1. Here is a second form of the Min-Max theorem:

\[ \lambda_k = \min_{S, \dim(S)=n-k+1} \max_{x \in S, x \neq 0} \frac{(Ax, x)}{(x, x)}. \]

Adapt the proof of the other result to establish this.

2. Let \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \) be the eigenvalues of the symmetric matrix

\[ A = \begin{pmatrix} B & c \\ c^T & d \end{pmatrix} \]

shown on the right and let the eigenvalues of \( B \) be: \( \mu_1 \geq \mu_2 \geq \cdots \geq \mu_n \). Show that:

\[ \mu_i \leq \lambda_i \quad \text{and} \quad \lambda_{i+1} \leq \mu_i \quad \text{for} \quad i = 1, 2, \cdots, n-1. \]

[Hint: show first that when \( x = [y^T, 0]^T \) we have \((Ax, x) = (By, y)\). Then use the two min-max theorems]

3. Show the interlacing property:

\[ \lambda_1 \geq \mu_1 \geq \lambda_2 \geq \mu_2 \geq \cdots \geq \mu_i \geq \lambda_{i+1} \geq \mu_{i+1} \geq \cdots \]