String matching

```
>>>
>>> import re
>>> r=re.compile(r"regexes\s(are|do)\s?(n[o']t)?\s(fun|boring)", re.I)
>>> def is_match(x): return x is not None
>>> is_match(re.match(r, "Regexes are fun!!!"))
True
>>> is_match(re.match(r, "Regexes are not fun!!!"))
True
>>> is_match(re.match(r, "Regexes aren't boring!!!"))
True
>>> is_match(re.match(r, "Obviously, regexes are boring."))
False
>>> is_match(re.search(r, "Obviously, regexes are boring."))
True
```

Announcements

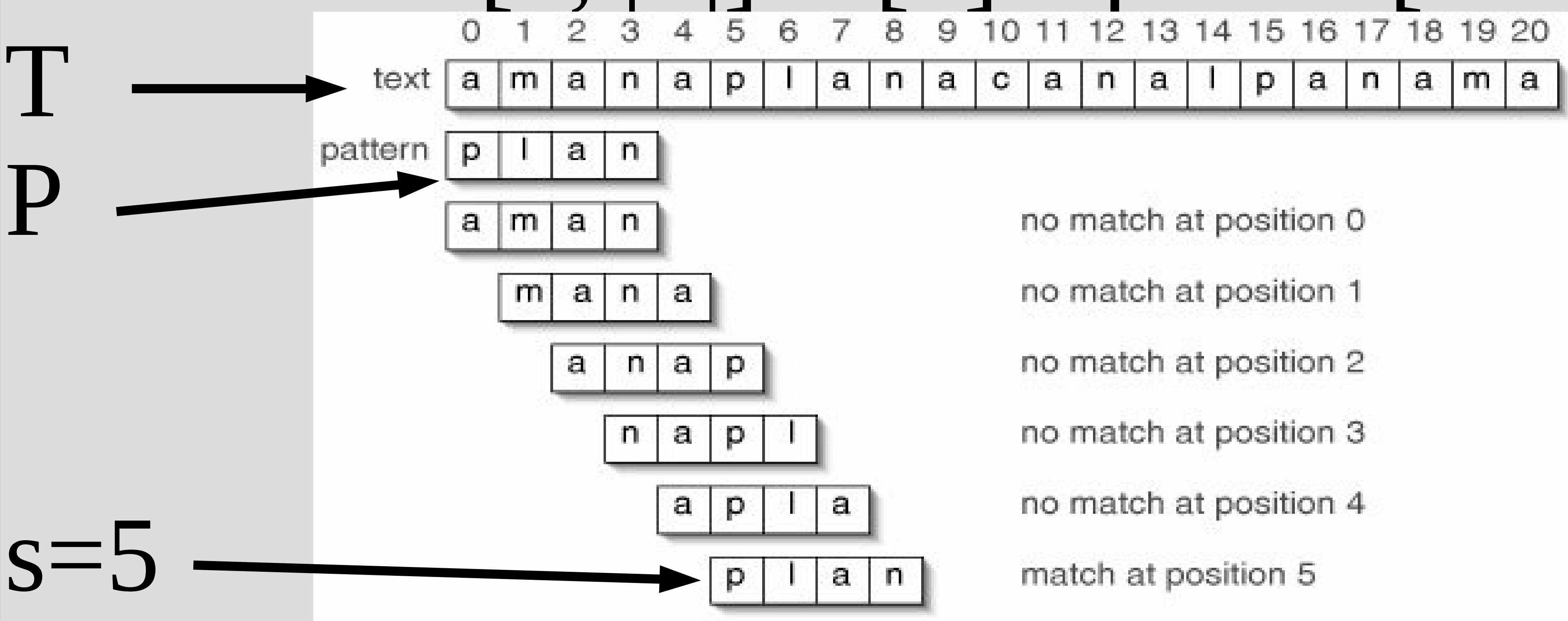
Programming assignment 1 posted
- need to submit a .sh file

The .sh file should just contain what you need to type to compile and run your program from the terminal

