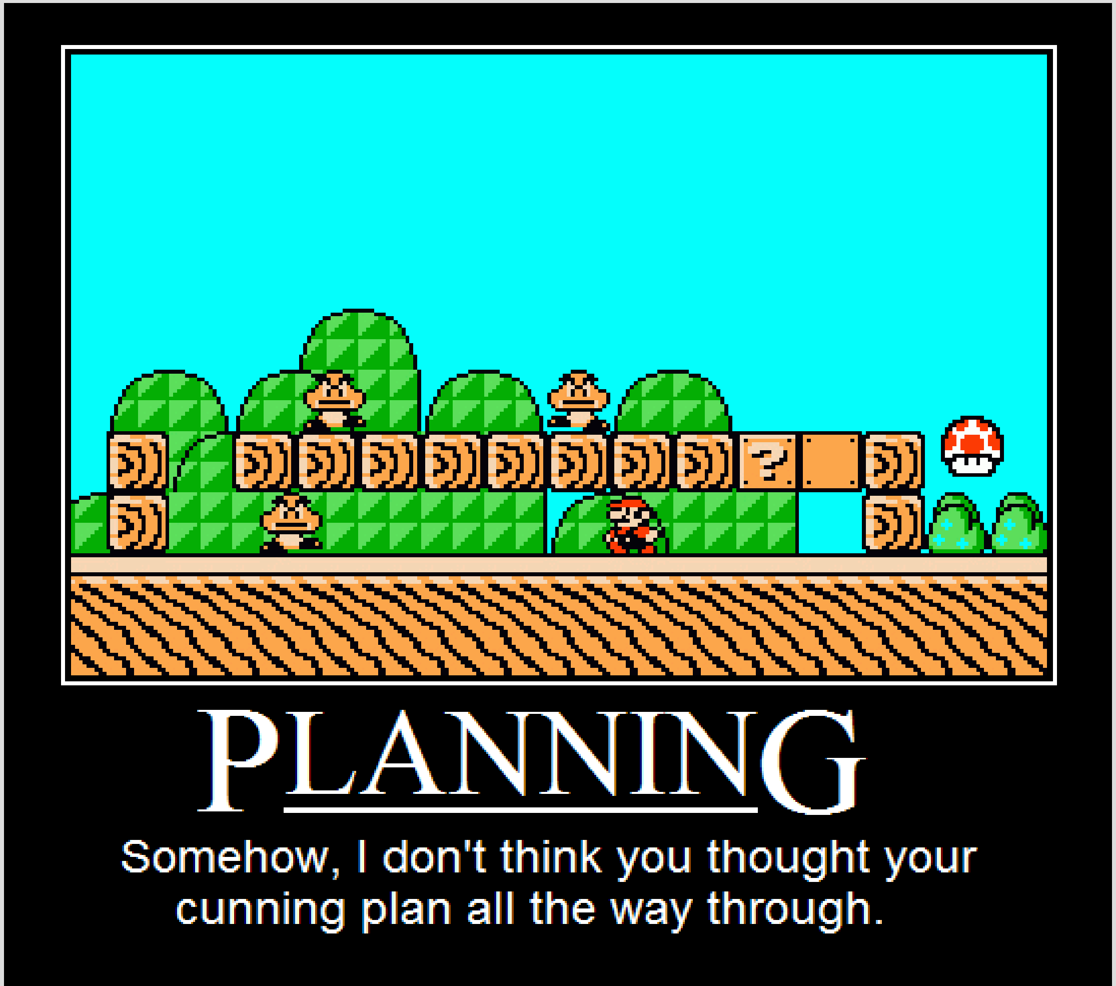
Planning (Ch. 10)



Graph Plan

A heuristic we will go over in detail is graph planning, which tries to do all possible actions at each step

The graph plan heuristic is nice because it is always admissible and computable in P time

The basic idea of graph plan is to track all the statements that could be true at any time

Graph Plan

Graph plan is an underestimate because once a relation/literal is added, it is never removed

Unlike the “remove negative effects” heuristic, we allow both negative and positive effects

But we can also use any preconditions that have been found anytime before (not quite as open as completely removing them)

Graph Plan

These simplifications/relaxations probably make the problem too easy

So we also track pairs of both actions and literals that are in conflict (called <u>mutex</u>es)

First, let's go over how to convert actions and relations into graph plan, then later we will add in the mutexes

Graph Plan

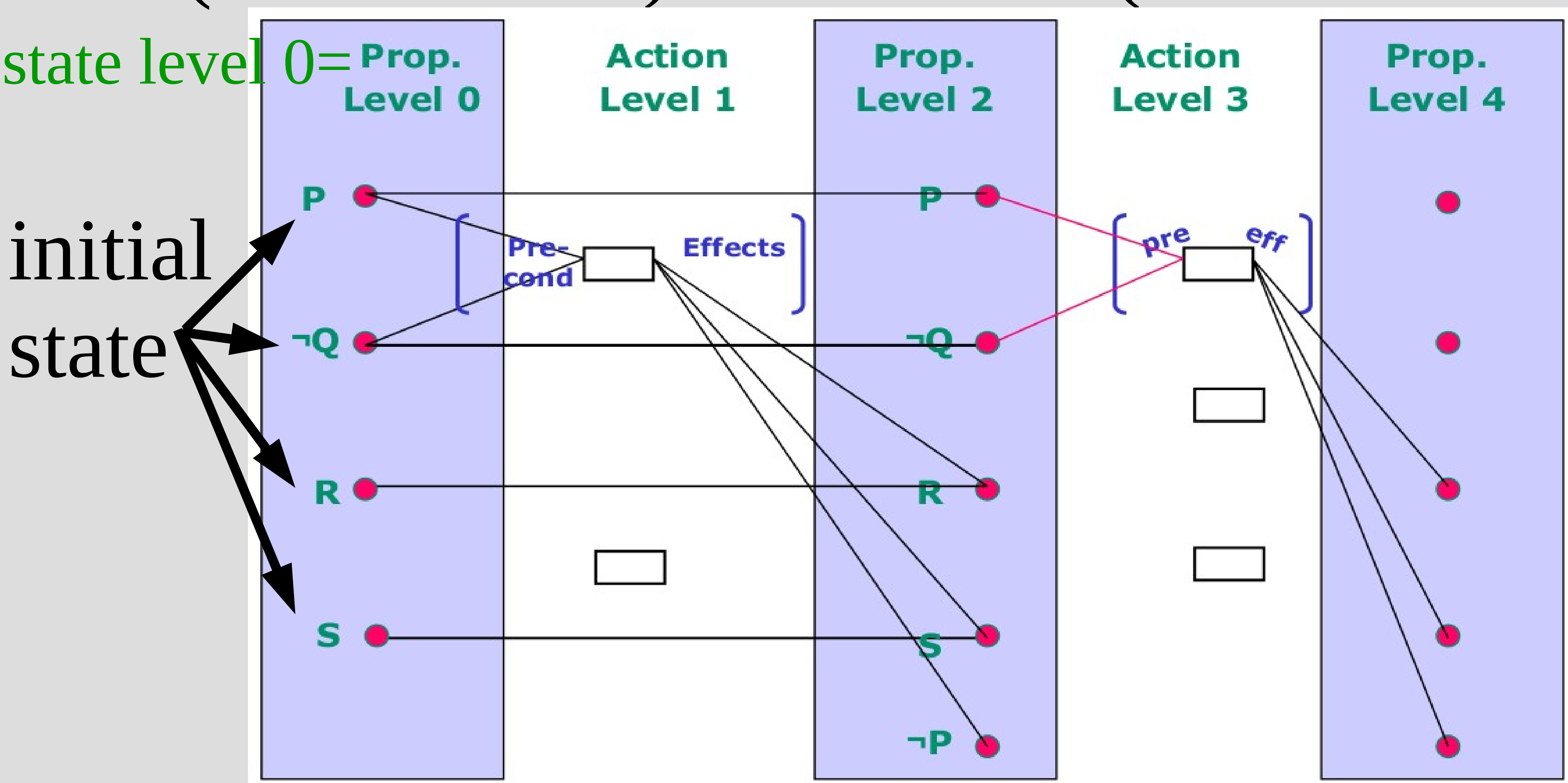
You start with the relations of the initial state on the left (now explicitly stating negatives)

Then you add “no actions” which simply keep all the relationships the same but move them to the right

Then you add actions, which you do by linking preconditions on the left to resulting effects on the right (adding any new ones)

Graph Plan

Graph plan will alternate between possible facts ("state level") and actions ("action level")



Graph Plan

Consider this problem:

Initial: $Sleepy(me) \wedge Hungry(me)$

Goal: $\neg Sleepy(me) \wedge \neg Hungry(me)$

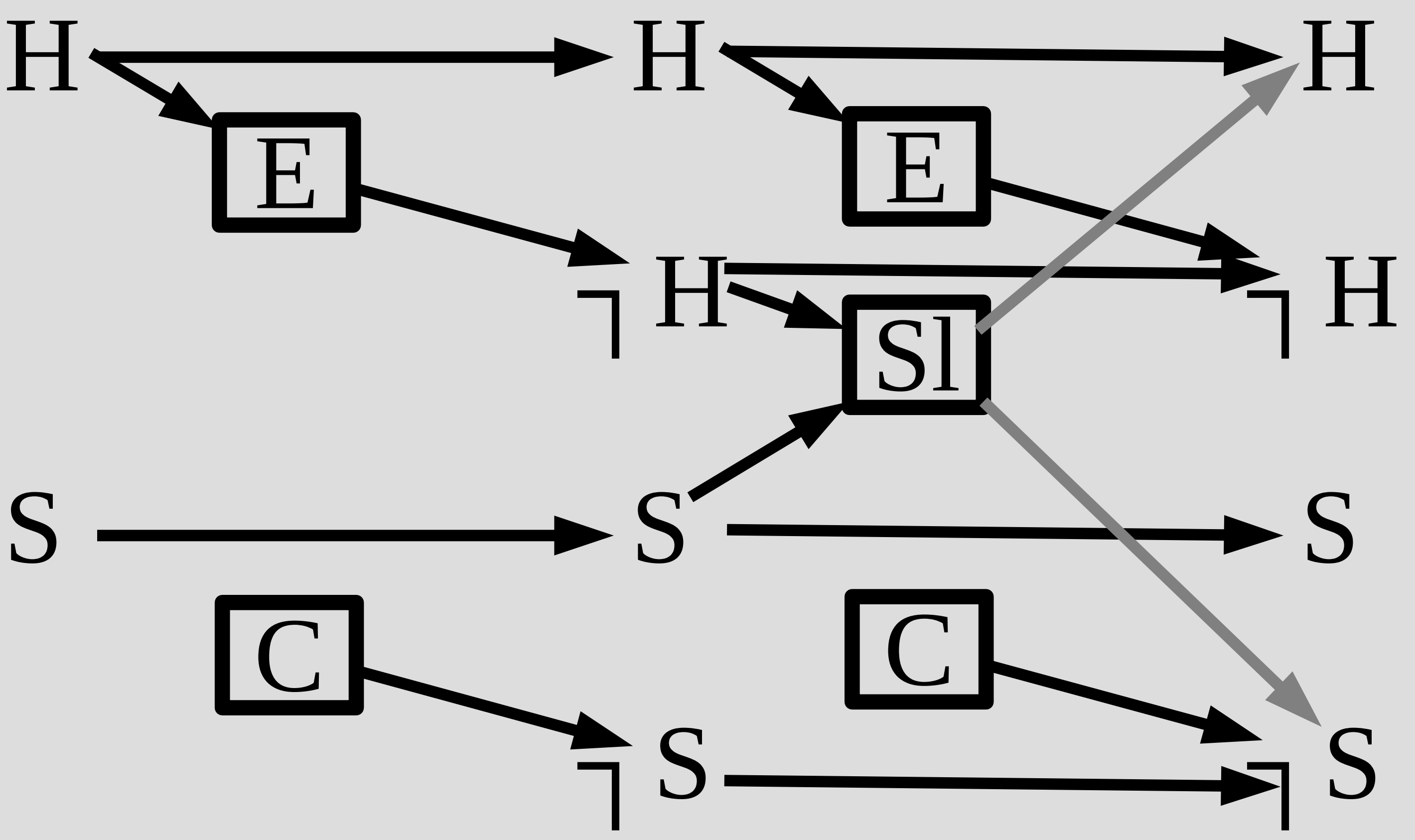
Action($Eat(x)$,
Precondition: $Hungry(x)$,
Effect: $\neg Hungry(x)$)

Action($Coffee(x)$,
Precondition: ,
Effect: $\neg Sleepy(x)$)

Action($Sleep(x)$,
Precondition: $Sleepy(x) \wedge \neg Hungr(x)$,
Effect: $\neg Sleepy(x) \wedge Hungry(x)$)

Graph Plan

Consider this problem:



Graph Plan

Each set of relations/literals are what we call levels of the graph plan, S = states, A = actions

