Physics-Based Animation

14 — Intro to continuum mechanics

October 20, 2016
Discrete vs. continuous systems

6 DOFs

3n DOFs

∞ DOFs?
A 1D example

\[ x = (x_0, x_1, x_2, \ldots, x_n) \]

\[ m = M/n \]

\[ f_{\text{spring}} = k_s (\| x_{i+1} - x_i \| - l_0) \]

\[ = k_s l_0 \left( \frac{\| x_{i+1} - x_i \|}{l_0} - 1 \right) \]

\[ = k_s l_0 \left( \frac{\| x_{i+1} - x_i \|}{X_{i+1} - X_i} - 1 \right) \]
A 1D example

\[ \mathbf{x} : [0, L] \rightarrow \mathbb{R}^3 \]

\[ dm = \rho \, dX \]

\[ \mathbf{f}_{\text{elastic}} = E \left( \left\| \frac{d\mathbf{x}}{dX} \right\| - 1 \right) \]
Continuous systems

State is some (continuous, differentiable) function of spatial coordinates

\( x: \) reference space \( \rightarrow \) world space

\( v: \) world space \( \rightarrow \) flow velocity
Continuous systems

Most relevant physical quantities are differential in nature

- Mass \((m)\) → density \((\rho)\),
- change in length \((\ell - \ell_0)\) → strain \((\frac{dx}{dX})\),
- etc.

\(\mathbf{f} = -\nabla U\) is still true for conservative forces… but working out \(\nabla\) requires variational calculus

- Don’t worry about it, we will just use direct formulas for \(\mathbf{f}\)

- Or apply \(\mathbf{f} = -\nabla U\) after discretizing
Spatial discretization

State variables (position, velocity, etc.) stored on nodes

Various ways to evaluate derivatives
Example: Wave equation

Scalar field $y$ (displacement, height, etc.) as a function of $x$ and $t$

\[
\dot{y} = v \\
\dot{v} = \partial_{xx} y
\]

(more commonly written as $\ddot{y} = \partial_{xx} y$)

• Spatial discretization: Grid $y = (y_0, y_1, y_2, \ldots, y_m)$, $v = (v_0, v_1, v_2, \ldots, v_m)$

• Time discretization: Given $y^n, v^n$, compute $y^{n+1}, v^{n+1}$
Example: Wave equation

\[ \dot{y} = v \]
\[ \dot{v} = \partial_{xy}y \]

...and boundary conditions (e.g. \( y(0) = 0, \partial_x y(1) = 0, \) etc.)

Spatial discretization: Grid \( \mathbf{y} = (y_0, y_1, y_2, \ldots, y_m), \mathbf{v} = (v_0, v_1, v_2, \ldots, v_m) \)

How to compute \( \dot{v} \)?
Next class

• Elastic bodies and finite elements