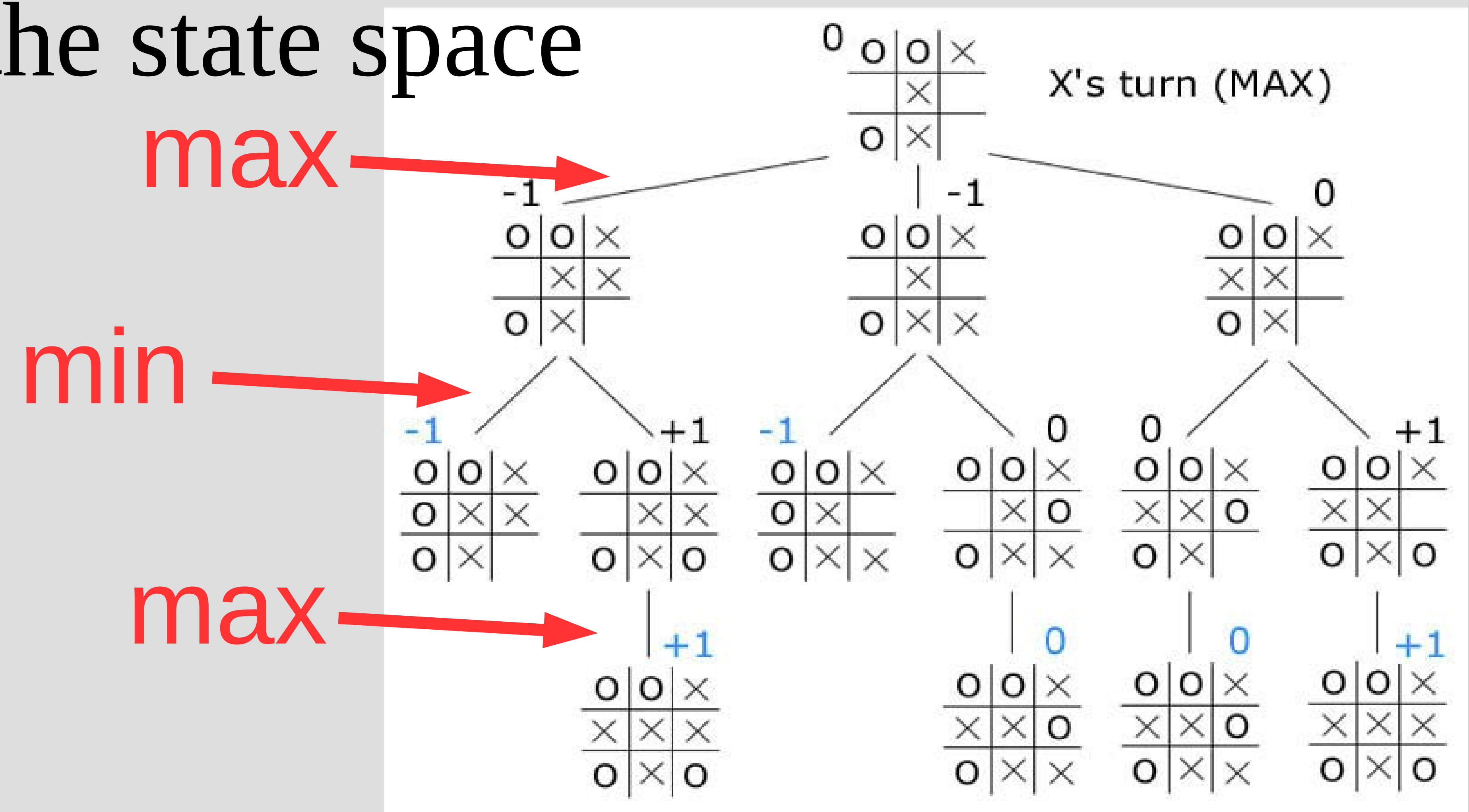
Unweighted directed graphs

BFS and DFS in trees

Solve problems by making a tree of the state space



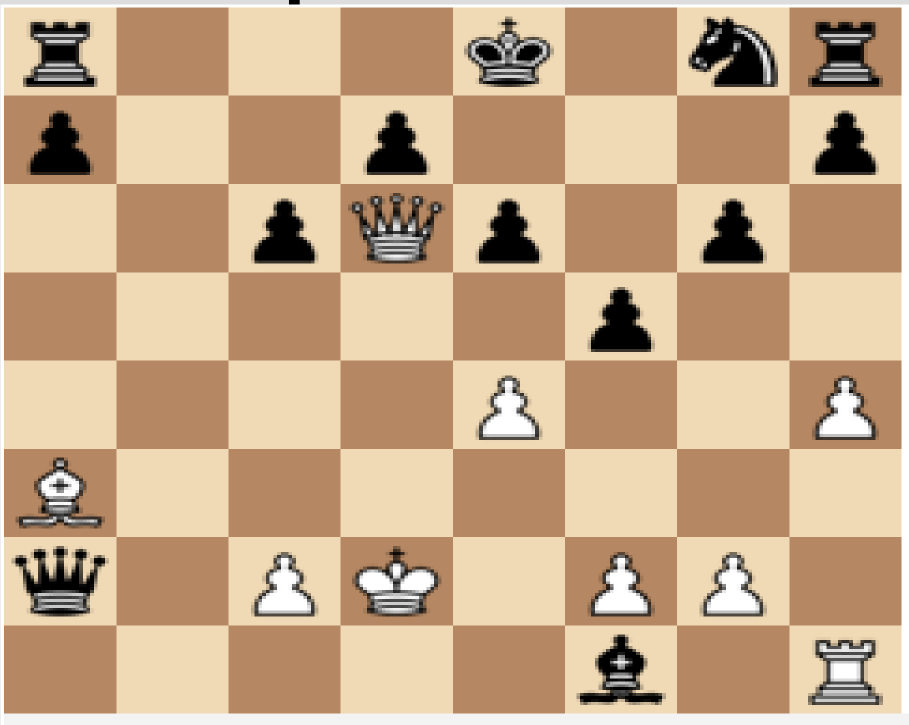
BFS and DFS in trees

Often times, fully exploring the state space is too costly (takes forever)