State level 0 is S_0 = {H, S}

A_0 = {C, E, all "no ops"}

S_1 = {H, ¬ H, S, ¬ S}

A_1 = {C, E, Sl, all "no ops"}

S_2 = {H, ¬ H, S, ¬ S}

Graph Plan

You do it! (show 3 state and 2 action levels)

Initial: $\neg Money(me) \wedge \neg Smart(me) \wedge \neg Debt(me)$

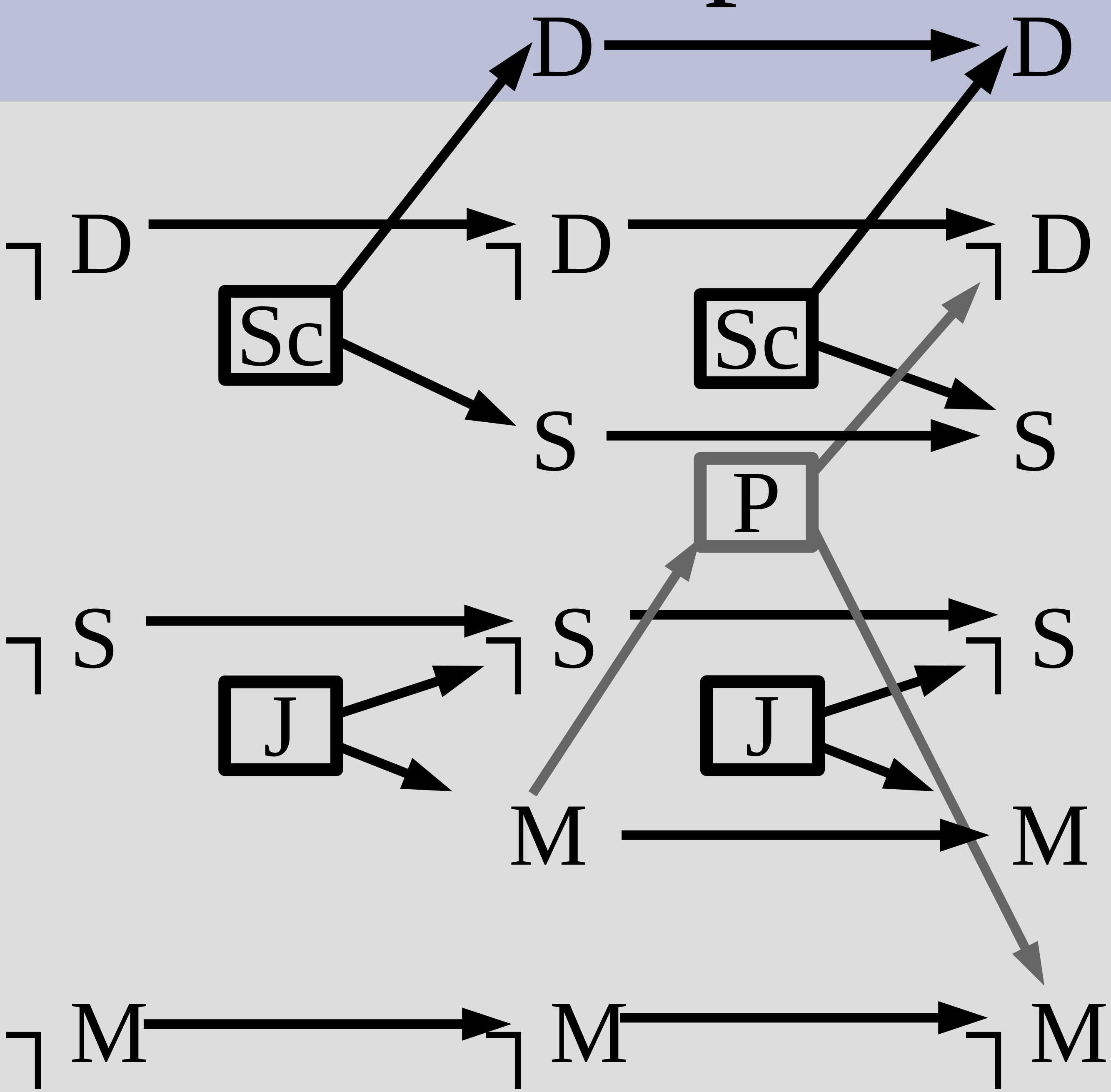
Goal: $Money(me) \wedge Smart(me) \wedge \neg Debt(me)$

Action($School(x)$,
Precondition: ,
Effect: $Debt(x) \wedge Smart(x)$)

Action($Job(x)$,
Precondition: ,
Effect: $Money(x) \wedge \neg Smart(x)$)

Action($Pay(x)$,
Precondition: $Money(x)$,
Effect: $\neg Money(x) \wedge \neg Debt(x)$)

Graph Plan



Graph Plan

The graph plan allows multiple actions to be done in a single turn, which is why S_1 has both $\neg$ Sleepy(me) and $\neg$ Hungry(me)

You keep building the graph until either:
(1) You find your goal (more on this later)
(2) The graph converges (i.e. states, actions and mutexes stop changing)

Mutexes

A <u>mutex</u> are two things that cannot be together (i.e. cannot happen or be true simultaneously)

You can put mutexes:

1. Between two relationships/literals
2. Between actions

There are different rules for doing mutexes between actions vs. relations

Mutexes: actions

For all of these cases I will assume actions two actions: A1 and A2

These actions have preconditions and effects: Pre(A1) and Effect(A1), respectively

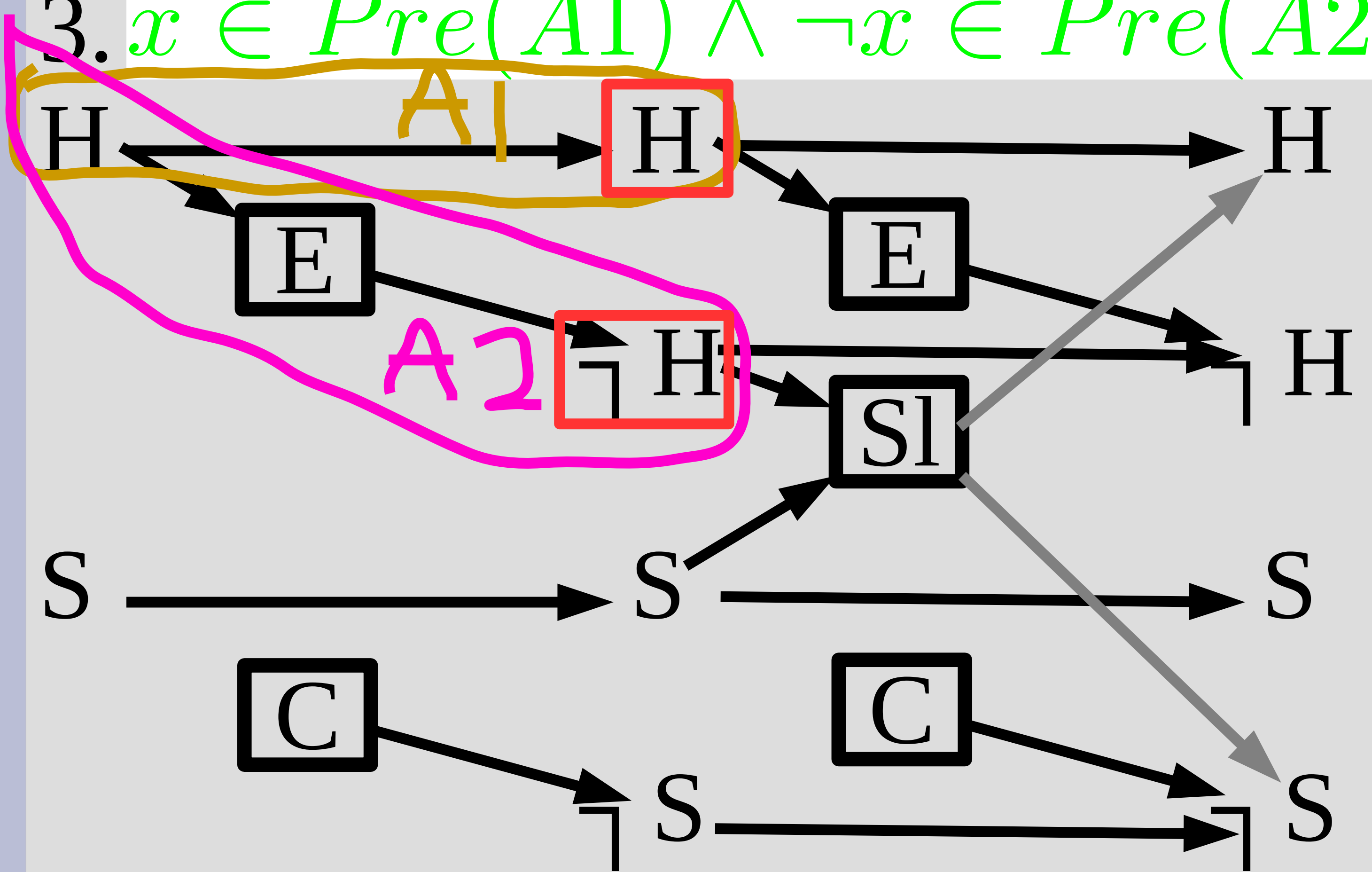
For example, I will abbreviate below as:

$$\begin{aligned} &\text{Action}(\ Eat(x), \\ &\text{Precondition: } Hungry(x), \\ &\text{Effect: } \neg Hungry(x)) \end{aligned} \longrightarrow \begin{aligned} &A1 = Eat \\ &\neg H \in Effect(A1) \\ &H \in Pre(A1) \end{aligned}$$

Mutexes: actions

Mutex Action rules:

1. $x \in Effect(A1) \wedge \neg x \in Effect(A2)$
2. $x \in Pre(A1) \wedge \neg x \in Effect(A2)$
3. $x \in Pre(A1) \wedge \neg x \in Pre(A2)$



Mutexes: actions

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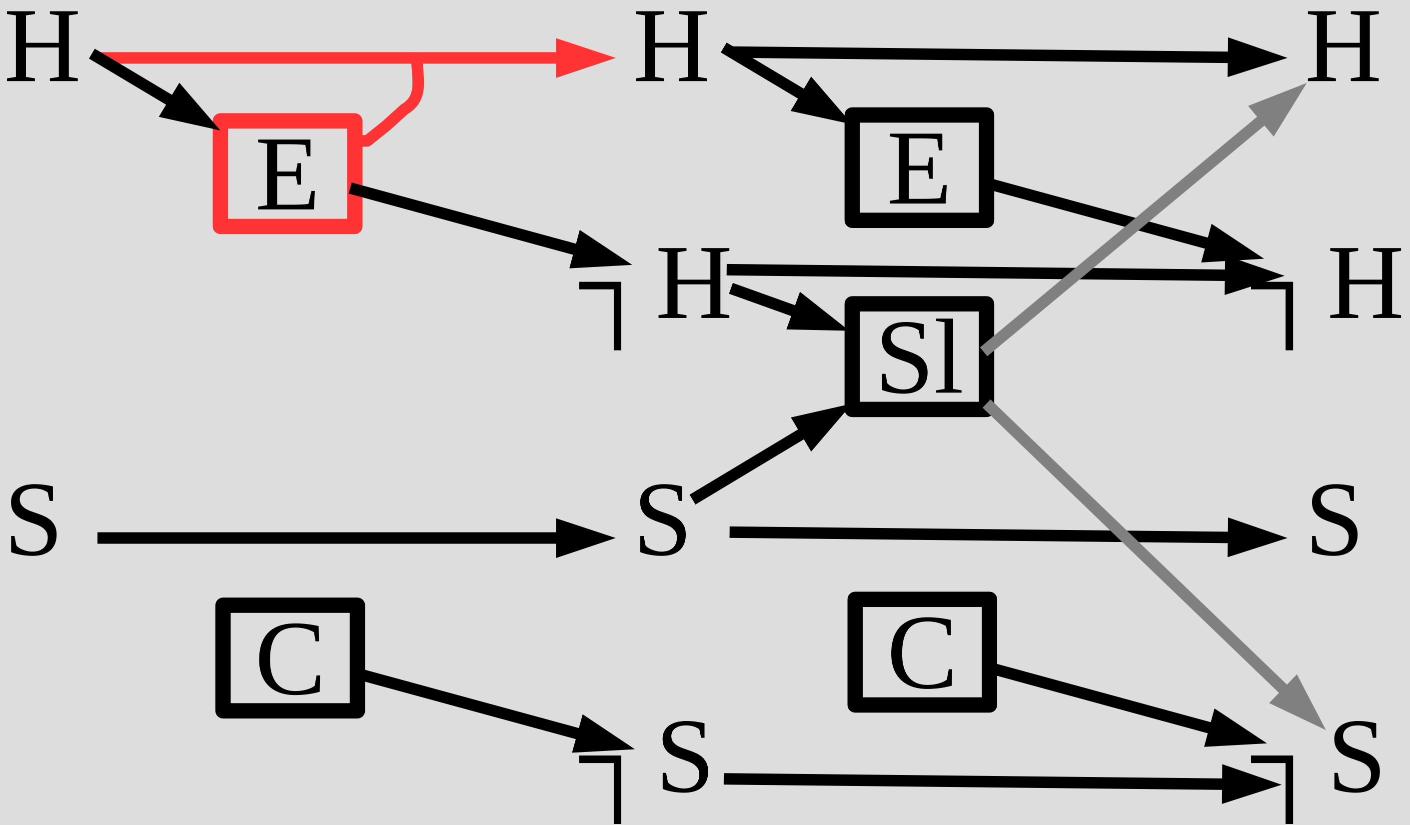

