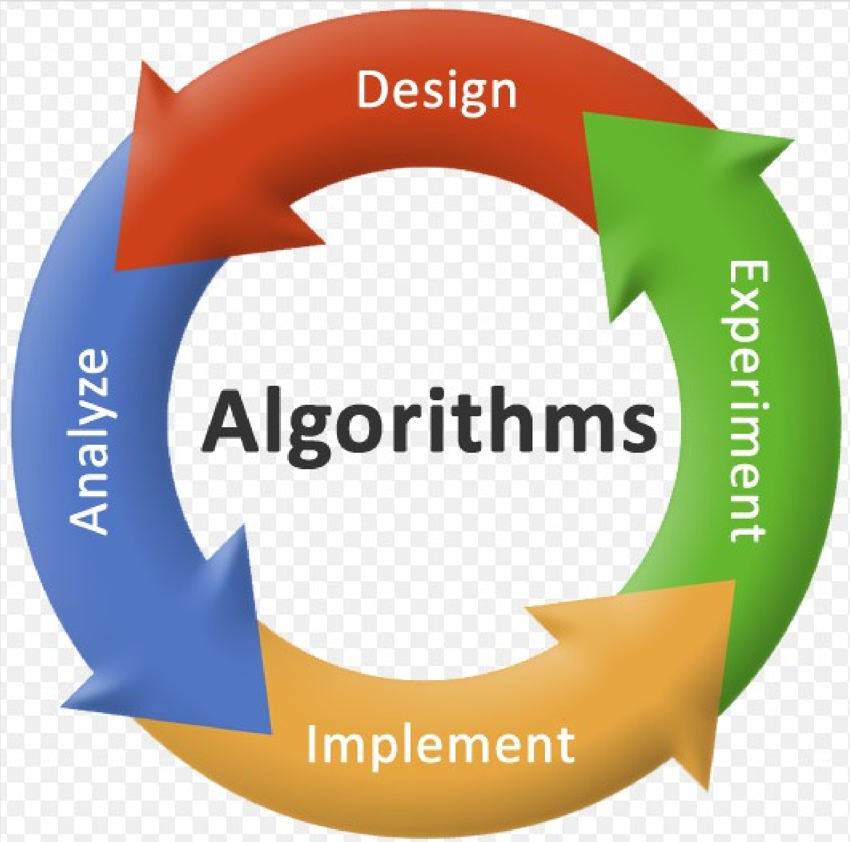
Introduction / Review



Moore's Law

Number of transistors double every two years

This trend has slowed a bit, closer to doubling every 2.5 years

First computer

Memory:
1 MB

CPU:
2.4 Mhz

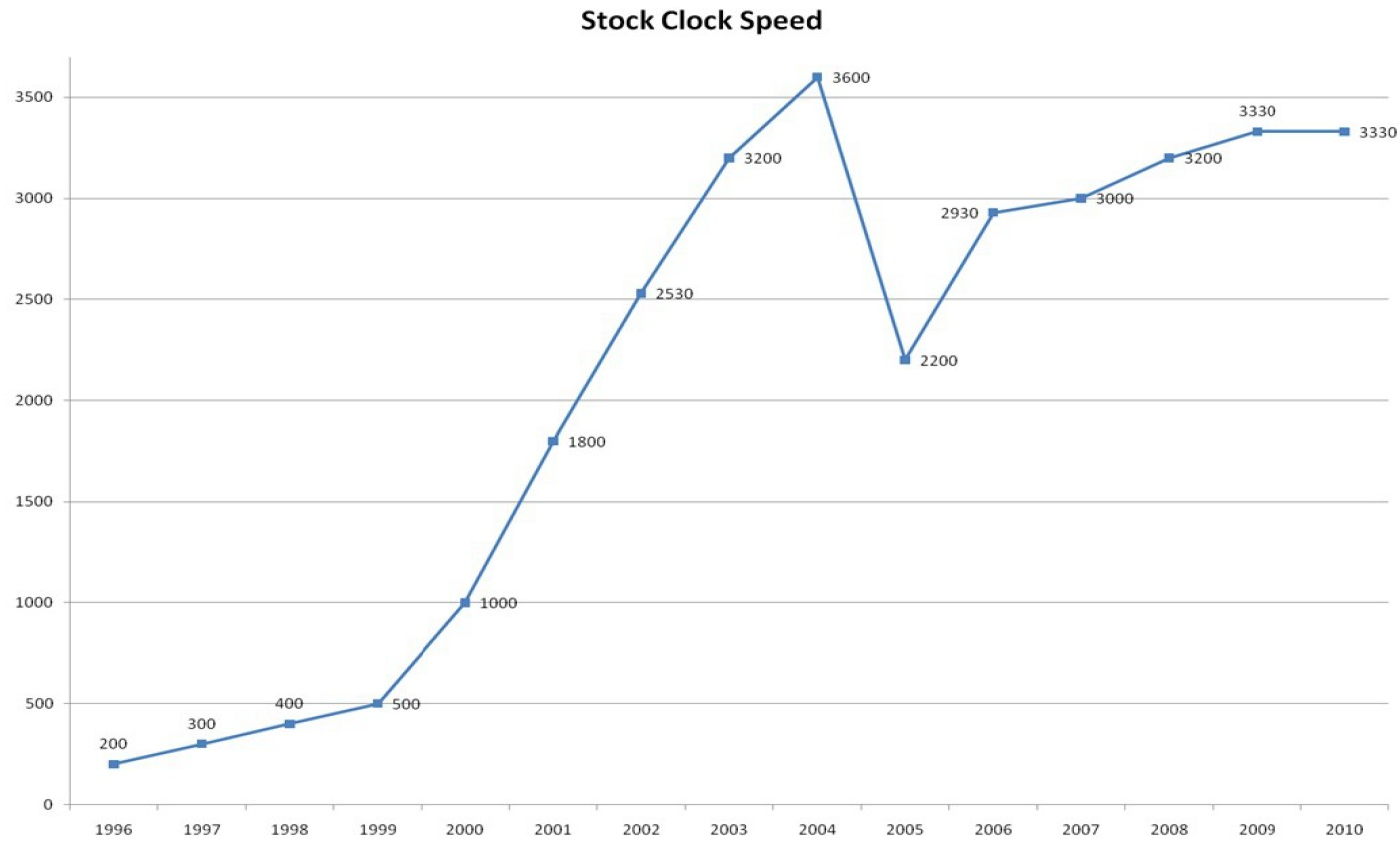
CPU trends

Do you remember how fast the CPU was on your first computer?

How about your current computer?

