1. Prove that two (right) eigenvectors $u_1, u_2$, associated with distinct eigenvalues $\lambda_1, \lambda_2$ are linearly independent. Generalize to a set $u_1, u_2, \cdots, u_k$ of $k$ eigenvectors associated with distinct eigenvalues $\lambda_i, i = 1 : k$ (with $\lambda_i \neq \lambda_j$ for $i \neq j$.)

2. (a) Find the nonzero singular values of the matrix $A$ shown on the right. (b) Let $A = U\Sigma V^T$ be the ’thin’ SVD of $A$ ($U \in \mathbb{R}^{4 \times 2}, \Sigma \in \mathbb{R}^{2 \times 2}, V \in \mathbb{R}^{2 \times 2}$). What is the matrix $V$? What is $\Sigma$? (c) Find the matrix $U$ defined in (b). (d) What is the pseudo-inverse of $A$? (e) Among all matrices $B \in \mathbb{R}^{4 \times 2}$ that are of rank 1, find the one that minimizes $\|A - B\|_2$. 

$A = \begin{pmatrix} 1 & 0 \\ 2 & -2 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$
3. Use Gershgorin’s theorem to locate the eigenvalues of the matrices

\[
A = \begin{pmatrix}
-1.5 & 0.5 & 0 \\
0.5 & 0 & -0.5 \\
0 & -0.5 & 1.5
\end{pmatrix}
\quad
B = \begin{pmatrix}
1 & -i & i \\
i & 2 & 0 \\
-i & 0 & 3
\end{pmatrix}
\]

Can you be specific as to the locations of each of the three eigenvalues of \( A \)? [Hint: You will need to use a continuity argument]