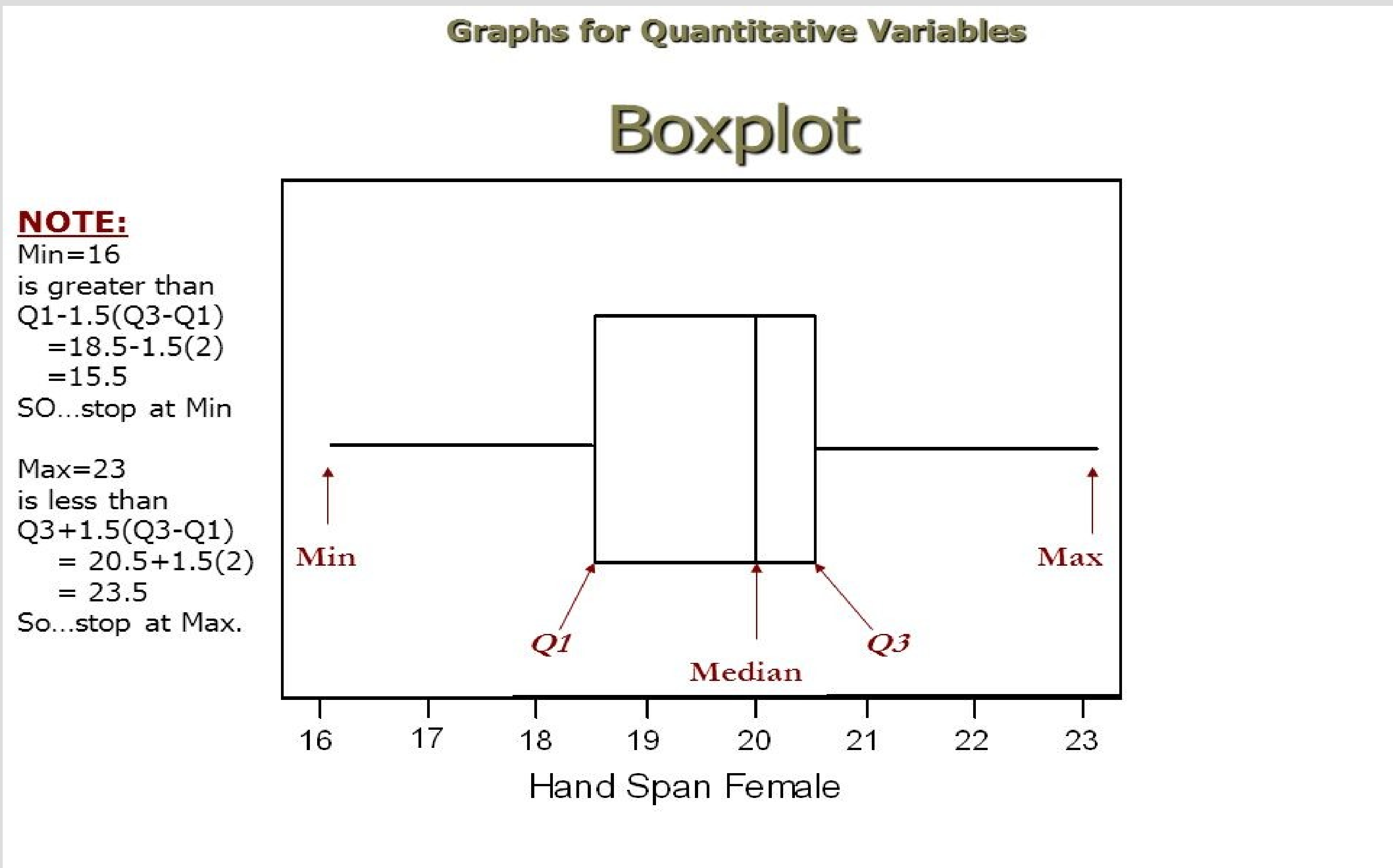
Selection


Graphs for Quantitative Variables
Boxplot
NOTE:
Min=16
is greater than
Q1-1.5(Q3-Q1)
=18.5-1.5(2)
=15.5
SO...stop at Min
Max=23
is less than
Q3+1.5(Q3-Q1)
= 20.5+1.5(2)
= 23.5
So...stop at Max.
Min
Max
Q1
Median
Q3
16
17
18
19
20
21
22
23
Hand Span Female

Selection

Selection given a set of (distinct) elements, finding the element larger than $i - 1$ other elements

Selection with...