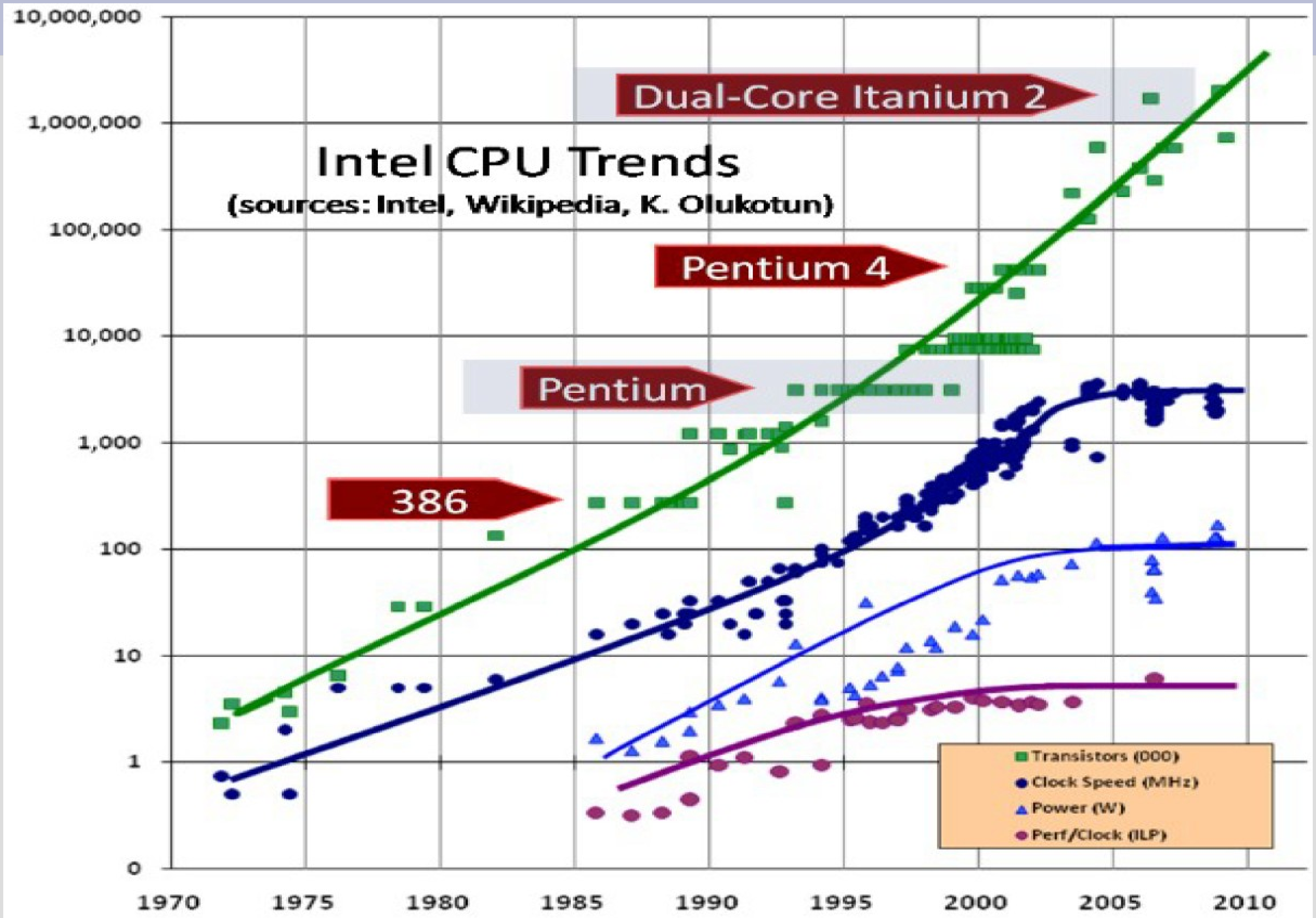
What about your previous computer?

CPU trends



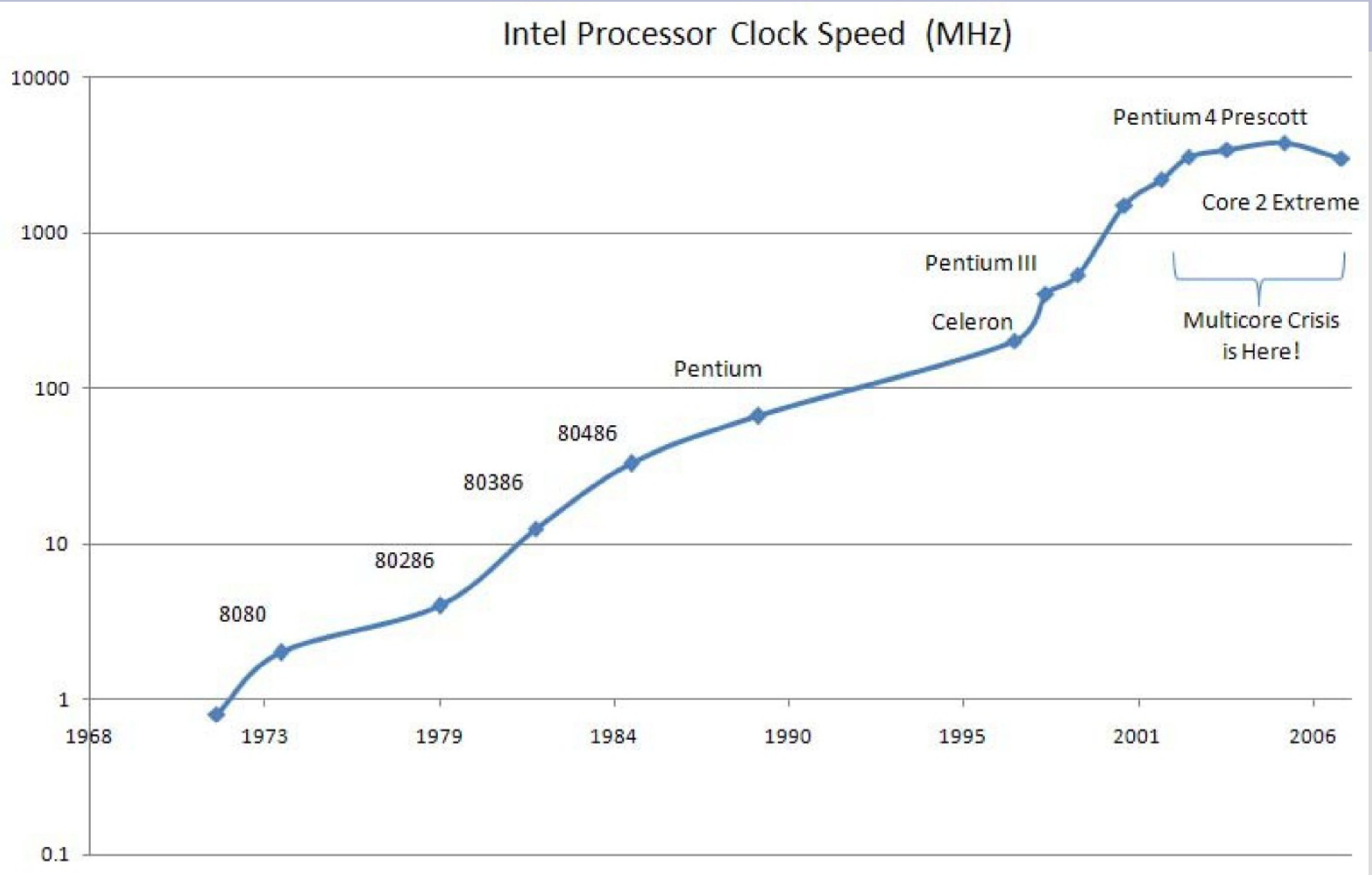
Stock Clock Speed
3500
3000
2500
2000
1500
1000
500
0
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
200
300
400
500
1000
1800
2530
3200
3600
2200
2930
3000
3200
3330
3330

