Physics-Based Animation

20 — Liquid surfaces

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Liquid surfaces

1. How to discretize the geometry of the liquid surface and volume?
2. How to discretize the physics on the geometrical representation?
Level set method

Surface represented implicitly as the level set \( \{ \mathbf{x} : \varphi(\mathbf{x}) = 0 \} \) of a SDF advected by velocity field


1. Advection: move \( \varphi \) according to velocity field, surface moves with it

2. Redistancing: after advection, \( \varphi \) is no longer an SDF; fix it
   - Fast marching, fast sweeping algorithms

Issues: Diffusion of surface detail, volume loss
Particles

Surface represented implicitly via presence/absence of particles in neighborhood

- **Color field**: interpolate a constant 1 over all particles, surface is where value is \( \frac{1}{2} \) [Müller et al. 2003, Yu & Turk 2010]

- **Signed distance function**: interpolate signed distance from particles [Zhu & Bridson 2005]

Again, surface is level set of scalar function
Meshes

Surface represented *explicitly* as triangle mesh

1. Advect vertices using velocity field
2. Fix topological changes

“Free surface” BCs: Air exerts no force on liquid

\[ p_{\text{air}} = 0 \]

- Naive approach: Assume \( p_{\text{nbr}} = 0 \) if neighbor is outside surface

- Second-order accurate: Assume \( p_{\text{nbr}} \) such that \( p = 0 \) at surface point between nodes [Gibou et al. 2001]

Similarly at slanted solid boundaries: Compute \( \nabla \cdot \mathbf{u} \) as net in/out-flow weighted by face area [Batty et al. 2007]
Surface tension

Curvature causes “pressure jump” at surface:

\[ \Delta p = 2\gamma H \text{ where } H = \text{mean curvature} \]

Compute \( H \), set pressure BC to \( p_{\text{air}} = 2\gamma H \) instead of 0. More details:

- Level set: [Hong and Kim 2003, Losasso et al. 2004]
- Mesh: [Thürey et al. 2010]
- SPH: [Akinci et al. 2013]
Next class