Mutexes: actions

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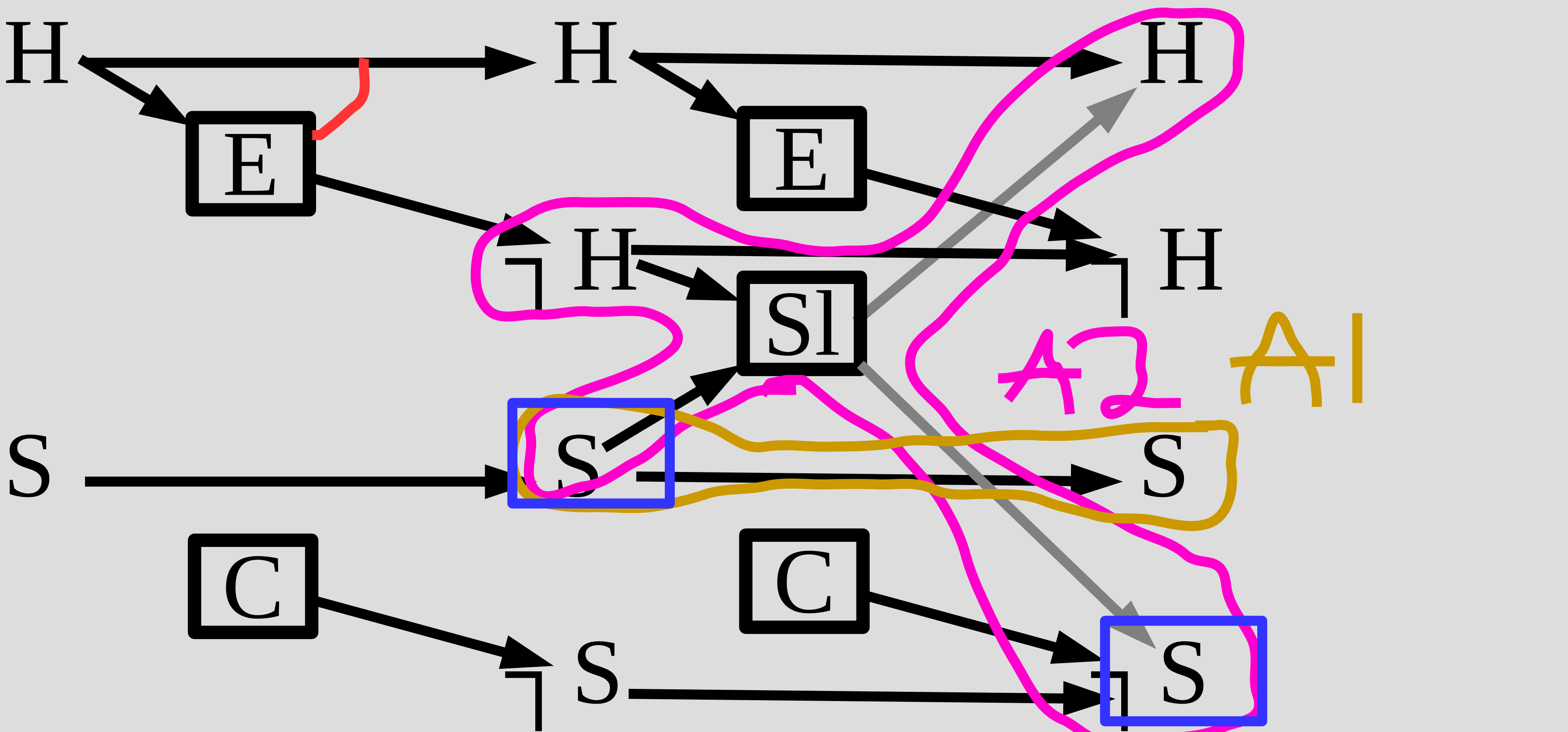

Mutexes: actions

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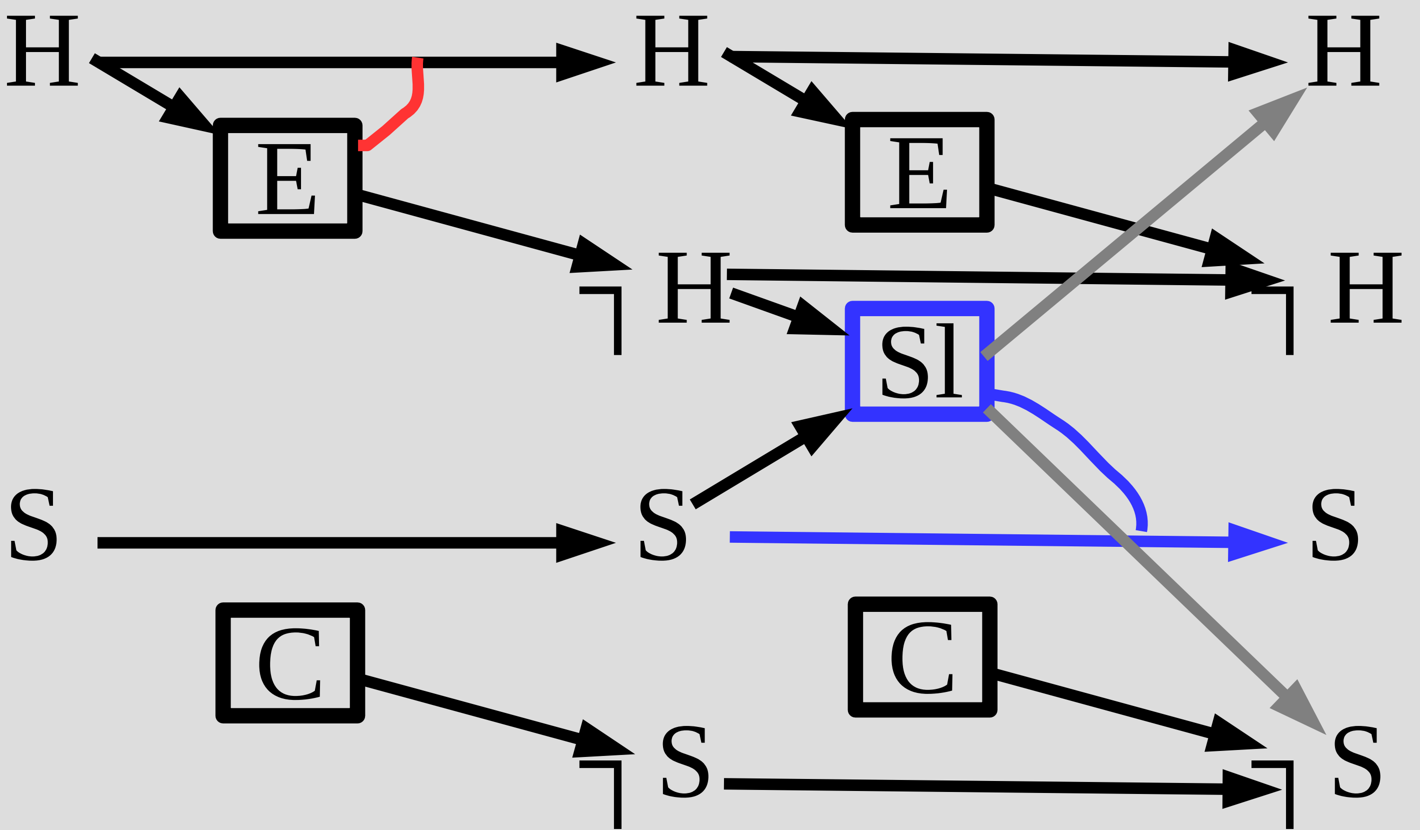

Mutexes: actions

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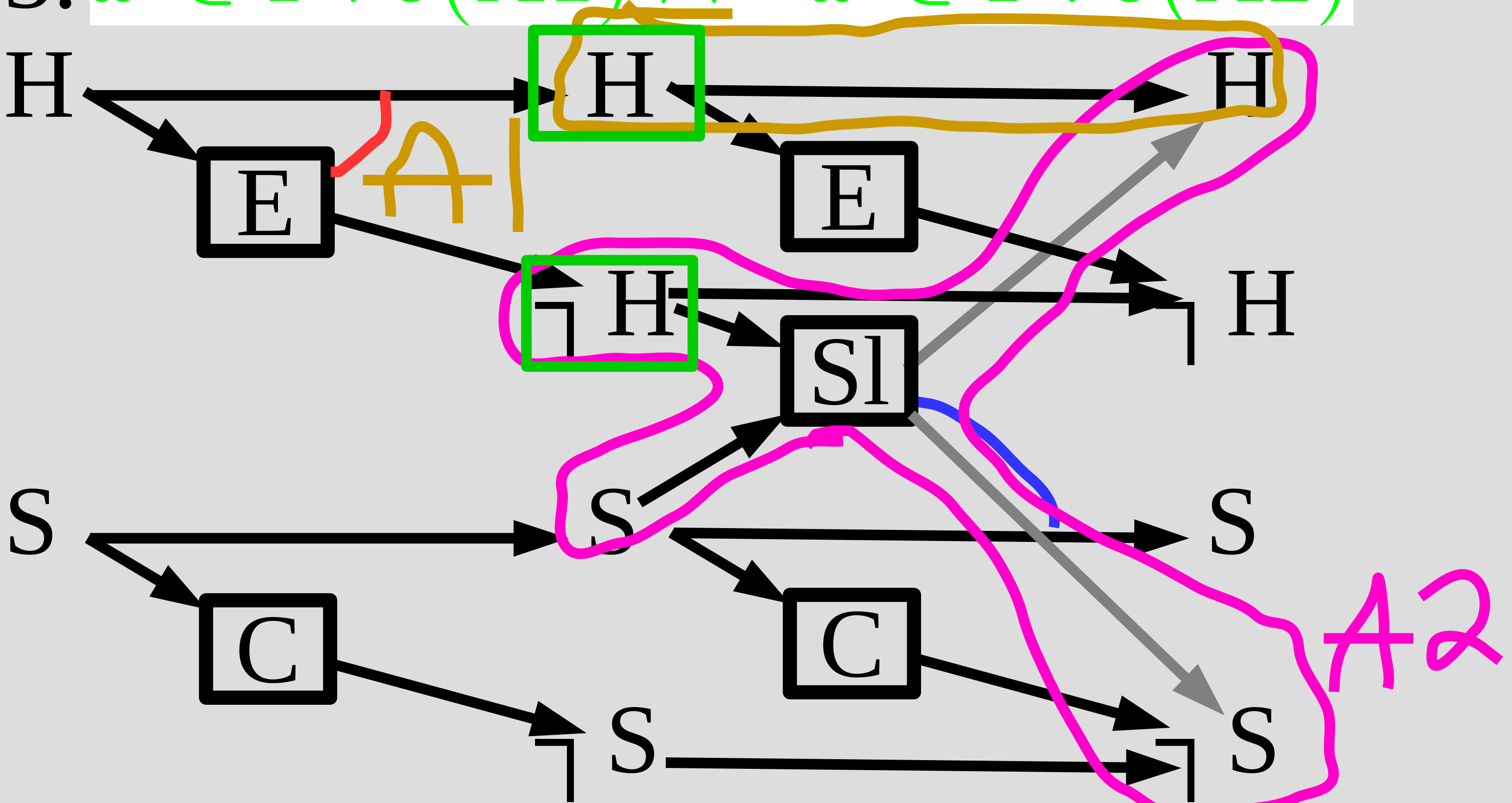