$i=n$ is finding maximum

$i=1$ is finding minimum

$i=n/2$ is finding median

Maximum

Selection for any i is O(n) runtime

Find max in O(n)?

Maximum

Selection for any i is O(n) runtime

Find max in O(n)?

```
max = A[ 1 ]
for i = 2 to A.length
   if ( A[ i ] > max )
      max = A[ i ]
```

Max and min

It takes about n comparisons to find max

How many would it take to find both max and min at same time?

Max and min

It takes about n comparisons to find max

How many would it take to find both max and min at same time?

Naïve = 2n
Smarter = 3/2 n

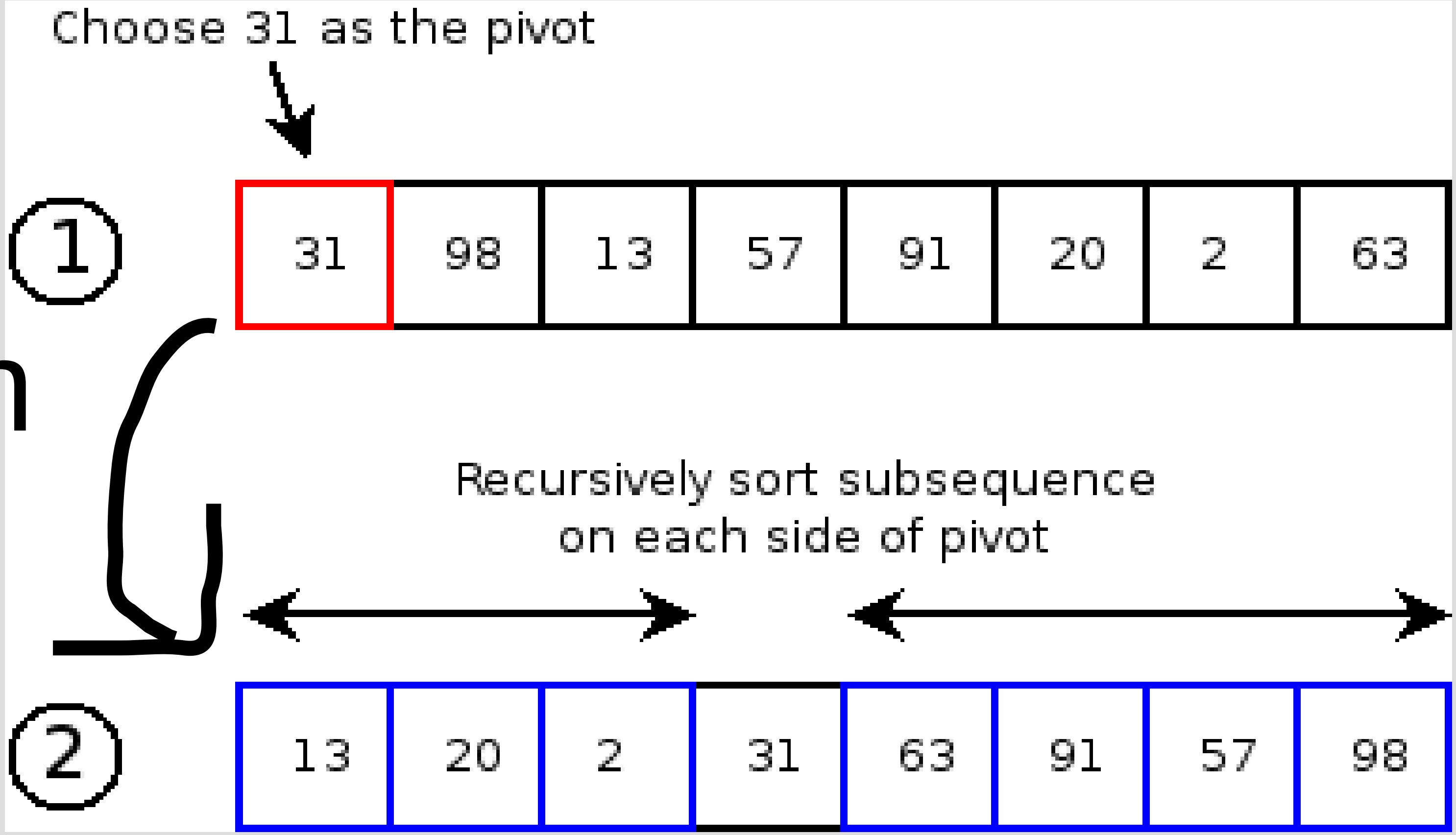
Max and min

```
smin = min(A[ 1 ], A[ 2 ])
smax = max(A[ 1 ], A[ 2 ])
for i = 3 to A.length step 2
    if (A[ i ] > A[ i+1 ])
        smax = max(A[ i ], smax)
        smin = min(A[ i+1], smin)
    else
        smax = max(A[ i+1], smax)
        smin = min(A[ i ], smin)
```