String matching

Some pattern/string P occurs with shift s in text/string T if:

for all k in [1, |P|]: P[k] equals T[s+k]



String matching

Both the pattern, P, and text, T, come from the same finite alphabet, ∑.

empty string (“”) = ε

$\sqsubset$

w is a prefix of x=w [x, means exists y s.t. wy = x (also implies |w| ≤ |x|)

(w] x = w is a suffix of x)

Prefix

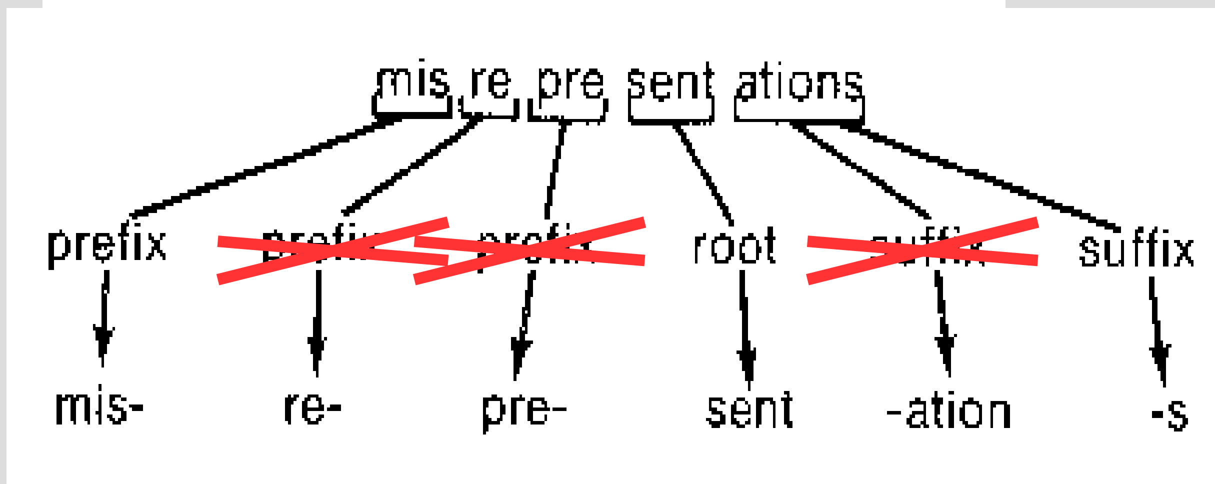
w prefix of x means: all the first letters of x are w

x → “bread”

prefixes of x → b , br , bre , brea

suffixes of x → read , ead , ad , d

not english!