Mutexes: actions

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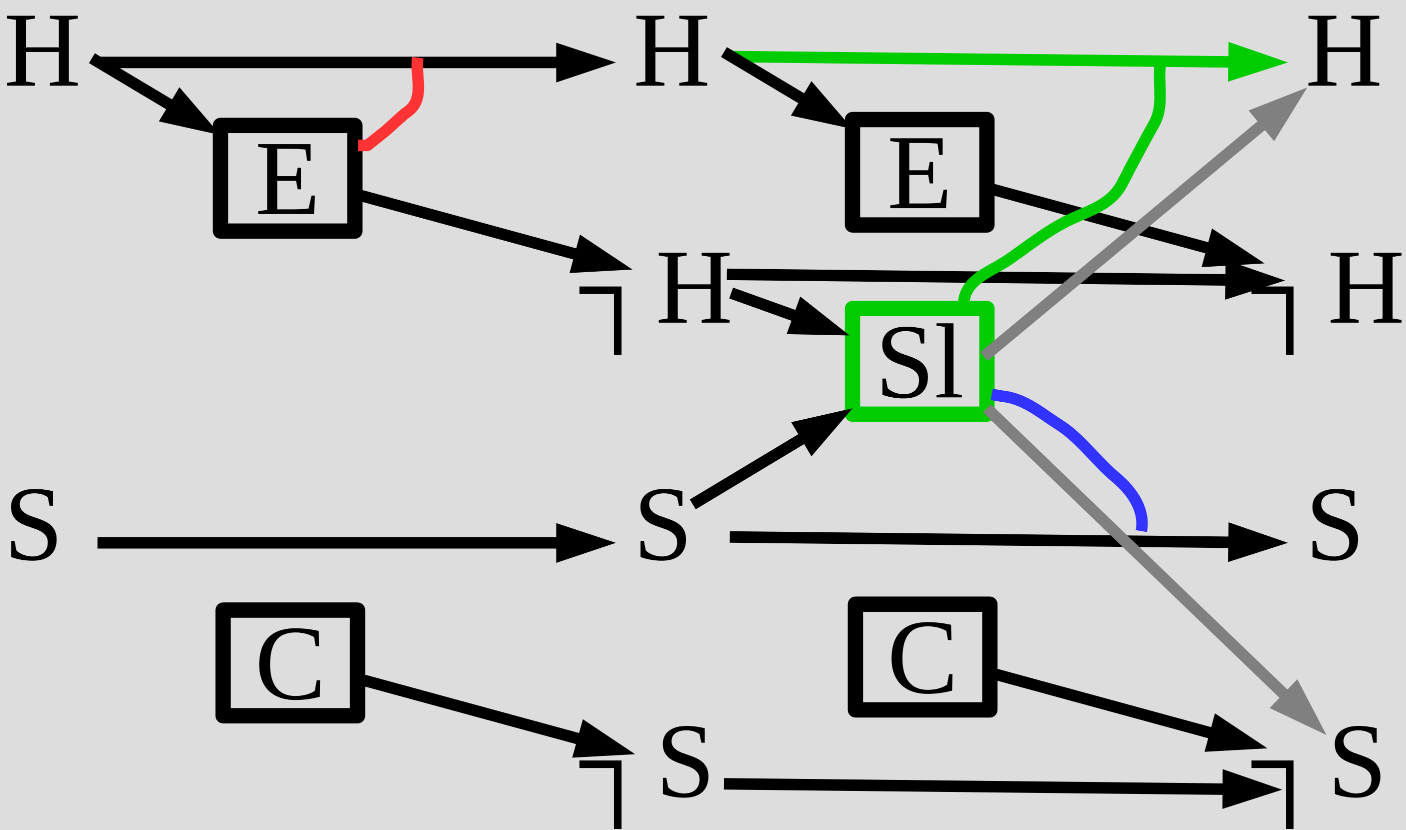

Mutexes: actions

Mutex Action rules:

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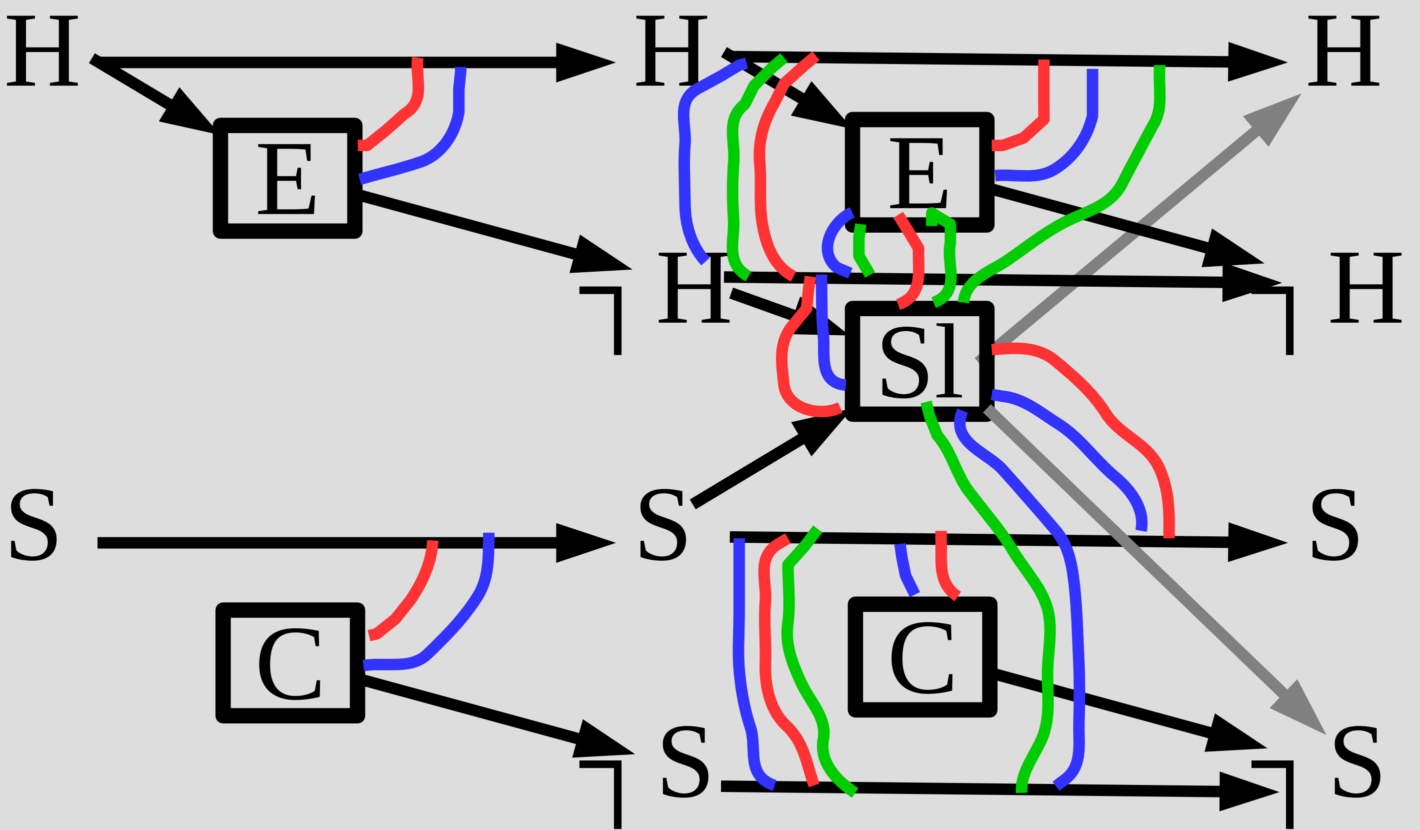


Mutexes: states

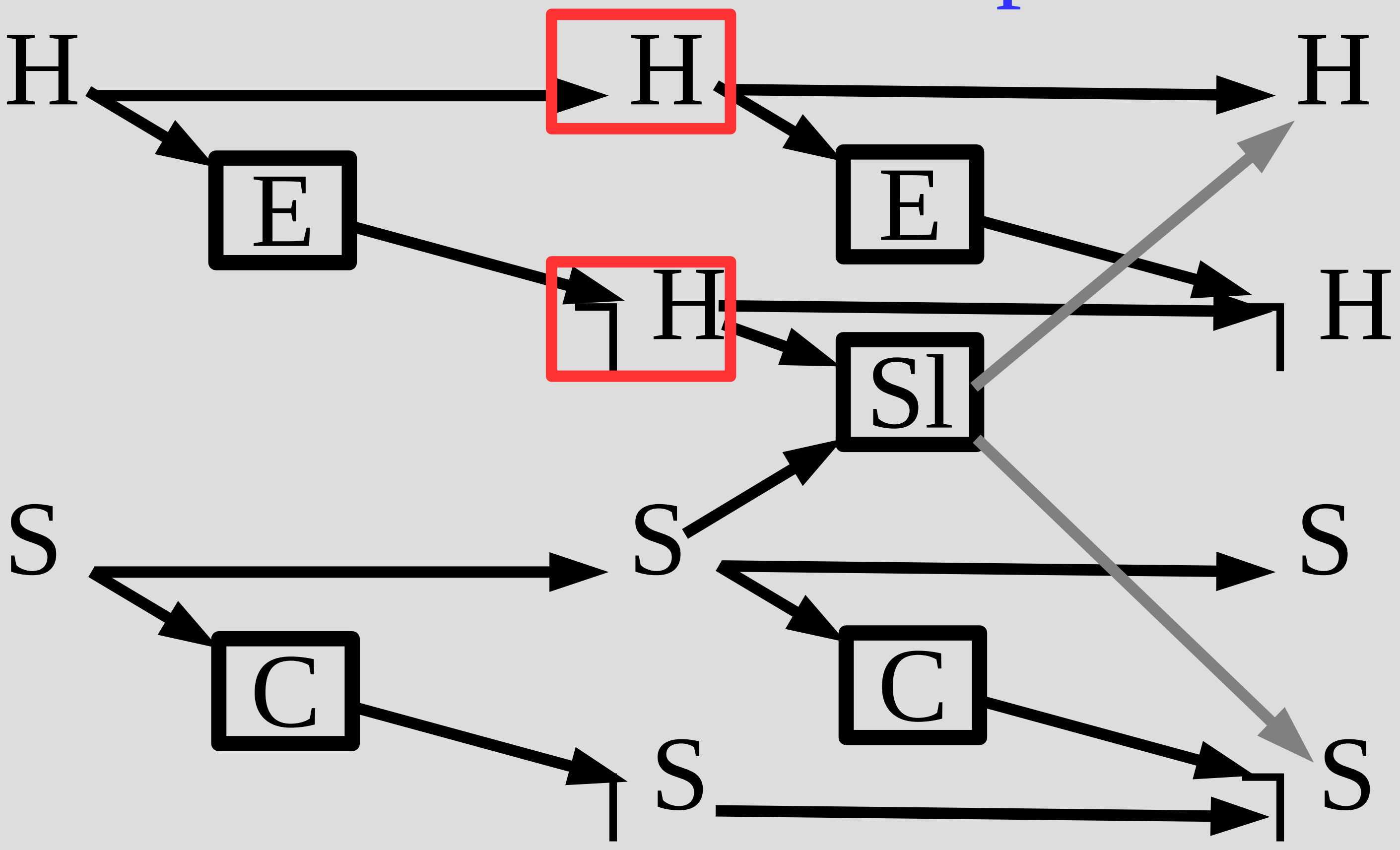
There are 2 (easier) rules for states, but unlike action mutexes they can change across levels

1. Opposite relations are mutexes (x and ¬ x)
2. If there are mutexes between all possible actions that lead to a pair of states

Can rephrase second rule: All pairs of states start with a mutex, but remove mutex if there are un-mutexes actions that lead to state pair

Mutexes: states

1. Opposite relations are mutexes (x and ┐ x)
2. If there are mutexes between all possible actions that lead to a pair of states



Mutexes: states

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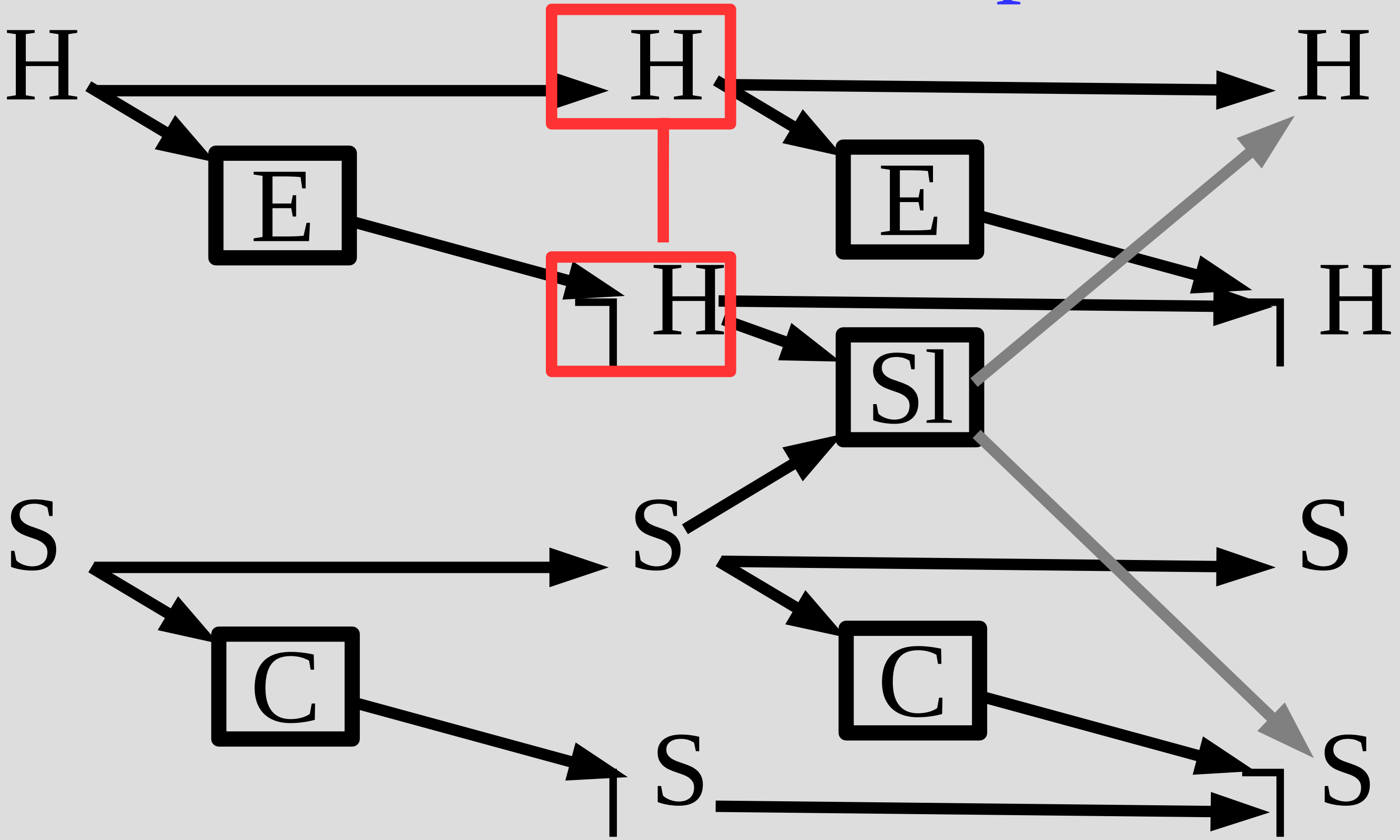