Randomized selection

Remember quicksort?

Partition step



Randomized selection

To select i:

1. Partition on random element
2. If partitioned element i, end otherwise recursively partition on side with i

Randomized selection

{2, 6, 4, 7, 8, 4, 7, 2} find i = 5

Randomized selection

{2, 6, 4, 7, 8, 4, 7, 2} find i = 5

Pick pivot = 4

{2, 6, 4, 7, 8, 2, 7, 4}

{2, 6, 4, 7, 8, 2, 7, 4}

{2, 6, 4, 7, 8, 2, 7, 4}

{2, 4, 6, 7, 8, 2, 7, 4}

{2, 4, 6, 7, 8, 2, 7, 4}

{2, 4, 6, 7, 8, 2, 7, 4}

Randomized selection

{2, 4, 6, 7, 8, 2, 7, 4}

{2, 4, 2, 7, 8, 6, 7, 4}

{2, 4, 2, 7, 8, 6, 7, 4}

{2, 4, 2, 4, 7, 8, 6, 7}

1, 2, 3, 4, 5, 6, 7, 8

i=5 on green side, recurse

Randomized selection

{7, 8, 6, 7} pick pivot = 6

{7, 8, 7, 6}

{7, 8, 7, 6}

{7, 8, 7, 6}

{7, 8, 7, 6}

{6, 7, 8, 7}

5, 6, 7, 8

found i=5, value = 6

Randomized selection

Quicksort runs in O(n lg n), but we only have sort one side and sometimes stop early

This gives randomized selection O(n) running time (proof in book, I punt)

Randomized selection

Just like quicksort, the worst case running time is $O(n^2)$

This happens when you want to find the min, but always partition on the max

Select

A worst case O(n) selection is given by Select: (see code)

1. Make n/5 groups of 5 and find their medians (via sorting)
2. Recursively find the median of the n/5 medians (using Select)
3. Partition on median of medians
4. Recursively Select correct side

Select

Proof of the general case:

$$T(n) = \sum_i T(k_i \cdot n + q_i) + O(n)$$

// assume T(n) is O(n)

$$T(n) = c \cdot n - c \cdot n + c \sum_i (k_i \cdot n + q_i) + a \cdot n$$

If T(n) is O(n) then...

$$-c \cdot n + c \sum_i (k_i \cdot n + q_i) + a \cdot n \leq 0$$

$$a \cdot n \leq c(n(1 - \sum_i k_i) - \sum_i q_i)$$

Select

$$a \cdot n \leq c(n(1 - \sum_i k_i) - \sum_i q_i)$$

$$\frac{a \cdot n}{n(1 - \sum_i k_i) - \sum_i q_i} \leq c$$

// Pick n > 2(sum$_i$ q$_i$/(1 – sum$_i$ k$_i$))

$$\frac{2a(\sum q_i/(1 - \sum_i k_i)}{\sum_i q_i} \leq c$$

$$\frac{2a}{1 - \sum_i k_i} \leq c$$

Done as sum$_i$ k$_i$ < 1 (just need show for this n, O(1)

Select

Select runs in:

$T(n) = T(\text{ceiling}(n/5))$
$\quad + T(7n/10 + 6) + O(n)$

By the previous proof this is O(n):

$\text{ceiling}(n/5) + 7n/10 + 6$

$\leq n/5 + 1 + 7n/10 + 6 = 9n/10 + 7$

$\text{sum}_i\ k_i = 9/10 < 1$, done

Select

Does this work for making:

(1) n/3 groups of 3?

(2) n/7 groups of 7?

(3) n/9 groups of 9?