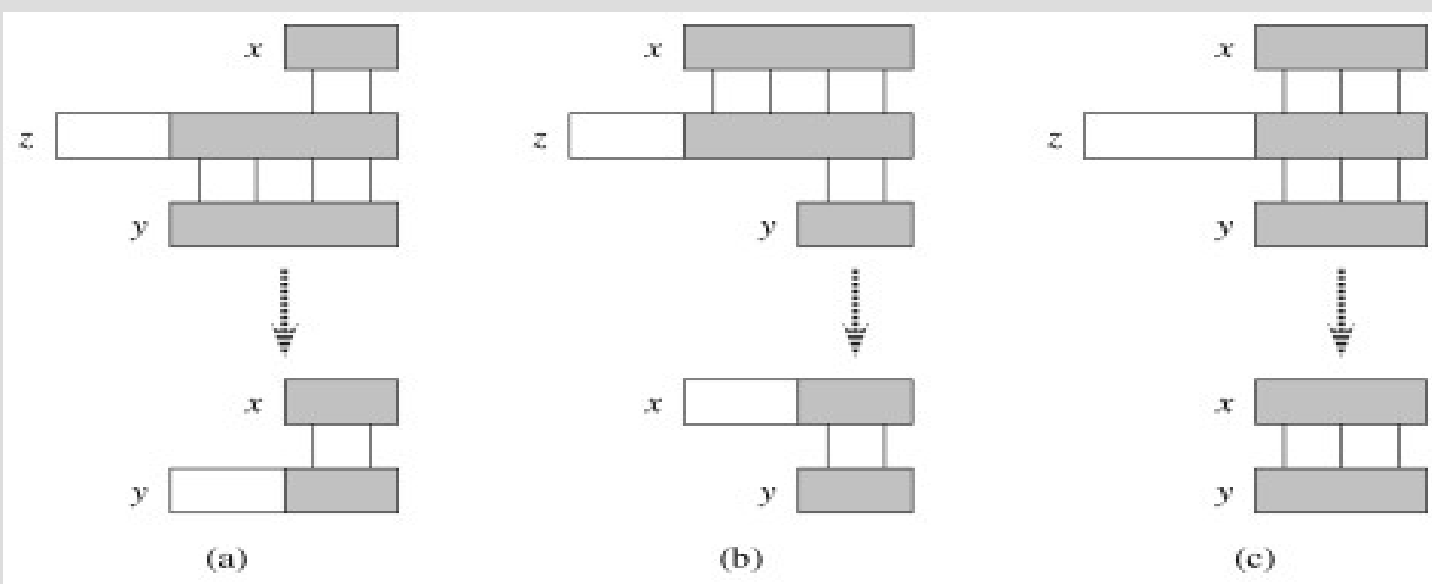
Suffix

If x] z and y] z, then:

(a) If |x| ≤ |y|, x] y

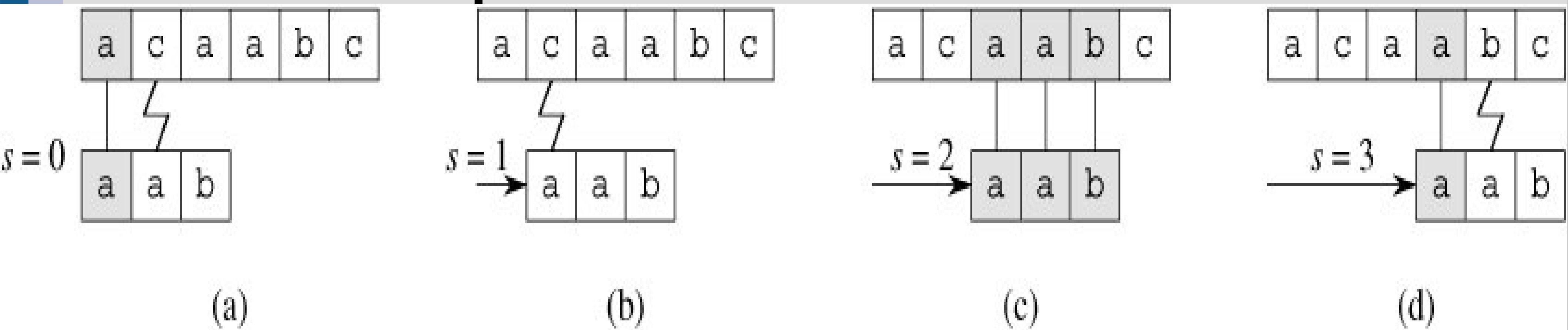
(b) If |y| ≤ |x|, y] x

(c) If |x| = |y|, x = y



Dumb matching

Dumb way to find all shifts of P in T?
Check all possible shifts!