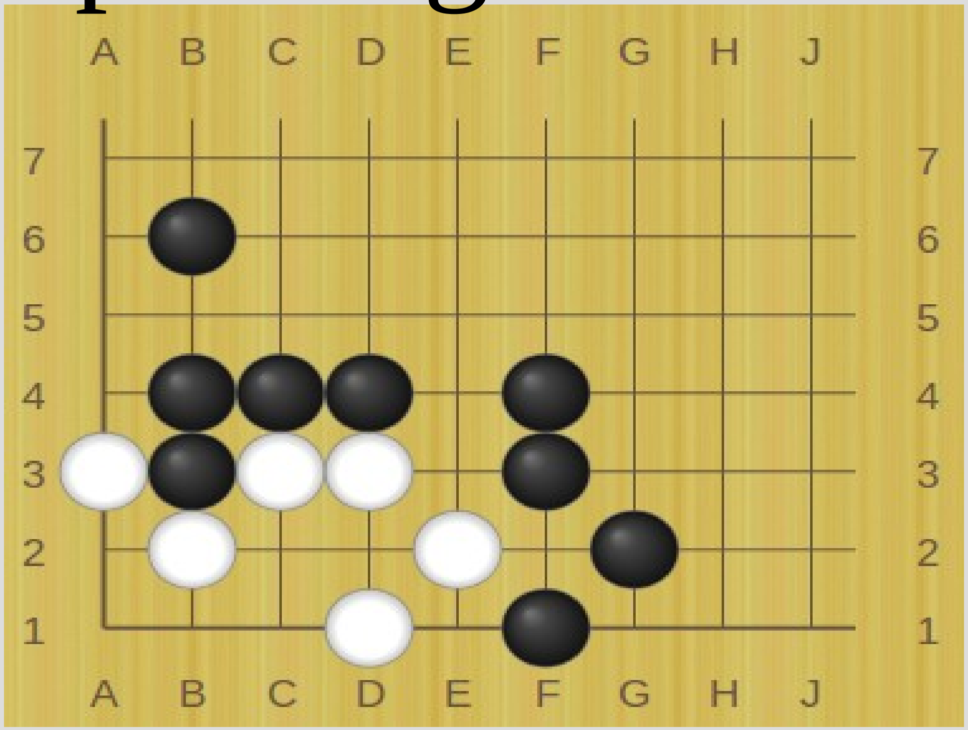
Chess: 10^{47} states (tree about 10^{123})
Go: 10^{171} states (tree about 10^{360})
At 1 million states per second...
Chess: 10^{109} years (past heat death
Go: 10^{346} years   of universe)