CPU trends



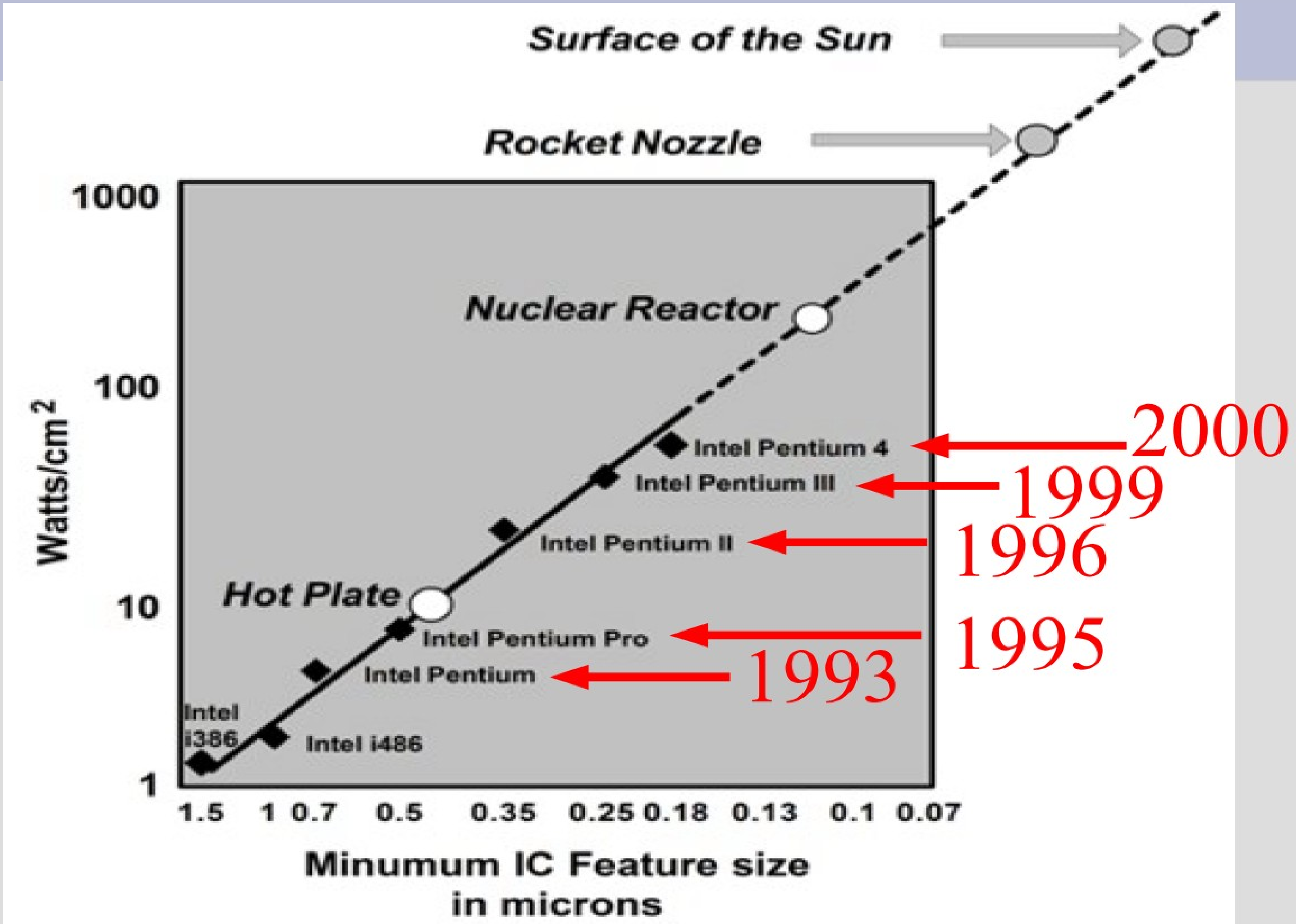
Intel CPU Trends
(sources: Intel, Wikipedia, K. Olukotun)
Dual-Core Itanium 2
Pentium 4
Pentium
386
Transistors (000)
Clock Speed (MHz)
Power (W)
Perf/Clock (ILP)
10,000,000
1,000,000
100,000
10,000
1,000
100
10
1
0
1970
1975
1980
1985
1990
1995
2000
2005
2010

CPU trends



Intel Processor Clock Speed (MHz)
10000
1000
100
10
1
0.1
1968
1973
1979
1984
1990
1995
2001
2006
8080
80286
80386
80486
Pentium
Celeron
Pentium III
Pentium 4 Prescott
Core 2 Extreme
Multicore Crisis is Here!

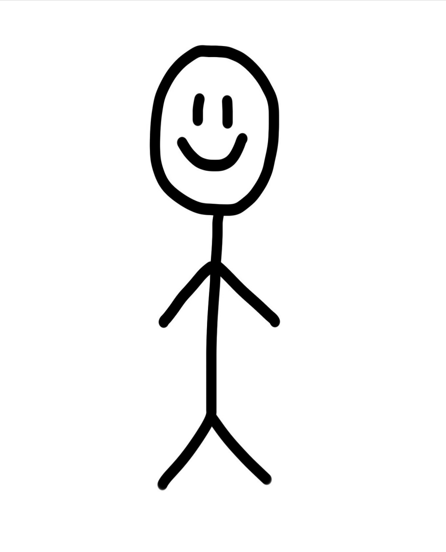
CPU trends

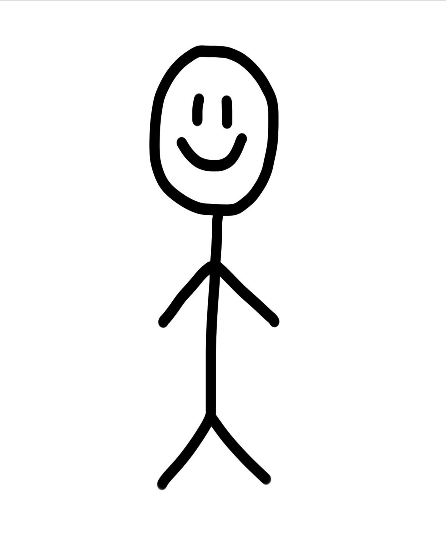


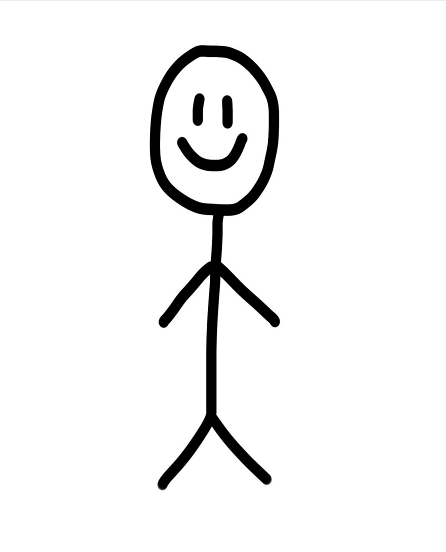
Surface of the Sun
Rocket Nozzle
Nuclear Reactor
Hot Plate
Watts/cm2
1000
100
10
1
Intel Pentium 4
Intel Pentium III
Intel Pentium II
Intel Pentium Pro
Intel Pentium
Intel i386
Intel i486
2000
1999
1996
1995
1993
1.5
1
0.7
0.5
0.35
0.25
0.18
0.13
0.1
0.07
Minumum IC Feature size in microns

Parallel processing (cooking)