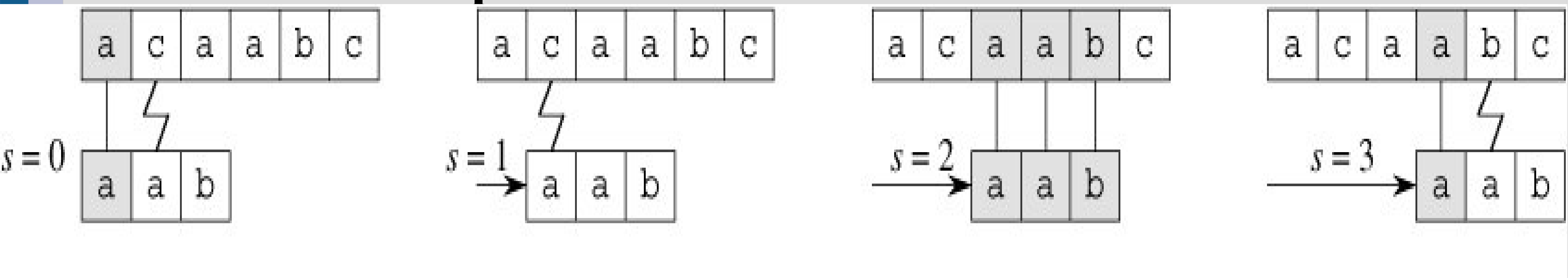


(see: naiveStringMatcher.py)
Run time?

Dumb matching

Dumb way to find all shifts of P in T?

Check all possible shifts!



(a) (b) (c) (d)

(see: naiveStringMatcher.py)

Run time?

O(|P| |T|)

Rabin-Karp algorithm

A better way is to treat the pattern as a single numeric number, instead of a sequence of letters

So if P = {1, 2, 6} treat it as 126 and check for that value in T

Rabin-Karp algorithm

The benefit is that it takes a(n almost) constant time to get the each number in T by the following:

(Let t_s = T[s, s+1, ..., s+|P|])

$t_{s+1} = d(t_s – T[s+1]h) + T[s+|P|+1]$

where $d = | \sum |$, $h = d^{|P|-1}$

Rabin-Karp algorithm

Example: $\Sigma = \{0, 1, ..., 9\}, |\Sigma| = 10$

$T = \{1, 2, 6, 4, 7, 2\}$

$P = \{6, 4, 7\}$

$t_0 = 126$

$t_1 = 10(126\text{-}T[0+1]10^{3-1}) + T[0+|P|+1]$

$t_1 = 10(126-100) + T[0+3+1]$

$t_1 = 264$

Rabin-Karp algorithm

This is a constant amount of work if the numbers are small...

So we make them small!
(using modulus/remainder)

Any problems?

Rabin-Karp algorithm

This is a constant amount of work
if the numbers are small...

So we make them small!
(using modulus/remainder)

Any problems?
x mod q=y mod q does not mean x=y

Hash functions

One way functions

Modulus is a <u>one way function</u>, thus computing the modulus is easy but recovering the original number is hard/impossible

127 % 5 = 2, or 127 mod 5 = 2 mod 5
However if we want to solve x%5=2, all we can say is x=2+5k or some k

One way functions

Other one way functions?

