Updates

**Assignment 2** will be posted soon

- Collisions between mass-spring systems, rigid bodies, static obstacles

**Presentations**

- List of suggested papers will be posted tomorrow
- 2 presentations per person. Send me 4 choices by end of Tuesday
Updates

Project

• Form pairs, start thinking of project ideas

• Send me a brief pre-proposal email next week, schedule a meeting to discuss the topic

• 2-page proposal due two weeks after that, should describe:
  — background and related work,
  — proposed method(s),
  — goals and evaluation
Constraints and collisions
Inequality constraints

\[ c(x) \leq 0 \]

If \( c(x) = 0 \) then \( J_c v \leq 0, \ f_{\text{con}} = J_c^T \lambda \)

If \( c(x) < 0 \) then \( J_c v = \text{anything}, \ f_{\text{con}} = 0 \)

\[ \|x_1\| - \ell_1 \leq 0 \]
Collisions as constraints

$$\varphi(x) \geq 0,$$

$$J_\varphi = n^T$$

If $$\varphi(x) = 0$$

then $$n \cdot v \geq 0,$$

$$f_{\text{con}} = \lambda n$$

(participant is on surface)

(participant cannot move into object)

(force is normal to surface)

If $$\varphi(x) > 0$$

then $$v = \text{anything},$$

$$f_{\text{con}} = 0$$

(participant is away from surface)

(no force if not in contact)
Collision processing

1. Collision detection
2. Collision response
Collision detection

Given *two objects in motion* (particles, deformable bodies, rigid bodies),
determine if they *are intersecting* or *will soon intersect*,
and return the collision geometry (*contact points* and *normal*).
Collision detection

Usual form: given initial and final configurations $x^n, x^{n+1}$ of the objects,

• Do they collide over the course of the time step?

• What are the contact points?
  What is the collision normal?
Collision detection

Usual form: given initial and final configurations $x^n, x^{n+1}$ of the objects,

- Do they collide over the course of the time step?
- What are the contact points? What is the collision normal?
Collision detection

Usual form: given initial and final configurations $x^n, x^{n+1}$ of the objects,

- Do they collide over the course of the time step?
- What are the contact points?
  What is the collision normal?
Disclaimer

Collision detection is a huge topic!

Lots of different algorithms, depending on whether the objects are:

- convex or non-convex
- represented as meshes or as implicit regions
  - whether the mesh is open (cloth) or closed (solids)
- rigid or deformable

We will only cover a few representative algorithms.

For more, see the *Collision Detection and Proximity Queries* SIGGRAPH 2004 course notes by Hadap et al.
Discrete vs. continuous collision detection
Geometrical tests

Signed distance fields

Vertex-face and edge-edge tests

Convex decomposition
Broad phase algorithms

Sweep and prune

Bounding volume hierarchies
Narrow phase
Signed distance fields

Is a point inside or outside a fixed shape?

- Analytical SDFs for simple shapes
e.g. sphere $\varphi(x) = \|x - c\| - r$, cylinders, boxes, …

- Precomputed SDFs on grids for arbitrary shapes
  Lookup: interpolation
  Gradient: finite differences

*Good for what kinds of collisions?*

*How to do CCD?*
Proximity queries for convex objects

Two convex shapes are disjoint iff there exists a plane that they are on opposite sides of.

- Finding a separating plane: Gilbert-Johnson-Keerthi (GJK) algorithm
- Finding the penetration depth: expanding polytope algorithm (EPA)

More info: van den Bergen, “Proximity Queries and Penetration Depth Computation on 3D Game Objects” (2001)

How to apply this to non-convex objects?