Mutexes: states

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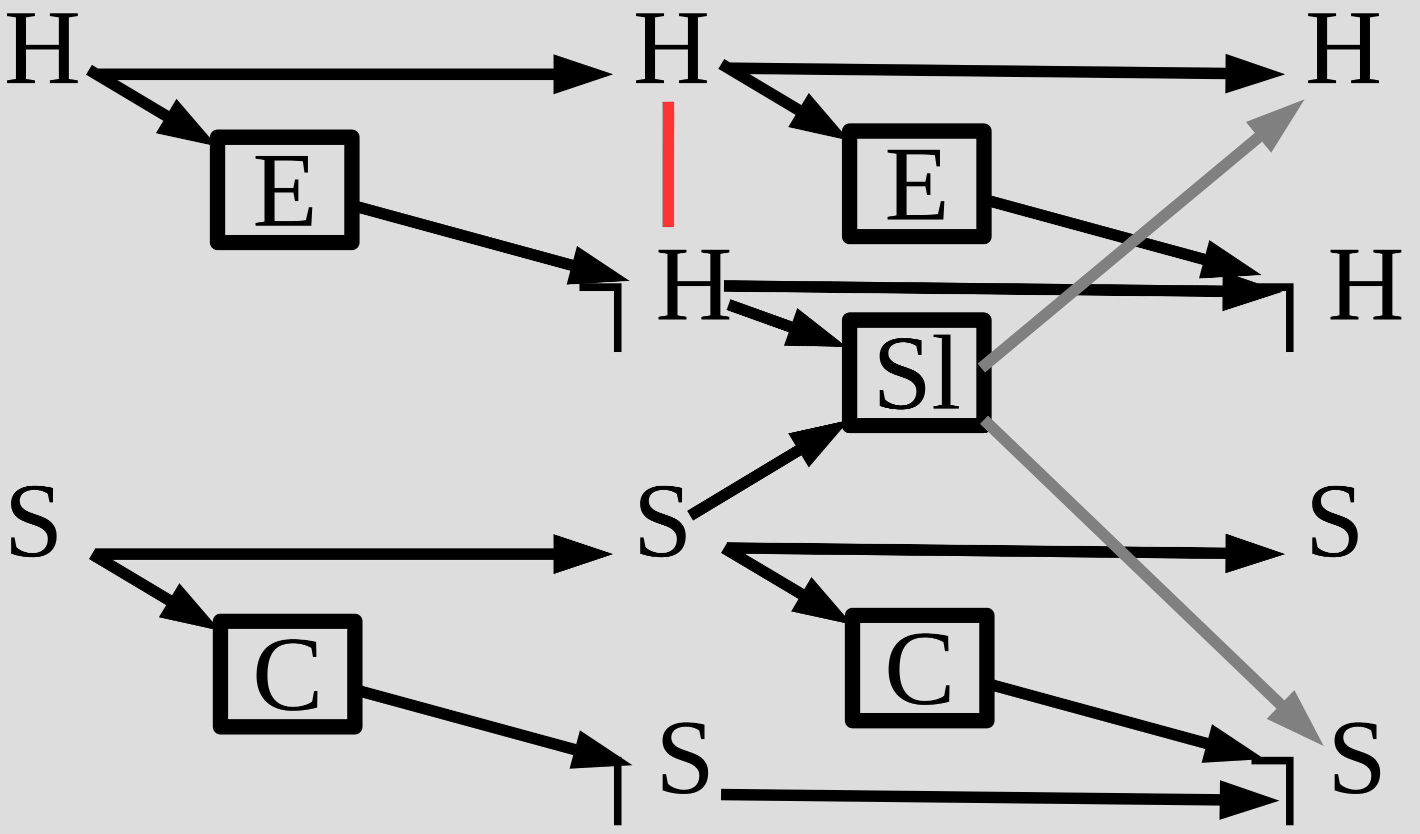
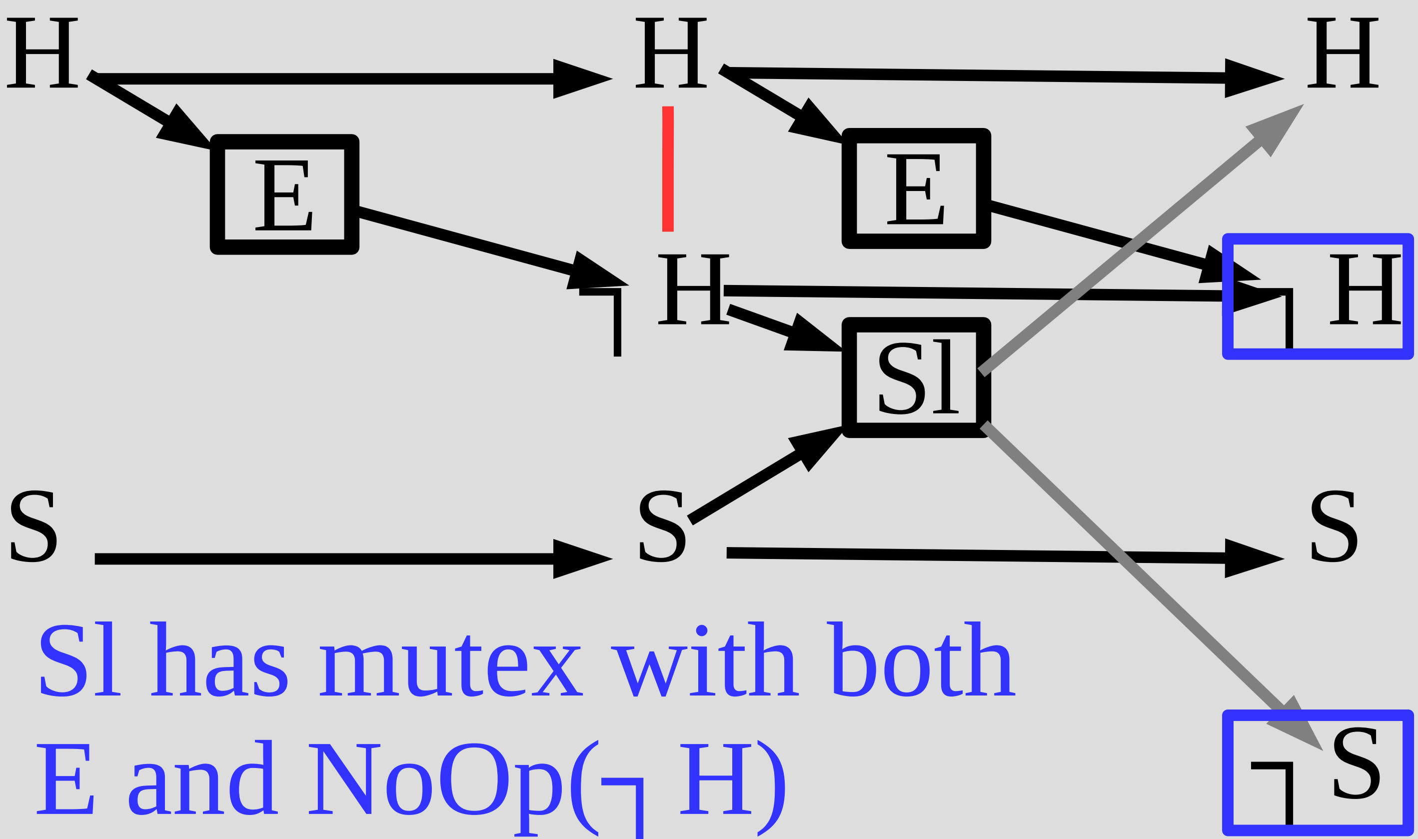


None... but if we remove coffee...

Mutexes: states

1. Opposite relations are mutexes (x and ¬ x)
2. If there are mutexes between all possible actions that lead to a pair of states

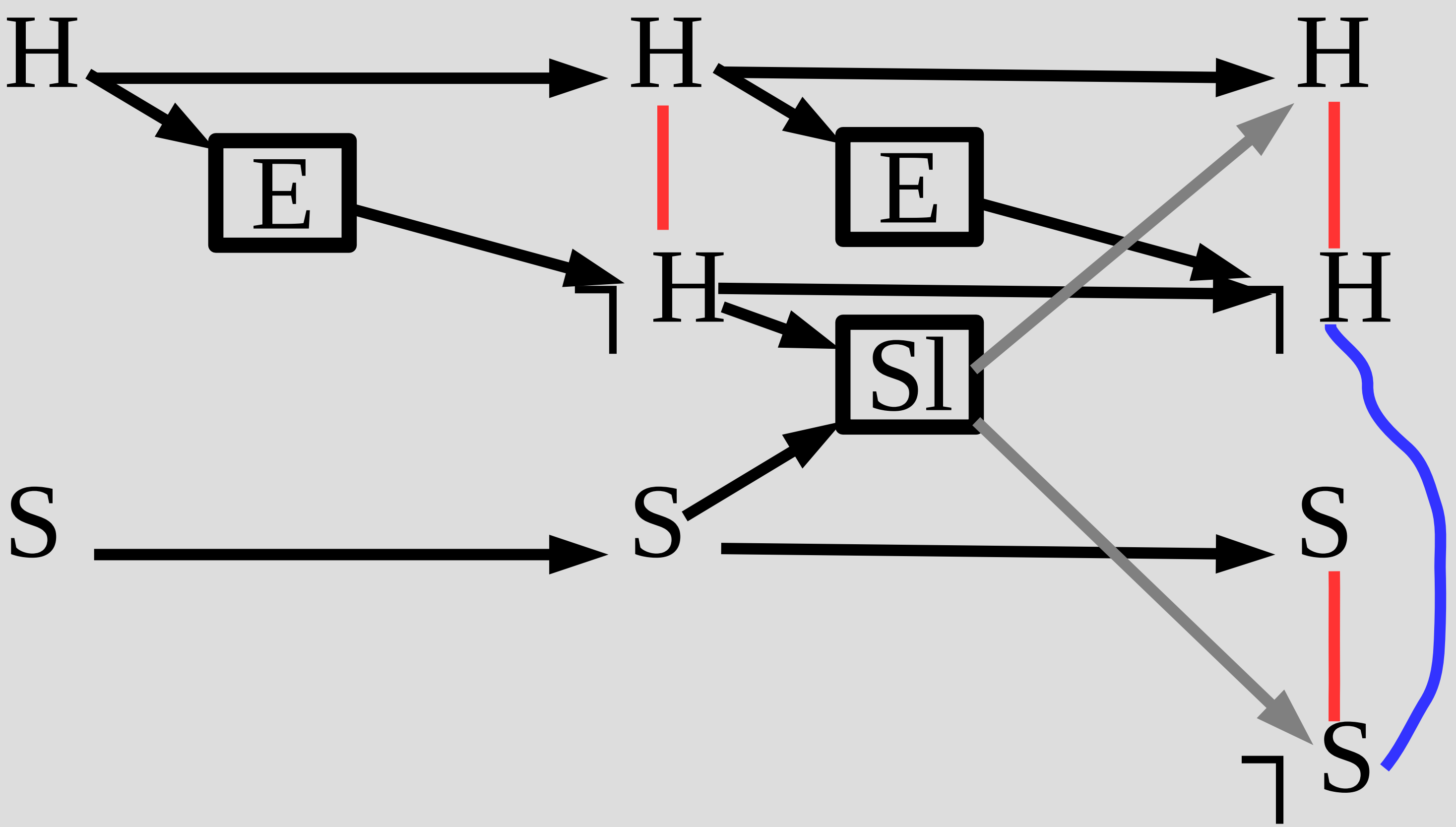


This mutex will be gone on the next level (as you can eat again)

Sl has mutex with both E and NoOp(¬ H)

Mutexes: states

1. Opposite relations are mutexes (x and ¬ x)
2. If there are mutexes between all possible actions that lead to a pair of states



Mutexes: actions

You do it!