One way functions

Other one way functions?
- multiplication
- hashing

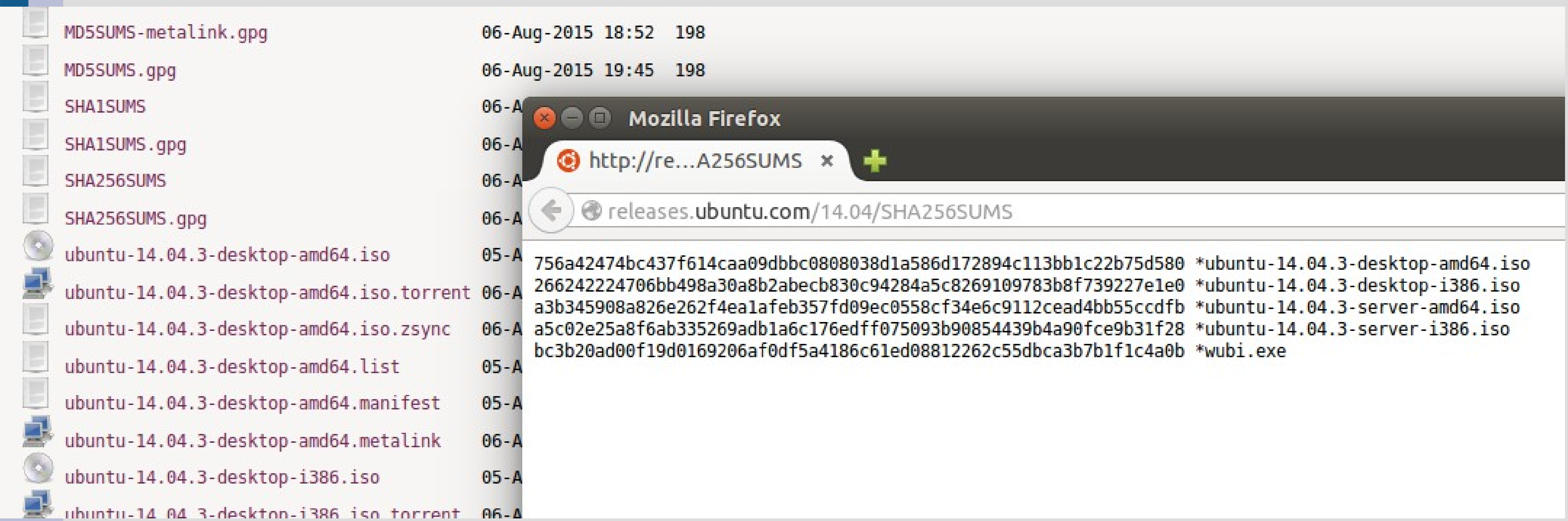
Multiplication is famous, as it is easy:
200*50 = 10,000
... yet factoring is hard:
132773= 31 * 4283 (what alg?)

One way functions

Hashing is another commonly used function for security/verification, as...

- -fast (low computation)
- -low collision chance
- -cannot easily produce a specific hash

One way functions



Hash functions

Rabin-Karp algorithm

Larger q (for mod):
- larger numbers = more computation
- less frequent errors

There are trade-offs, but we often pick $q > |P|$ but not $q >> |P|$

Pick a prime number as q

Rabin-Karp algorithm

Kabin-Karp-Matcher(T,P,|∑|,q,)

d=|∑|, h=d$^{|P|-1}$ mod q, p=0, t_0 = 0

for i=1 to |P| // "preprocessing"