You and your siblings are going to make dinner







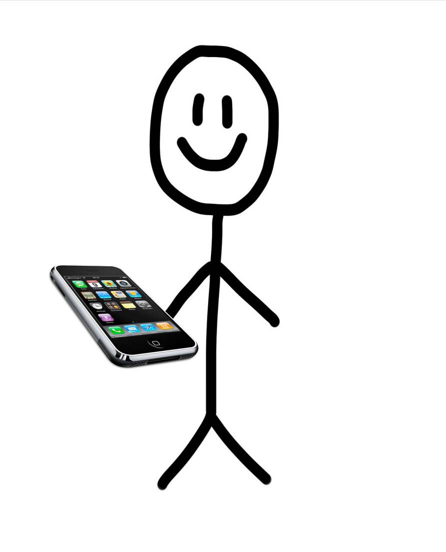
How would all three of you make... :

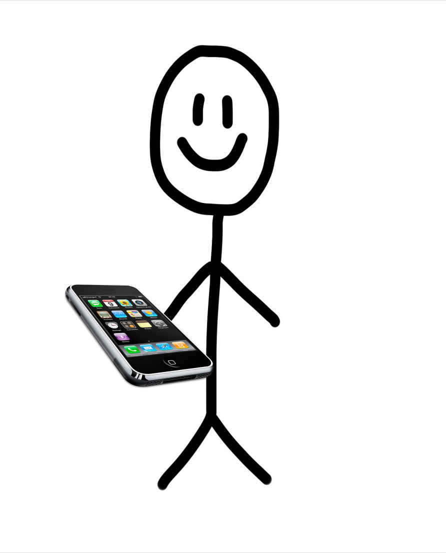
(1) turkey?

(2) a salad?

Parallel processing (cooking)

If you make turkey....

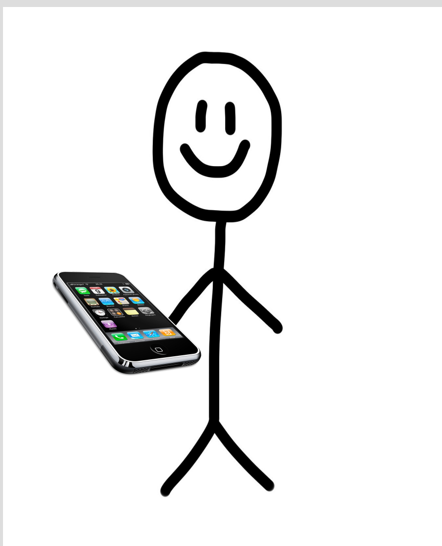


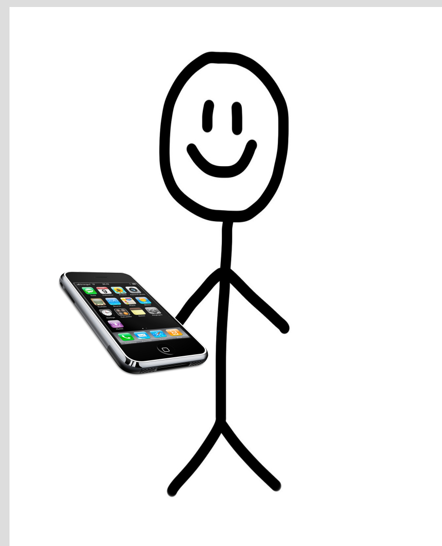


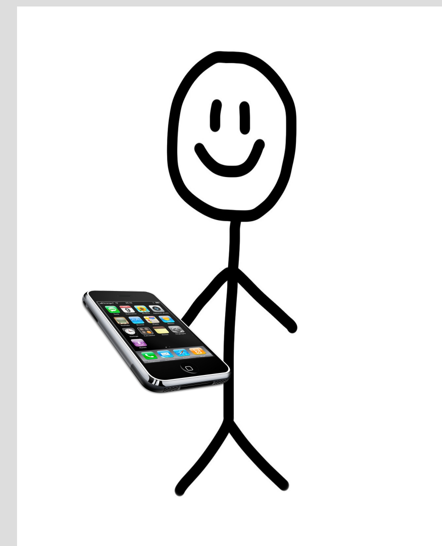
preheat

Parallel processing (cooking)

If you make turkey....





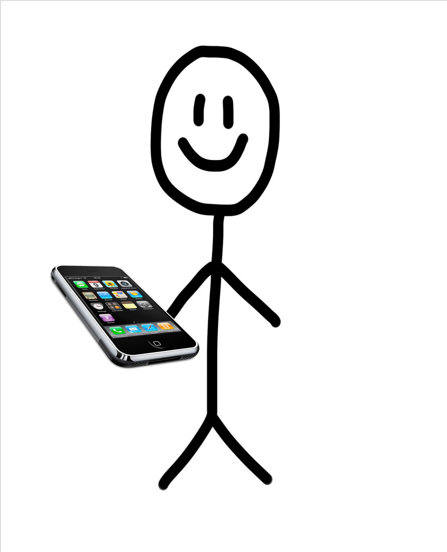


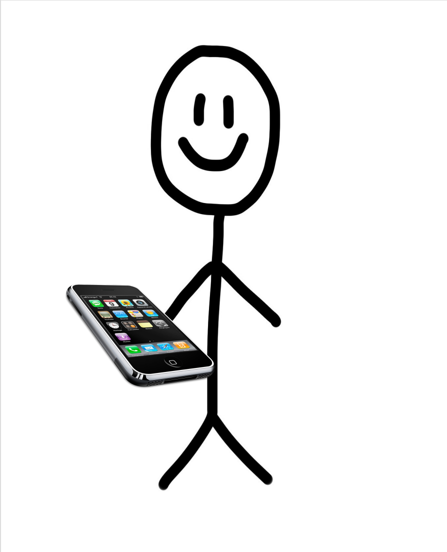
preheat

WAIT

Parallel processing (cooking)

If you make turkey....

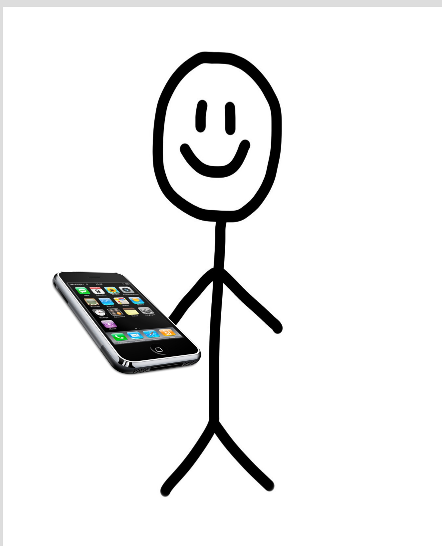


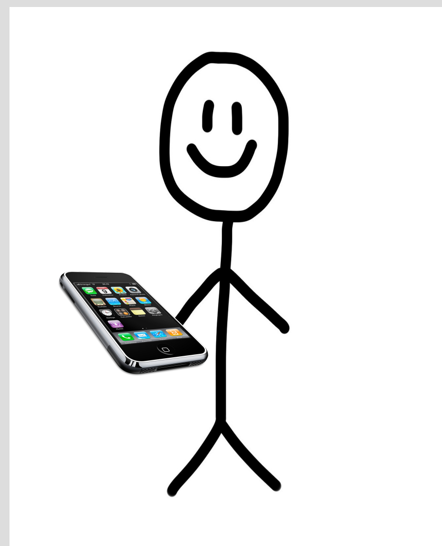


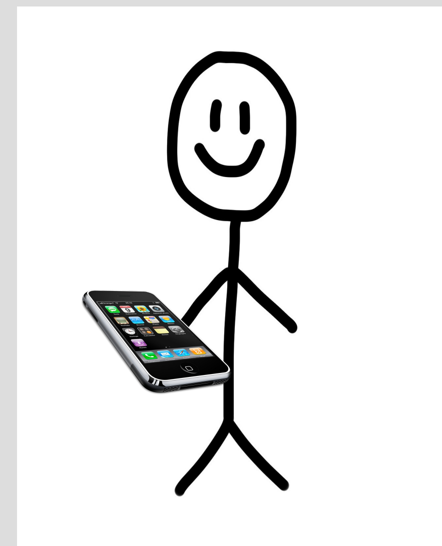
put in turkey

Parallel processing (cooking)

If you make turkey....