Initial: $\neg Money(me) \land \neg Smart(me) \land \neg Debt(me)$

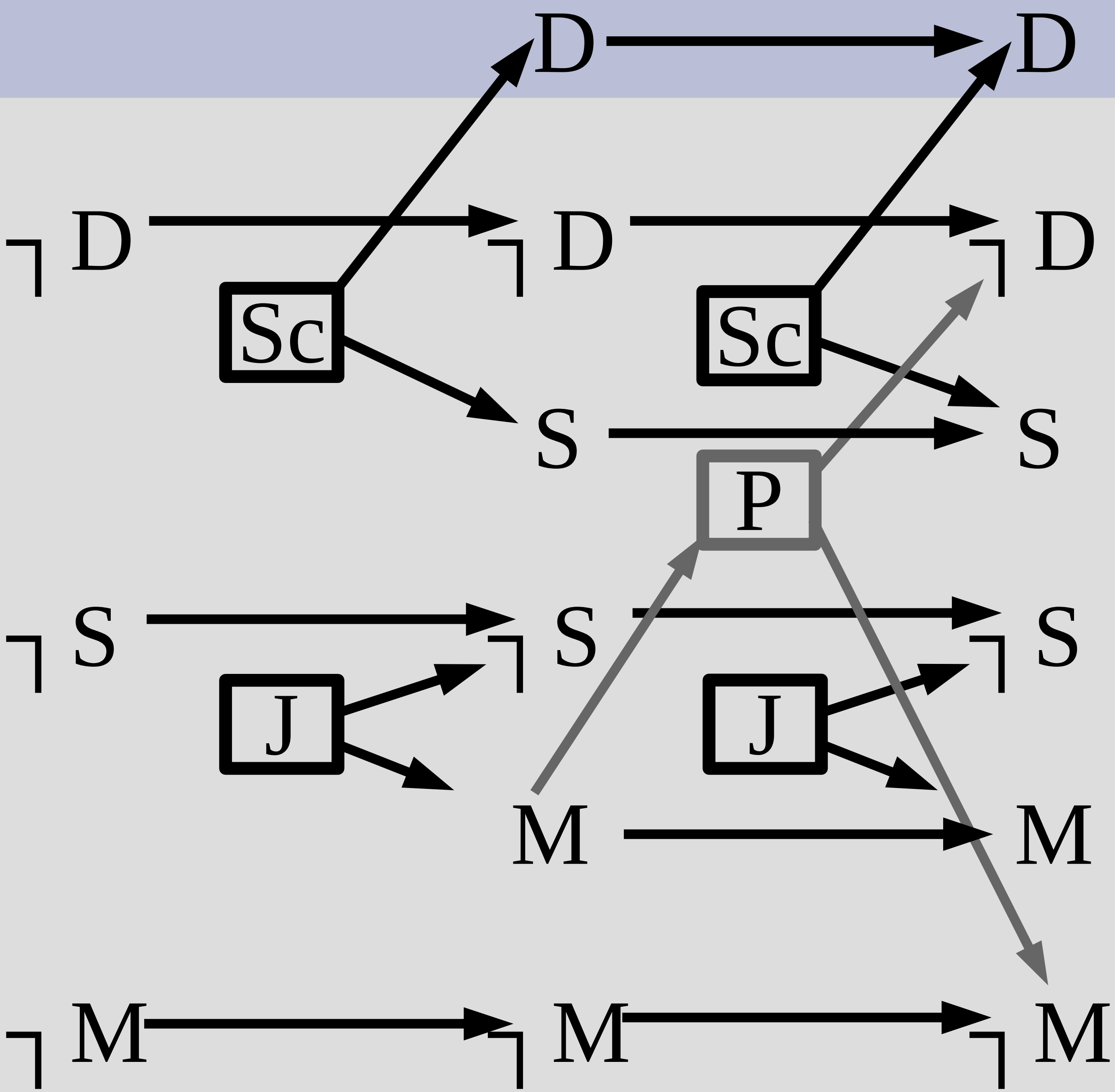
Goal: $Money(me) \land Smart(me) \land \neg Debt(me)$

Action($School(x)$,
Precondition: ,
Effect: $Debt(x) \land Smart(x)$)

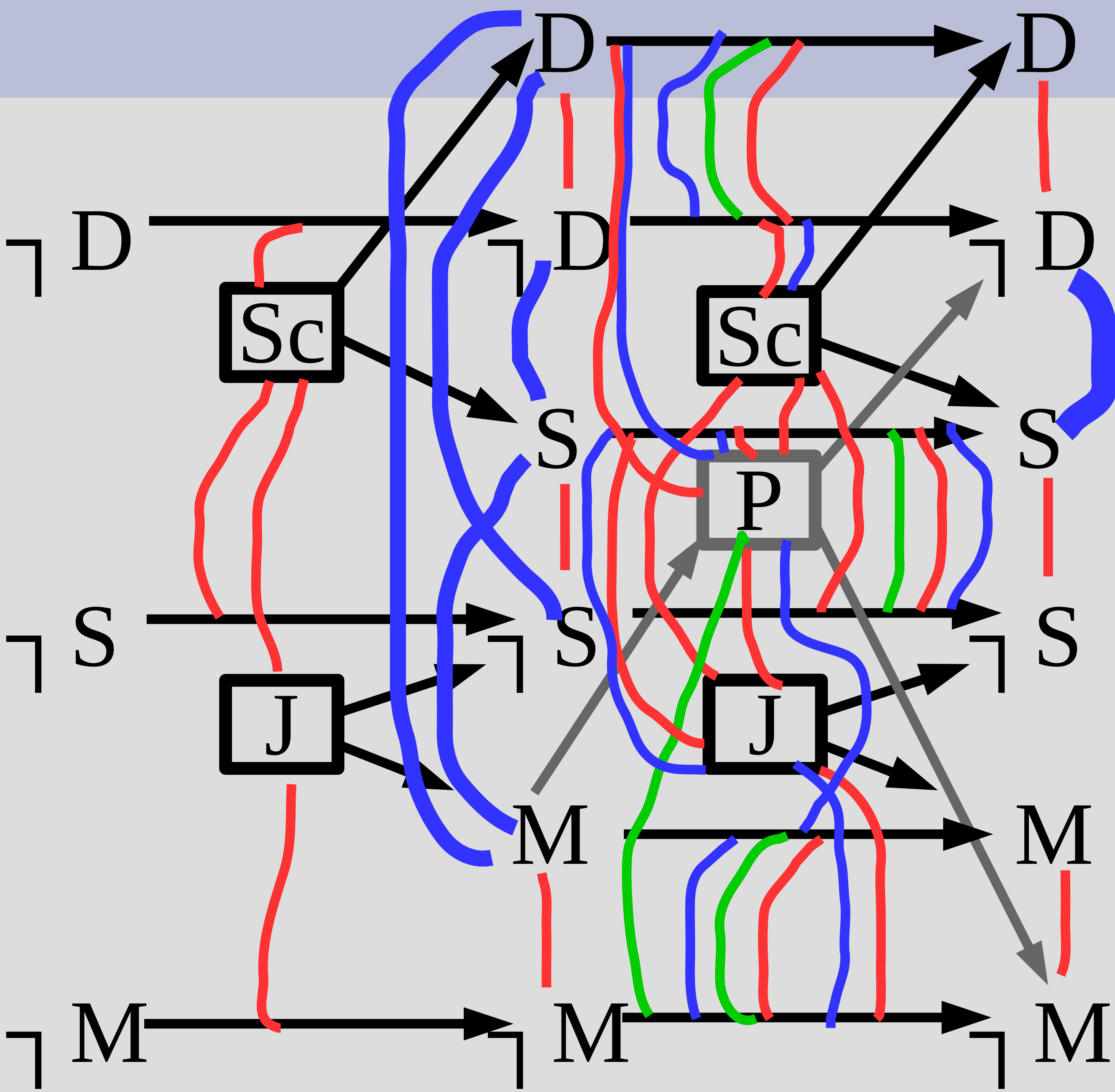
Action($Job(x)$,
Precondition: ,
Effect: $Money(x) \land \neg Smart(x)$)

Action($Pay(x)$,
Precondition: $Money(x)$,
Effect: $\neg Money(x) \land \neg Debt(x)$)

Mutexes: actions



Mutexes: actions



Non-trivial mutexes:
(SC, P),
(J, P),
(SC, J),
(P,¬ D&M),
(SC,¬ D&¬ S),
(J,¬ M&S)

GraphPlan

GraphPlan can be computed in $O(n(a+l)^2)$,
where n = levels before convergence
a = number of actions
l = number of relations/literals/states
(square is due to needing to check all pairs)

The original planning problem is PSPACE,
which is known to be harder than NP

GraphPlan: states

Let's consider this problem:

Initial: $Clean \wedge Garbage \wedge Quiet$

Goal: $Food \wedge \neg Garbage \wedge Present$

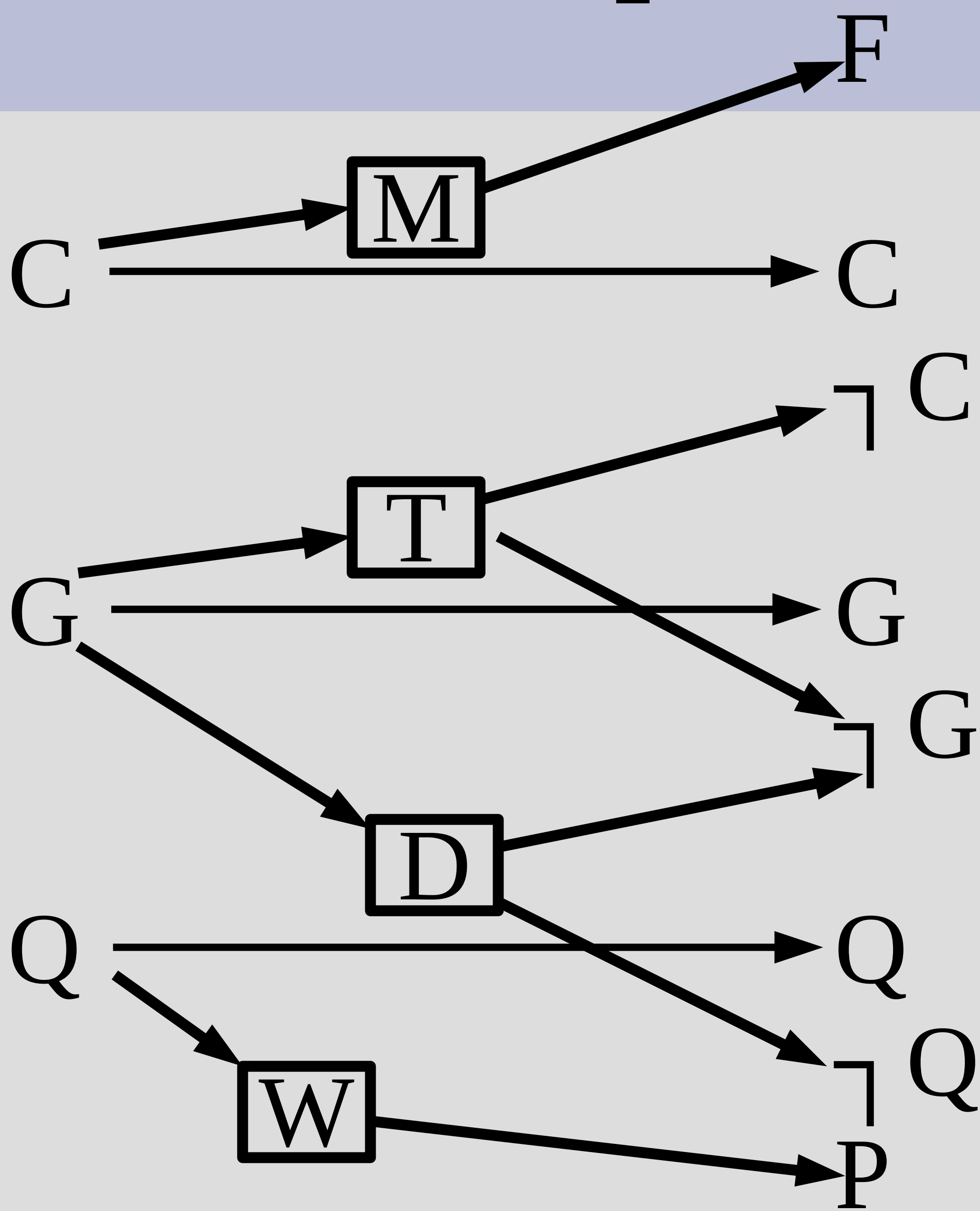
Action: ($MakeFood$,
Precondition: $Clean$,
Effects: $Food$)

Action: ($Takeout$,
Precondition: $Garbage$,
Effects: $\neg Garbage \wedge \neg Clean$)