p = (dp + P[i]) mod q // for P

t_0 = (dt_0 + T[i]) mod q // for T

for s = 0 to |T| - |P|

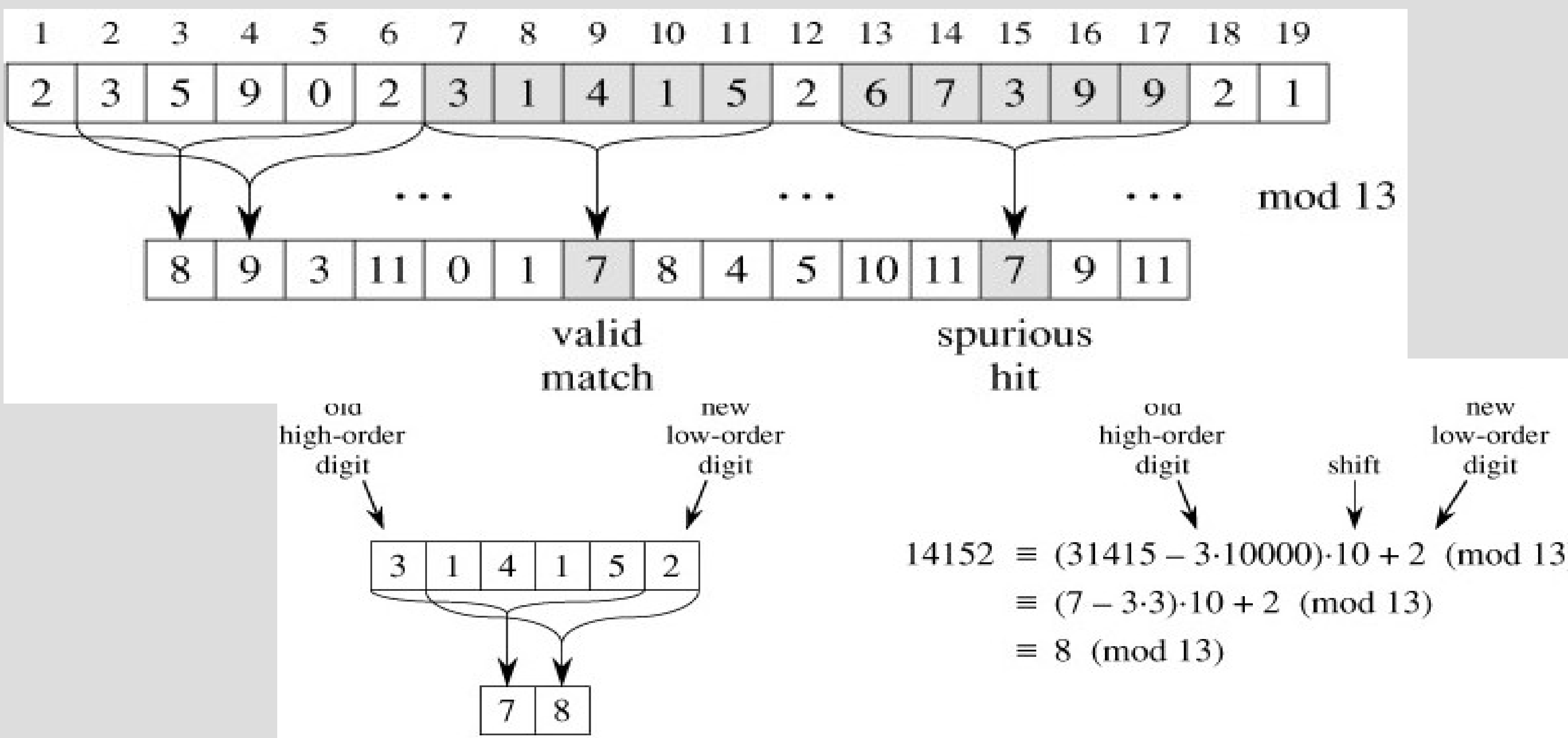
if p == t_s, check brute-force match at s

if s < |T| - |P| then compute t_{s+1}

Rabin-Karp algorithm

To compute t_{s+1}:

$t_{s+1}=(d(t_s\text{-}t[s+1]h)+T[s+|P|+1]) \bmod q$



Rabin-Karp algorithm

Example: T = {1, 2, 5, 3, 5, 2, 6, 3}

P = {2, 5}, q = 5, assume base 10

Rabin-Karp algorithm

Example: T = {1, 2, 5, 3, 5, 2, 6, 3}

P = {2, 5}, q = 5, assume base 10

<u>P = 25 mod 5 = 0</u>, t_0 = 12 mod 5 = 2

t_{i+1}=10*(t_i-T[i+1]*10)+T[i+|P|+1]%q

t_1 = 25 mod 5 = 0, true match!

t_2 = 53 mod 5 = 3,

t_3 = 35 mod 5 = 0, false match

Rabin-Karp algorithm

$T = \{1, 2, 5, 3, 5, 2, 6, 3\}$, $P = \{2, 5\}$

$t_5 = 52 \bmod 5 = 2$,

$t_6 = 26 \bmod 5 = 1$,

$t_7 = 63 \bmod 5 = 3$

$t_{i+1}=10*(t_i\text{-}T[i+1]*10)+T[i+|P|+1]\%q$

So only s=1 is match

Rabin-Karp algorithm

Run time? (Average? Worst case?)

Rabin-Karp algorithm

Run time?

- “preprocessing” (first loop)= O(|P|)
- “matching” (second loop) = O(|T|)

So O(|T|+|P|) and as n>m, O(|T|) on average

Worst case: always a match O(|T| |P|)