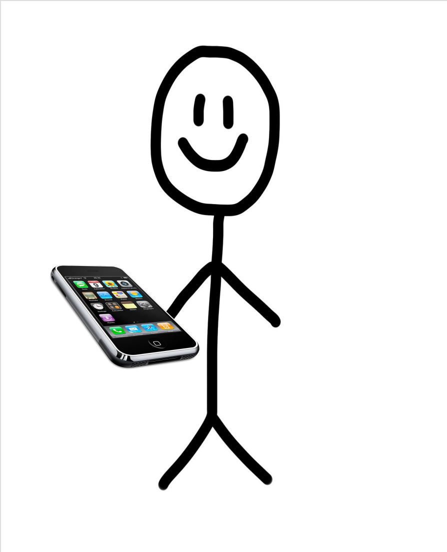
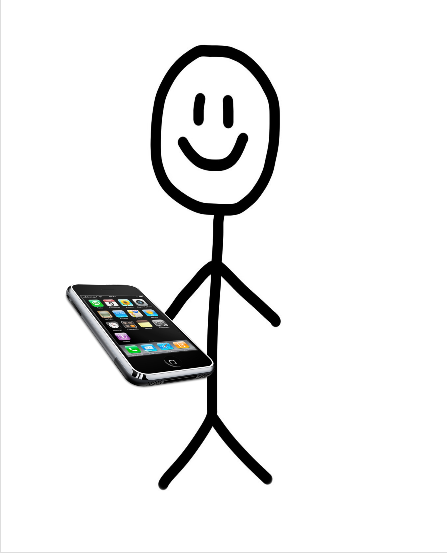


put in turkey

WAIT A LOT

Parallel processing (cooking)

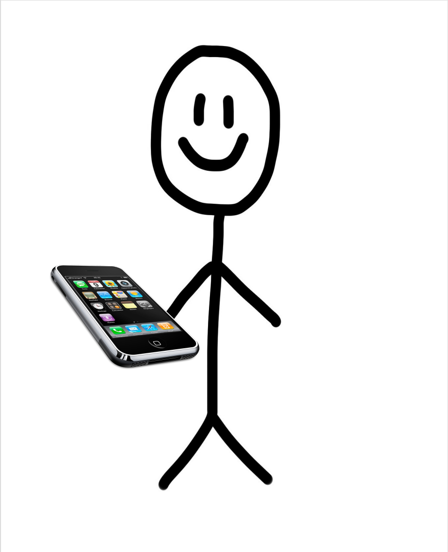
If you make turkey....

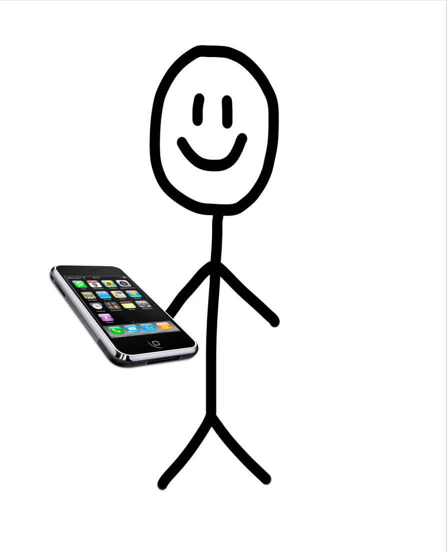




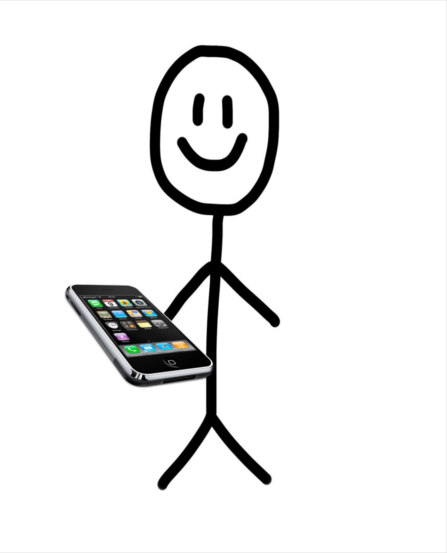
take
out


Parallel processing (cooking)

If you make turkey....







take
out

WAIT

Parallel processing (cooking)

If you make a salad...

chop

grate

cut

Parallel processing (cooking)

If you make a salad...



chop

grate

cut



Parallel processing (cooking)

If you make a salad...



dump together

Parallel processing (cooking)

To make use of last 15 years of technology, need to have algorithms like salad

Multiple cooks need to work at the same time to create the end result

Computers these days have 4-8 “cooks” in them, so try not to make turkey

Correctness

An algorithm is <u>correct</u> if it takes an <u>input</u> and always halts with the correct <u>output</u>.

Many hard problems there is no known correct algorithm and instead approximate algorithms are used

Asymptotic growth

What does $O(n^2)$ mean?

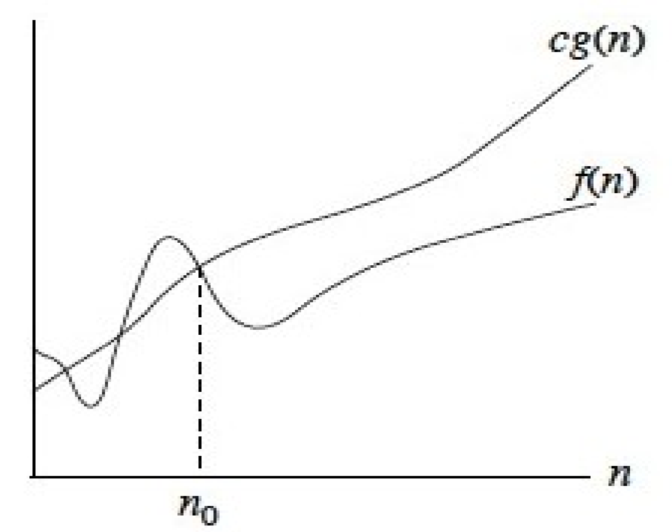
$\Theta(n^2)$?

$\Omega(n^2)$?

Asymptotic growth

If our algorithm runs in f(n) time, then our algorithm is O(g(n)) means there is an n_0 and c such that

$0 \le f(n) \le c\, g(n)$ for all $n \ge n_0$



O(g(n)) can be used for more than run time

Asymptotic growth

$f(n)=O(g(n))$ means that for large inputs (n), g(n) will not grow slower than f(n)

$n = O(n^2)$?