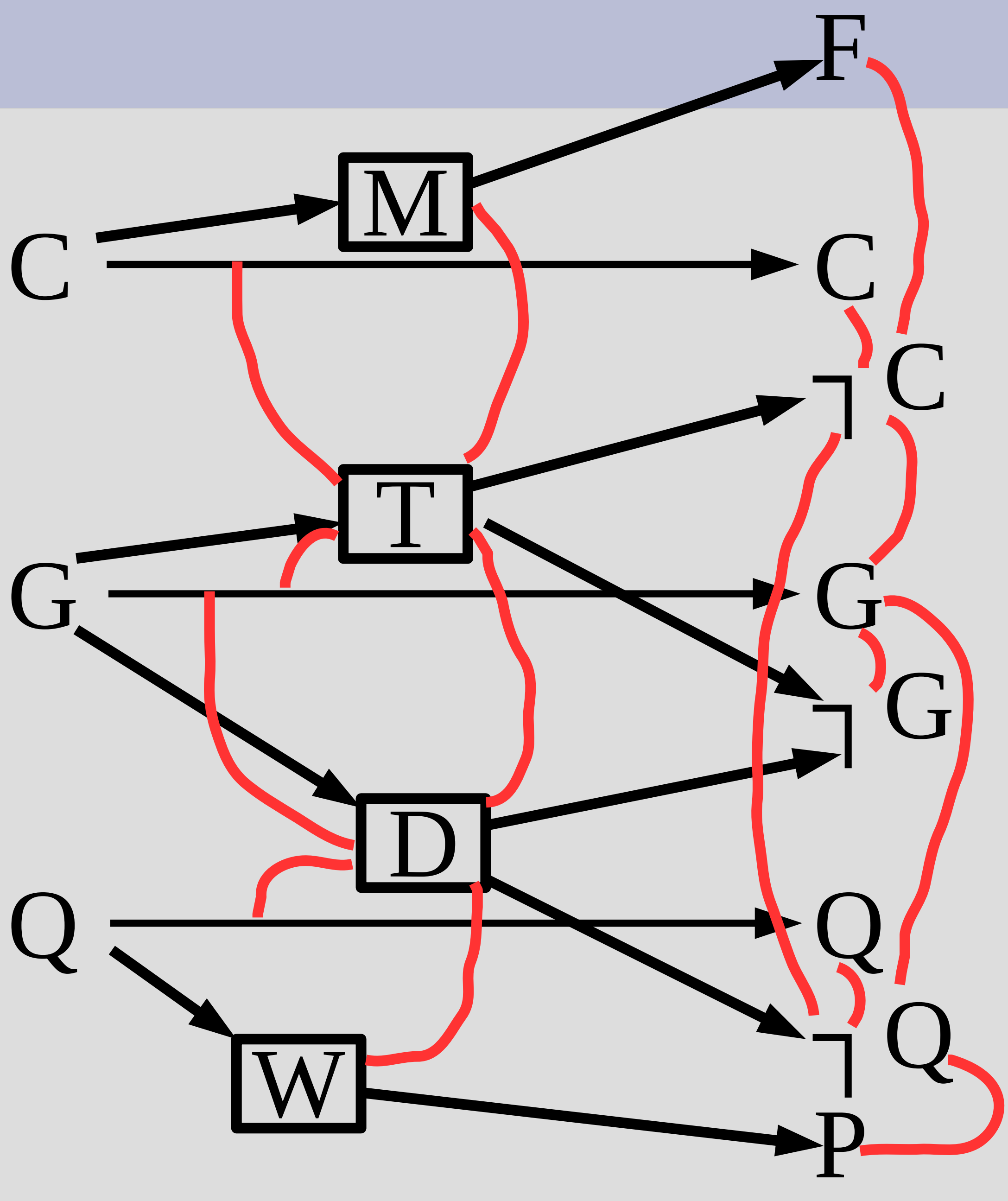
Action: ($Wrap$,
Precondition: $Quiet$,
Effects: $Present$)

Action: ($Dolly$,
Precondition: $Garbage$,
Effects: $\neg Garbage \wedge \neg Quiet$)

GraphPlan: states



Mutexes

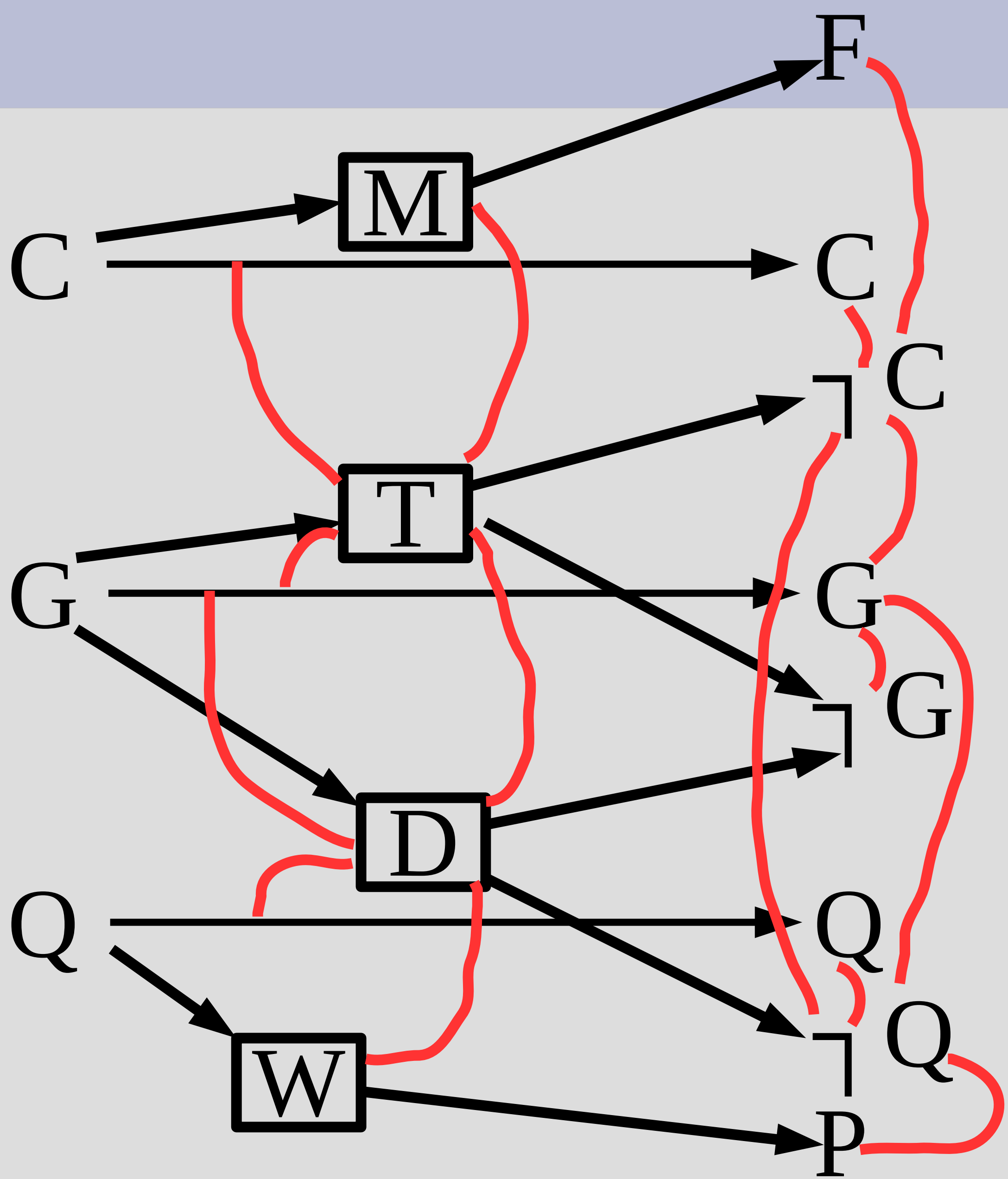


Possible state pairs:

F, C
~~F, ¬C~~
F, G
F, ¬G
F, Q
F, ¬Q
F, P
~~C, ¬C~~
C, G
C, ¬G

C, Q
C, ¬Q
C, P
~~¬C, G~~
¬C, ¬G
¬C, Q
~~¬C, ¬Q~~
¬C, P
... (more)

Mutexes

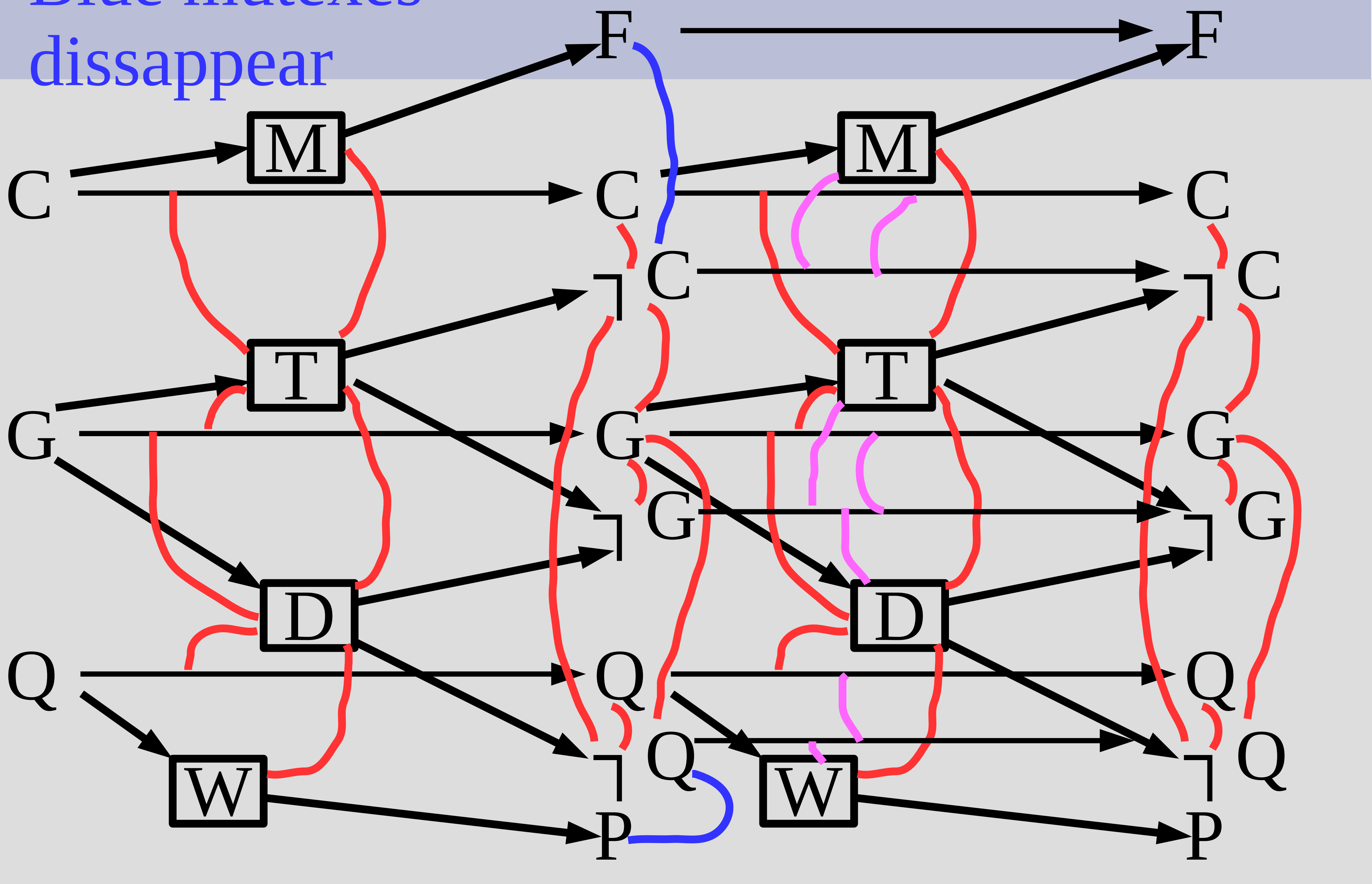


Make one more level here!

