector $x$ in $\mathbb{R}^m$ can be written as $x = Qy + w$ where $y \in \mathbb{R}^n$ and $w \perp \text{span}\{Q\}$. Show a geometric illustration of this decomposition.

2 (Continuation) What are all solutions of the system of equations $Q^T x = b$ for a given $b$? Which one of these has the shortest length? Show a geometric illustration.