$n = O(n)$?

$n^2 = O(n)$?

Asymptotic growth

$f(n)=O(g(n))$ gives an upper bound for the growth of $f(n)$

$f(n)=\Omega(g(n))$ gives a lower bound for the growth of $f(n)$, namely:
there is an n_0 and c such that
$0 \leq c\, g(n) \leq f(n)$ for all $n \geq n_0$

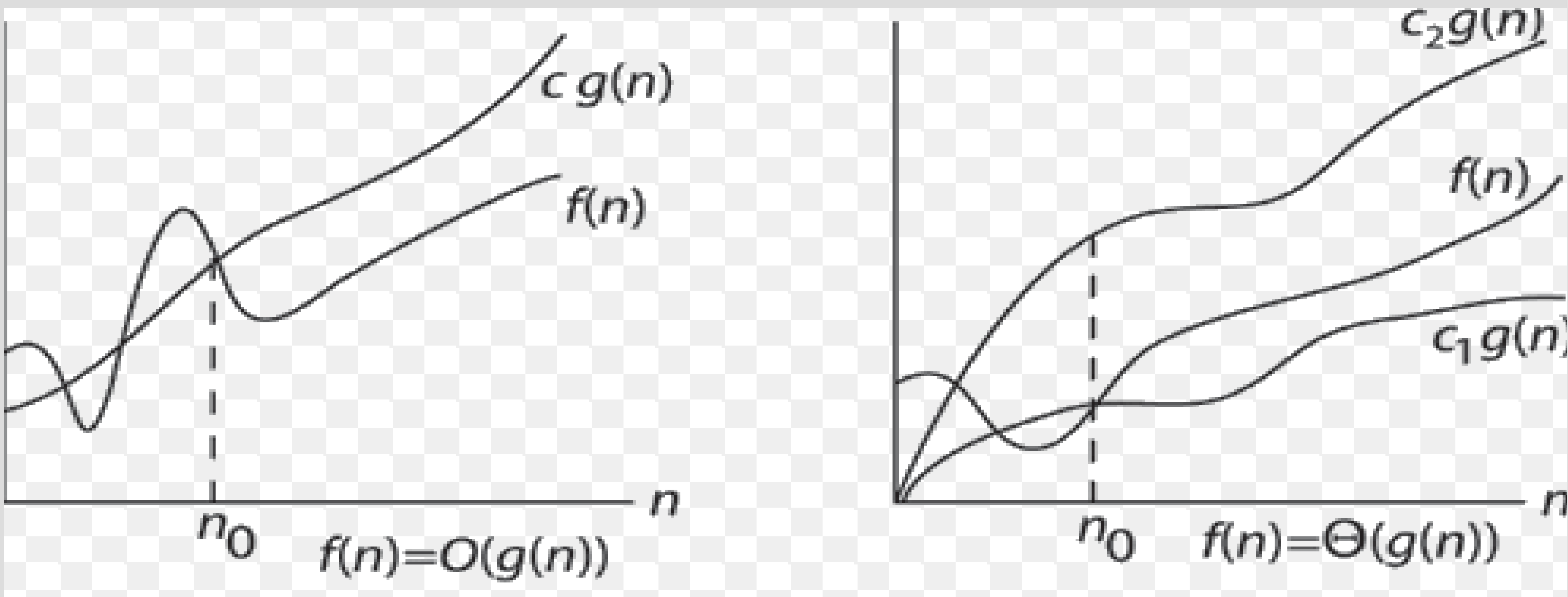
Asymptotic growth

f(n)=Θ(g(n)) is defined as:

there is an n_0, c_1 and c_2 such that

$0 \leq c_1\ g(n) \leq f(n) \leq c_2\ g(n)$ for all

$n \geq n_0$



Asymptotic growth

Suppose $f(n) = 2n^2 – 5n + 7$

Show $f(n) = O(n^2)$:

we need to find 'c' and 'n_0' so that

$c\ n^2 > 2n^2 – 5n + 7$, guess c=3

$3\ n^2 > 2n^2 – 5n + 7$

$n^2 > -\ 5n + 7$

$n > 2$, so $c=3$ and $n_0=2$ proves this

Asymptotic growth

Suppose $f(n) = 2n^2 – 5n + 7$
Show $f(n) = \Omega(n^2)$:

For any general $f(n)$ show:
$f(n)=\Theta(g(n))$ if and only if
$f(n)=O(g(n))$ and $f(n)=\Omega(g(n))$

Asymptotic growth

Suppose $f(n) = 2n^2 – 5n + 7$

Show $f(n) = \Omega(n^2)$:

again we find a 'c' and 'n_0'

$cn^2 < 2n^2 – 5n + 7$, guess c=1

$1\ n^2 < 2n^2 – 5n + 7$

$0 < n^2 -5n +7$, or $n^2 > 5n -7$

$n > 4$, so $c=1$ and $n_0=4$ proves this

Asymptotic growth

$f(n)=\Theta(g(n))$ implies

$f(n)=O(g(n))$ and $f(n)=\Omega(g(n))$:

by definition we have 'c_1', 'c_2', 'n_0' so

$0 \leq c_1\ g(n) \leq f(n) \leq c_2\ g(n)$ after n_0

$0 \leq c_1\ g(n) \leq f(n)$ after n_0 is $\Omega(g(n))$

$0 \leq f(n) \leq c_2\ g(n)$ after n_0 is $O(g(n))$

Asymptotic growth

$f(n)=O(g(n))$ and $f(n)=\Omega(g(n))$ implies $f(n)=\Theta(g(n))$:

by definition we have c_1, c_2, n_0, n_1

$\Omega(g(n))$ is $0 \leq c_1\ g(n) \leq f(n)$ after n_0

$O(g(n))$ is $0 \leq f(n) \leq c_2\ g(n)$ after n_1

$0 \leq c_1\ g(n) \leq f(n) \leq c_2\ g(n)$ after $\max(n_0,n_1)$

Asymptotic growth

There are also o(g(n)) and w(g(n)) but are rarely used

f(n)=o(g(n)) means for any c there is an n_0: $0 \leq f(n) < c\, g(n)$ after n_0

$\lim(n \to \infty) \quad f(n)/g(n) = 0$

w(g(n)) is the opposite of o(g(n))

Asymptotic growth

Big-O notation is used very frequently to describe run time of algorithms

It is fairly common to use big-O to bound the worst case and provide empirical evaluation of runtime with data

Asymptotic growth

What is the running time of the following algorithms for n people:

1. Does anyone share my birthday?
2. Does any two people share a birthday?
3. Does any two people share a birthday (but I can only remember and ask one date at a time)?