BFS and DFS in trees

BFS prioritizes “exploring”
DFS prioritizes “exploiting”



White to move



Black to move

BFS and DFS in trees

BFS benefits?

DFS benefits?

BFS and DFS in trees

BFS benefits?

-can evaluate best path

DFS benefits?

-uses less memory on complete search

BFS and DFS in graphs

BFS: shortest path from origin to any node

DFS: find graph structure

Both running time of $O(V+E)$

Breadth first search

```
BFS(G,s) // to find shortest path from s
for all v in V
    v.color=white, v.d=∞,v.π=NIL
s.color=grey, v.d=0
Enqueue(Q,s)
while(Q not empty)
    u = Dequeue(Q,s)
    for v in G.adj[u]
        if v.color == white
            v.color=grey, v.d=u.d+1, v.π=u
            Enqueue(Q,v)
    u.color=black
```

Breadth first search

Let $\delta(s,v)$ be the shortest path from s to v

After running BFS you can find this path as: $v.\pi$ to $(v.\pi).\pi$ to ... s

(pseudo code on p. 601, recursion)

BFS correctness

Proof: contradiction

Assume $\delta(s,v) \neq v.d$

$v.d \geq \delta(s,v)$ (Lemma 22.2, induction)

Thus $v.d > \delta(s,v)$

Let u be previous node on $\delta(s,v)$

Thus $\delta(s,v) = \delta(s,u)+1$

and $\delta(s,u) = u.d$

Then $v.d > \delta(s,v) = \delta(s,u)+1 = u.d+1$

BFS correctness

$v.d > \delta(s,v) = \delta(s,u)+1 = u.d+1$

Cases on color of v when u dequeue,
all cases invalidate top equation

Case white: alg sets $v.d = u.d + 1$

Case black: already removed

thus $v.d \leq u.d$ (corollary 22.4)

Case grey: exists w that dequeued v,

$v.d = w.d+1 \leq u.d+1$ (corollary 22.4)

Depth first search

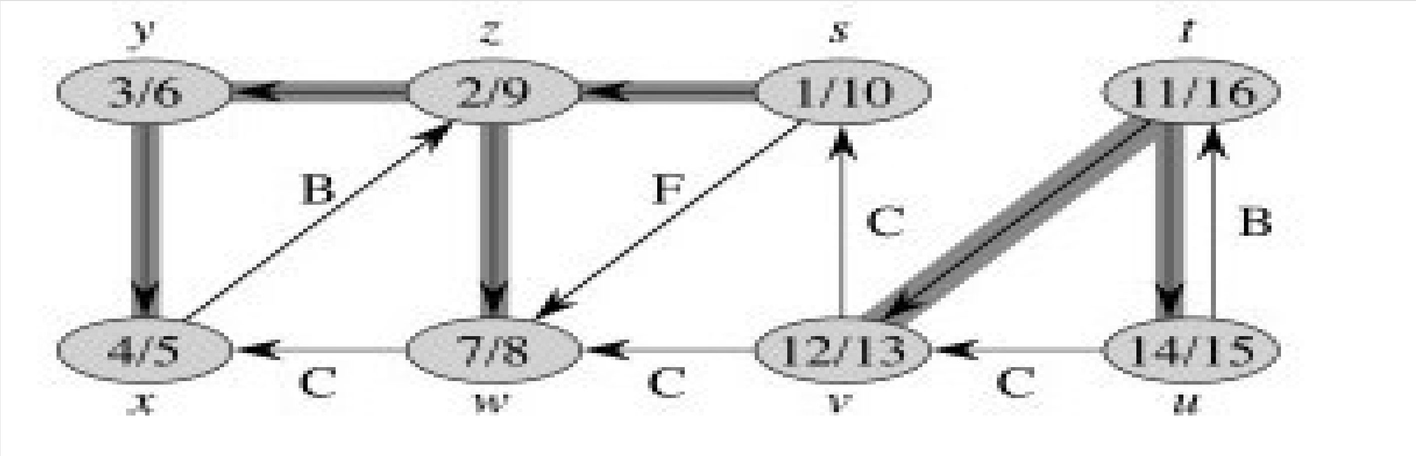
```
DFS(G)
for all v in V
   v.color=white, v.π=NIL
time=0
for each v in V
   if v.color==white
      DFS-Visit(G,v)
```