Asymptotic growth

1. O(n) or just n
2. O(n) or just n for small n (https://en.wikipedia.org/wiki/Birthday_problem)

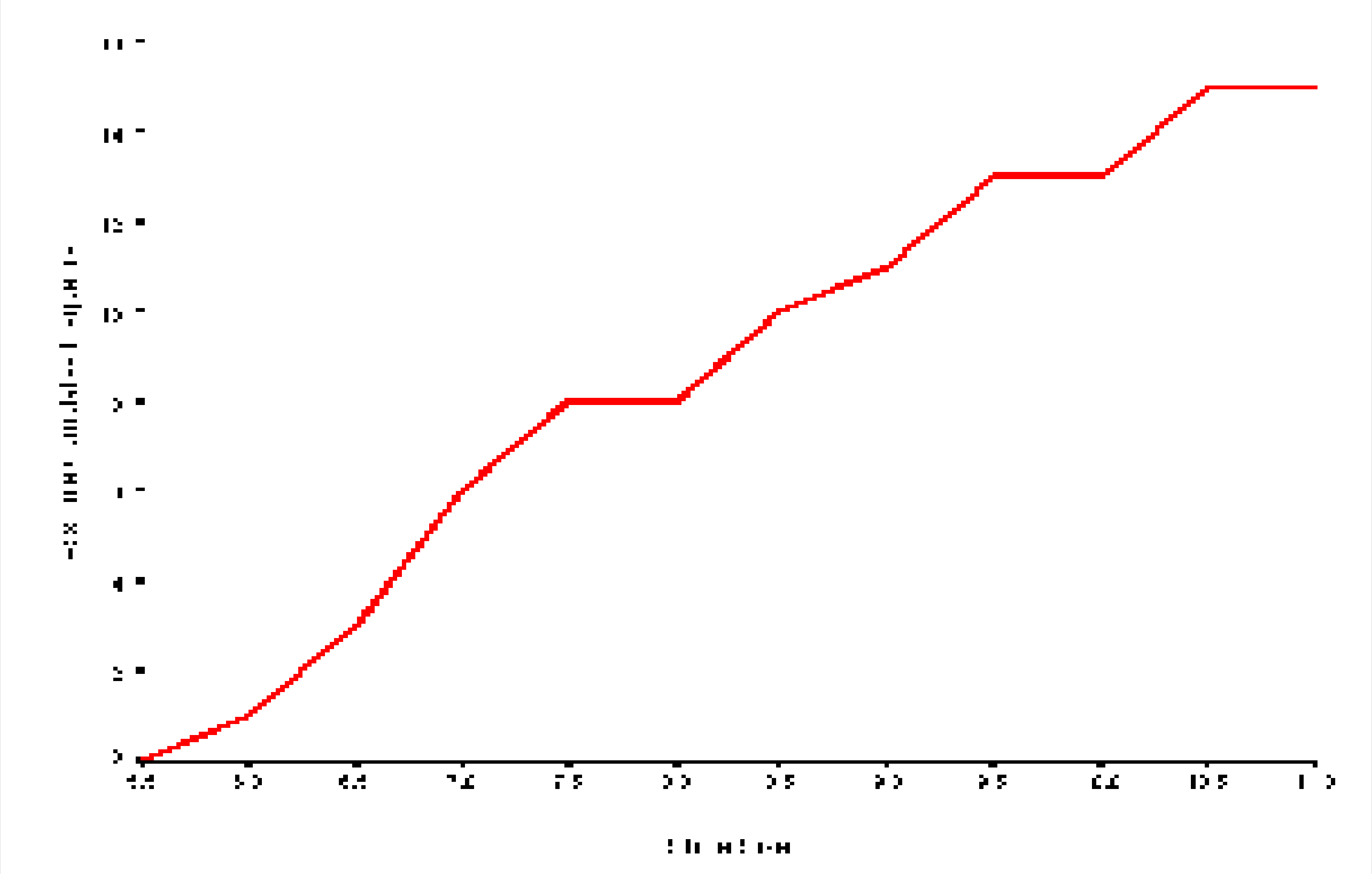
Worst case: 365 (technically 366)

Average run time: 24.61659

3. $O(n^2)$ or n^2

Math review

Monotonically increasing means:
for all m $\leq$ n implies f(m) $\leq$ f(n)



Math review

Monotonically decreasing means:
for all $m \leq n$ implies $f(m) \geq f(n)$

Strictly increasing means:
for all $m < n$ implies $f(m) < f(n)$

In proving it might be useful to use monotonicity of $f(n)$ or $d/dn\, f(n)$

Math review

floor/ceiling?
modulus?
exponential rules and definition?
logs?
factorials?

Floors and ceilings

floor is “round down”

floor(8/3) = 2

ceiling is “round up”

ceiling(8/3) = 3

(both are monotonically increasing)

Prove: floor(n/2) + ceiling(n/2) = n

Floors and ceilings

Prove: floor(n/2) + ceiling(n/2) = n

Case: n is even, n = 2k

floor(2k/2) + ceiling(2k/2) = 2k

k + k = 2k

Case: n is odd, n = 2k+1

floor((2k+1)/2) + ceiling((2k+1)/2)

floor(k+1/2) + ceiling(k+1/2)

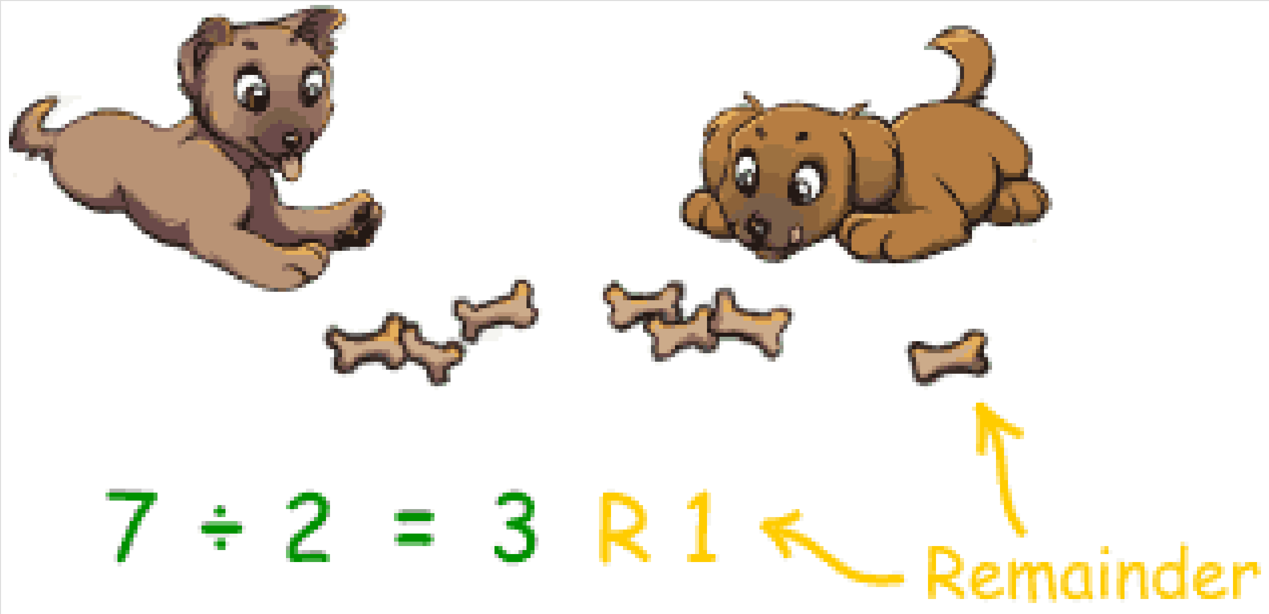
k + k+1 = 2k + 1

Modulus

Modulus is the remainder of the quotient a/n:

a mod n = a – n floor(a/n)

7 % 2 = 1



Factorial

$n! = 1 \times 2 \times 3 \times \ldots \times n$

$4! = 4 \times 3 \times 2 \times 1 = 24$

Guess the order (low to high):

1,000 1,000,000 1,000,000,000

2^5 2^{10} 2^{20} 2^{25} 2^{30}

5! 10! 15! 20!