Depth first search

```
DFS-Visit(G,u)
time=time+1
u.d=time, u.color=grey
for each v in G.adj[u]
   if v.color == white
      v.π=u
      DFS-Visit(G,v)
u.color=black, time=time+1, u.f=time
```

Depth first search

Edge markers:



Consider edge u to v

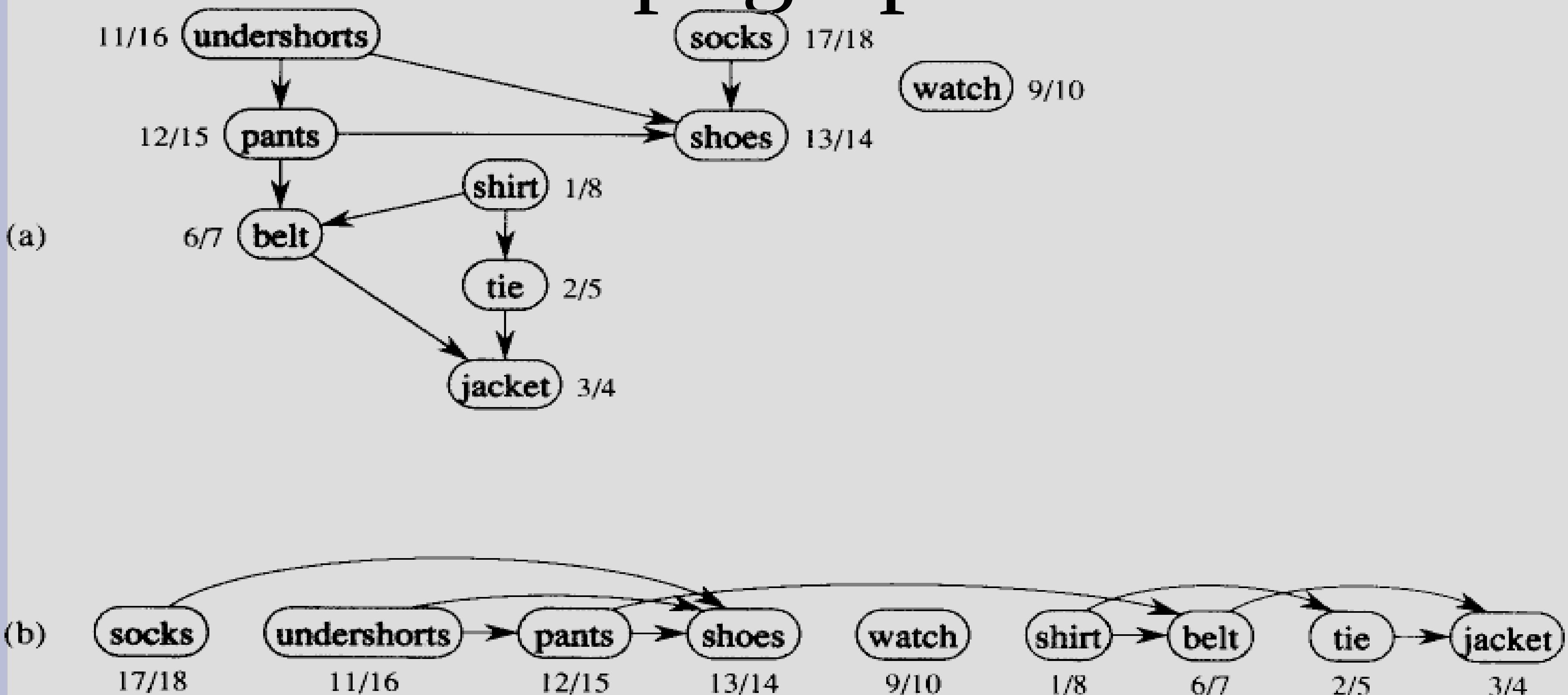
C = Edge to black node (u.d > v.f)

B = Edge to grey node (u.f < v.f)

F = Edge to black node (u.f > v.f)

Depth first search

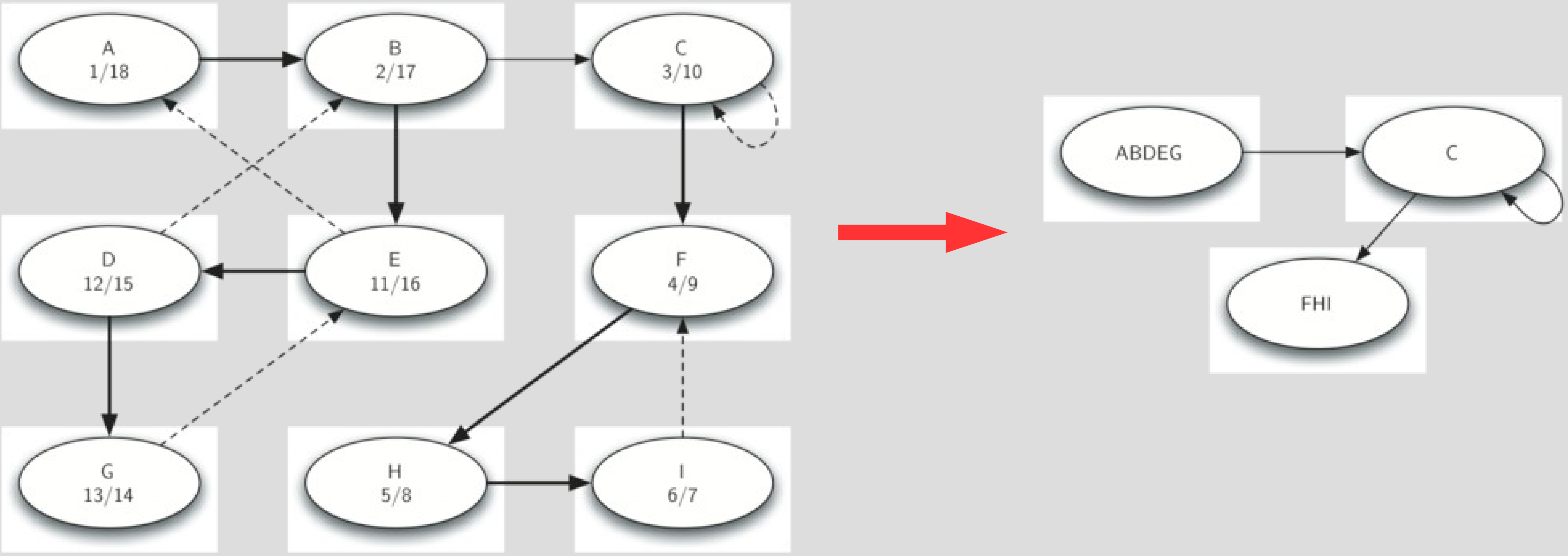
DFS can do topographical sort



Run DFS, sort in decreasing finish time

Depth first search

DFS can find strongly connected components



Depth first search

Let G^T be G with edges reversed

Then to get strongly connected:

1. DFS(G) to get finish times
2. Compute G^T
3. DFS(G^T) on vertex in decreasing finish time
4. Each tree in forest SC component