Factorial

The order is (low to high):
$\{2^5$, 5!, (1,000), 2^{10}, 2^{15},
(1,000,000), 2^{20}, 10!,
(1,000,000,000), 15!, 20!$\}$
10! = 3,628,800
15! ≈ 1,307,674,400,000
20! ≈ 2,432,902,000,000,000,000
($2^{10} = 1024 \approx 1{,}000 = 10^3$)

Factorial

Find $g(n)$ such that $(g(n) \neq n!)$:

1. $n! = \Omega(g(n))$

2. $n! = O(g(n))$

Factorial

1. $n! = \Omega(g(n))$
   - $n! = \Omega(1)$ is a poor answer
   - $n! = \Omega(2^n)$ is decent

2. $n! = O(g(n))$
   - $n! = O(n^n)$

Exponentials

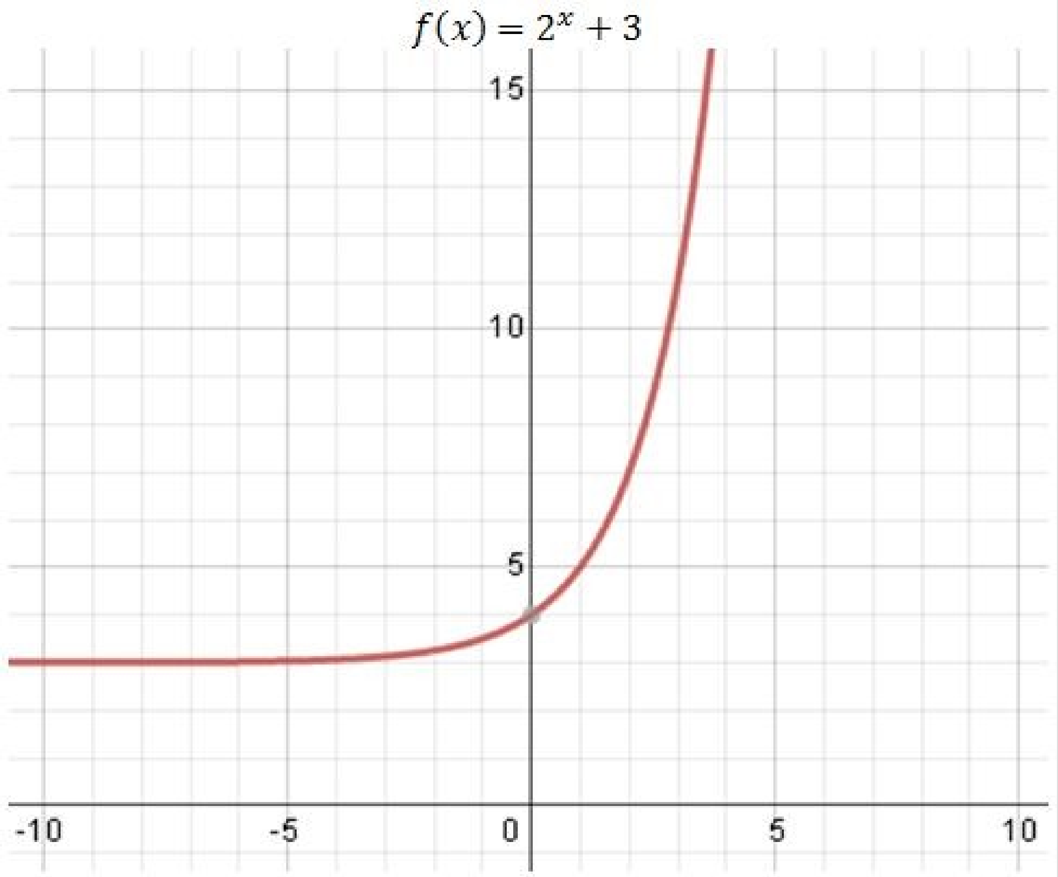
$(a^n)^m = a^{nm}$: $(2^3)^4 = 8^4 = 4096 = 2^{12}$

$a^n a^m = a^{n+m}$: $2^3 2^4 = 8x16 = 128 = 2^7$

$a^0 = 1$

$a^1 = a$

$a^{-1} = 1/a$



Exponentials

for all constants: $a>1$ and b:

$\lim(n \to \infty)\ n^b / a^n = 0$

What does this mean in big-O notation?

Exponentials

What does this mean in big-O notation?

$n^b = O(a^n)$ for any $a>1$ and b

i.e. the exponential of anything eventually grows faster than any polynomials

Exponentials

Sometimes useful facts:

$e^x = \text{sum}(i=0 \text{ to } \infty)\ x^i / i!$

$e^x = \lim(n \to \infty)\ (1 + x/n)^n$

Recurrence relationships

Write the first 5 numbers, can you find a pattern:

1. $F_i = F_i + 2$ with $f_0 = 0$
2. $F_i = 2F_i$ with $f_0 = 3$
3. $F_i = F_{i-1} + F_{i-2}$, with $f_0=0$ and $f_1=1$

Recurrence relationships

1. $F_i = F_i + 2$ with $f_0 = 0$
   - $F_0=0$, $F_1=2$, $F_2=4$, $F_3=6$, $F_4=8$
   - $F_i = 2i$
2. $F_i = 2F_i$ with $f_0 = 3$
   - $F_0=3$, $F_1=6$, $F_2=12$, $F_3=24$, $F_4=48$
   - $F_i = 3 \times 2^i$

Recurrence relationships

3. $F_i = F_{i-1} + F_{i-2}$, with $f_0=0$ and $f_1=1$
   - $F_0=0$, $F_1=1$, $F_2=1$, $F_3=2$, $F_4=3$
   - $F_0=5$, $F_1=8$, $F_2=13$, $F_3=21$, $F_4=34$

   - Fi = $[(1+\mathrm{sqrt}(5))^i – (1-\mathrm{sqrt}(5))^i]/(2^i\mathrm{sqrt}(5))$

Recurrence relationships

3. $F_i = F_{i-1} + F_{i-2}$ is homogeneous

We as $F_i = cF_{i-1}$ is exponential,

we guess a solution of the form:

$F^i = F^{i-1} + F^{i-2}$, divide by F^{i-2}

$F^2 = F + 1$, solve for F

$F = (1 \pm \mathrm{sqrt}(5))/2$, so have the form

$a[(1 + \mathrm{sqrt}(5))/2]^i+b[(1 - \mathrm{sqrt}(5))/2]^i$

Recurrence relationships

$a[(1 + \mathrm{sqrt}(5))/2]^i + b[(1 – \mathrm{sqrt}(5))/2]^i$

with $F_0=0$ and $F_1=1$

2x2 System of eqations → solve

i=0: a[1] + b[1] = 0 → a = -b

i=1: a[1+sqrt(5)/2] – a[1-sqrt(5)/2]

a[sqrt(5)] = 1

a = 1/sqrt(5) = -b

Recurrence relationships

$F_i = 2F_{i-1} - F_{i-2}$, change to exponent

$F^i = 2F^{i-1} - F^{i-2}$, divide by F^{i-2}

$F^2 = 2F - 1 \rightarrow (F-1)(F-1) = 0$

This will have solution of the form:

$1^i + i \times 1^i$

Next week sorting

- Insert sort
- Merge sort
- Bucket sort