Mutexes

Blue mutexes dissappear

Pink = new mutex



GraphPlan as heuristic

GraphPlan is optimistic, so if any pair of goal states are in mutex, the goal is impossible

3 basic ways to use GraphPlan as heuristic:

(1) Maximum level of all goals
(2) Sum of level of all goals (not admissible)
(3) Level where no pair of goals is in mutex

(1) and (2) do not require any mutexes, but are less accurate (quick 'n' dirty)

GraphPlan as heuristic

For heuristics (1) and (2), we relax as such:

1. Multiple actions per step, so can only take fewer steps to reach same result
2. Never remove any states, so the number of possible states only increases

This is a valid simplification of the problem, but it is often too simplistic directly

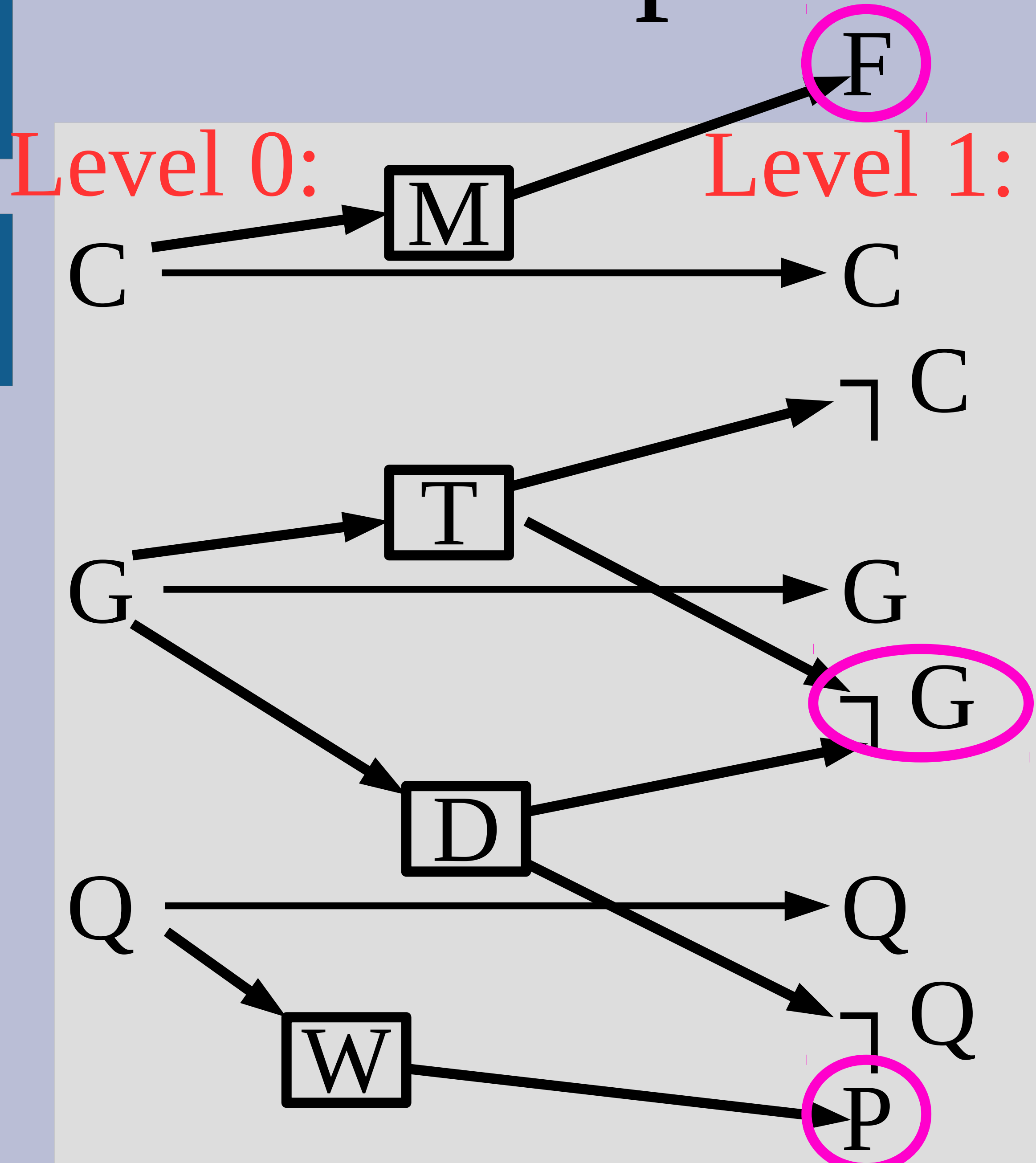
GraphPlan as heuristic

Heuristic (1) directly uses this relaxation and finds the first time when all 3 goals appear at a state level

(2) tries to sum the levels of each individual first appearance, which is not admissible (but works well if they are independent parts)

Our problem: goal={Food, ¬ Garbage, Present}
First appearance: F=1, ¬ G=1, P=1

GraphPlan: states



Heuristic (1):
Max(1,1,1) = 1

Heuristic (2):
1+1+1=3

GraphPlan as heuristic

Often the problem is too trivial with just those two simplifications

So we add in mutexes to keep track of invalid pairs of states/actions

This is still a simplification, as only impossible state/action pairs in the original problem are in mutex in the relaxation

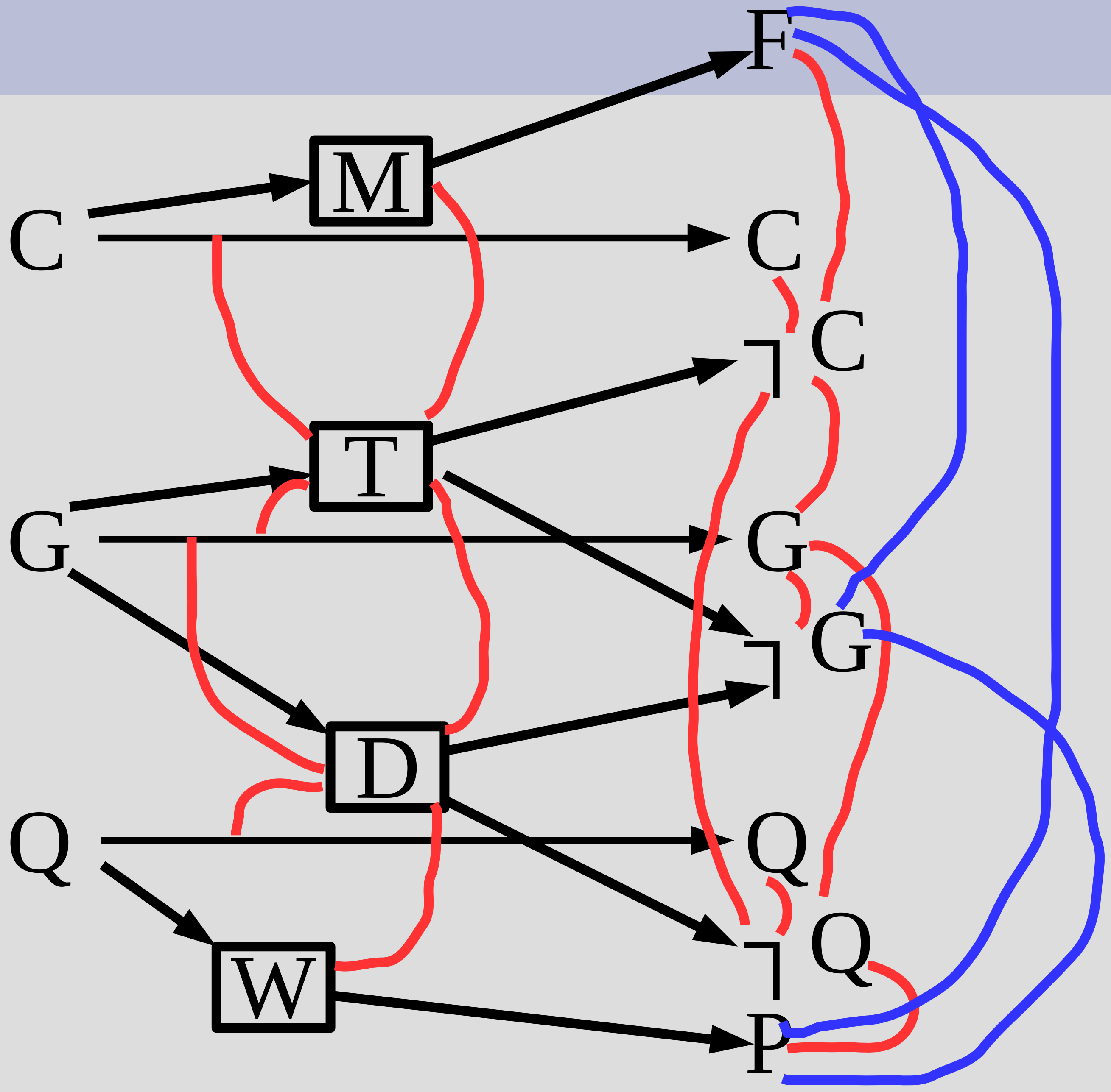
GraphPlan as heuristic

Heuristic (3) looks to find the first time none of the goal pairs are in mutex

For our problem, the goal states are:
(Food, ¬ Garbage, Present)

So all pairs that need to have no mutex:
(F, ¬ G), (F, P), (¬ G, P)

Mutexes



None of the pairs are in mutex at level 1

This is our heuristic estimate

Finding a solution

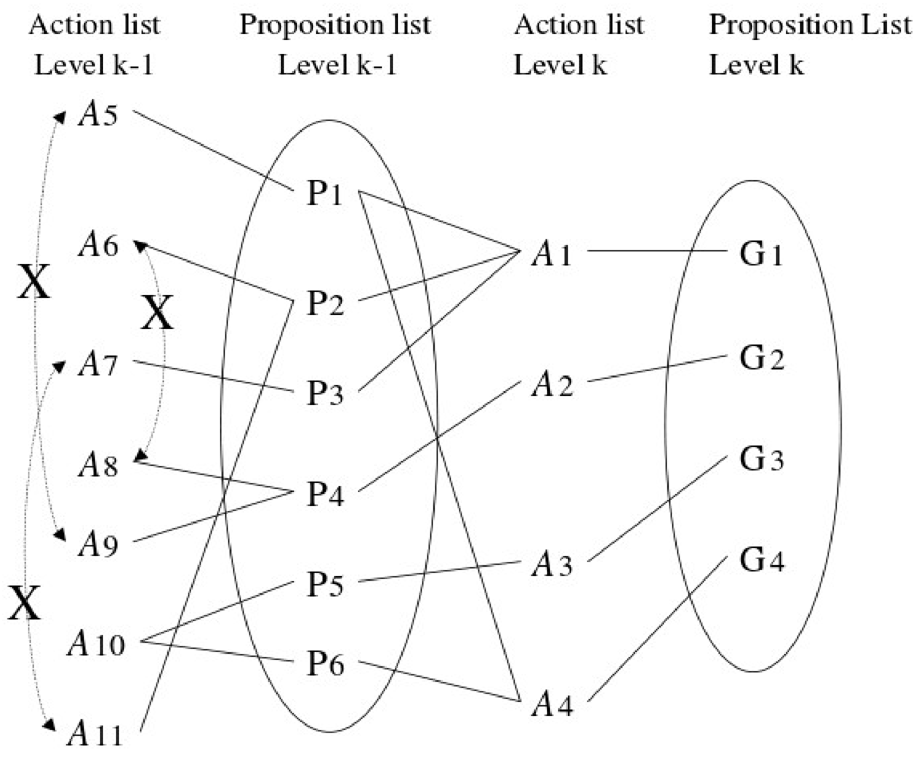
GraphPlan can also be used to find a solution:
(1) Converting to a CSP
(2) Backwards search

Both of these ways can be run once GraphPlan has all goal pairs not in mutex (or converges)

Additionally, you might need to extend it out a few more levels further to find a solution (as GraphPlan underestimates)

GraphPlan as CSP

Variables = states, Domains = actions out of
Constraints = mutexes & preconditions



(a) Planning Graph

Variables: $G_1, \cdots, G_4, P_1 \cdots P_6$

Domains: $G_1: \{A_1\}, G_2: \{A_2\} G_3: \{A_3\} G_4: \{A_4\}$
$P_1: \{A_5\} P_2: \{A_6, A_{11}\} P_3: \{A_7\} P_4: \{A_8, A_9\}$
$P_5: \{A_{10}\} P_6: \{A_{10}\}$

Constraints (normal): $P_1 = A_5 \Rightarrow P_4 \neq A_9$
$P_2 = A_6 \Rightarrow P_4 \neq A_8$
$P_2 = A_{11} \Rightarrow P_3 \neq A_7$

Constraints (Activity): $G_1 = A_1 \Rightarrow Active\{P_1, P_2, P_3\}$
$G_2 = A_2 \Rightarrow Active\{P_4\}$
$G_3 = A_3 \Rightarrow Active\{P_5\}$
$G_4 = A_4 \Rightarrow Active\{P_1, P_6\}$

Init State: $Active\{G_1, G_2, G_3, G_4\}$

(b) DCSP

from Do & Kambhampati

Finding a solution

For backward search, attempt to find arrows back to the initial state(without conflict/mutex)

This backwards search is similar to backward chaining in first-order logic (depth first search)

If this fails to find a solution, mark this level and all the goals not satisfied as: (level, goals)

(level, goals) stops changing, no solution

Graph Plan

Remember this from last time...

Initial: $\neg Money(me) \wedge \neg Smart(me) \wedge \neg Debt(me)$

Goal: $Money(me) \wedge Smart(me) \wedge \neg Debt(me)$

Action($School(x)$,
Precondition: ,
Effect: $Debt(x) \wedge Smart(x)$)

Action($Job(x)$,
Precondition: ,
Effect: $Money(x) \wedge \neg Smart(x)$)

Action($Pay(x)$,
Precondition: $Money(x)$,
Effect: $\neg Money(x) \wedge \neg Debt(x)$)

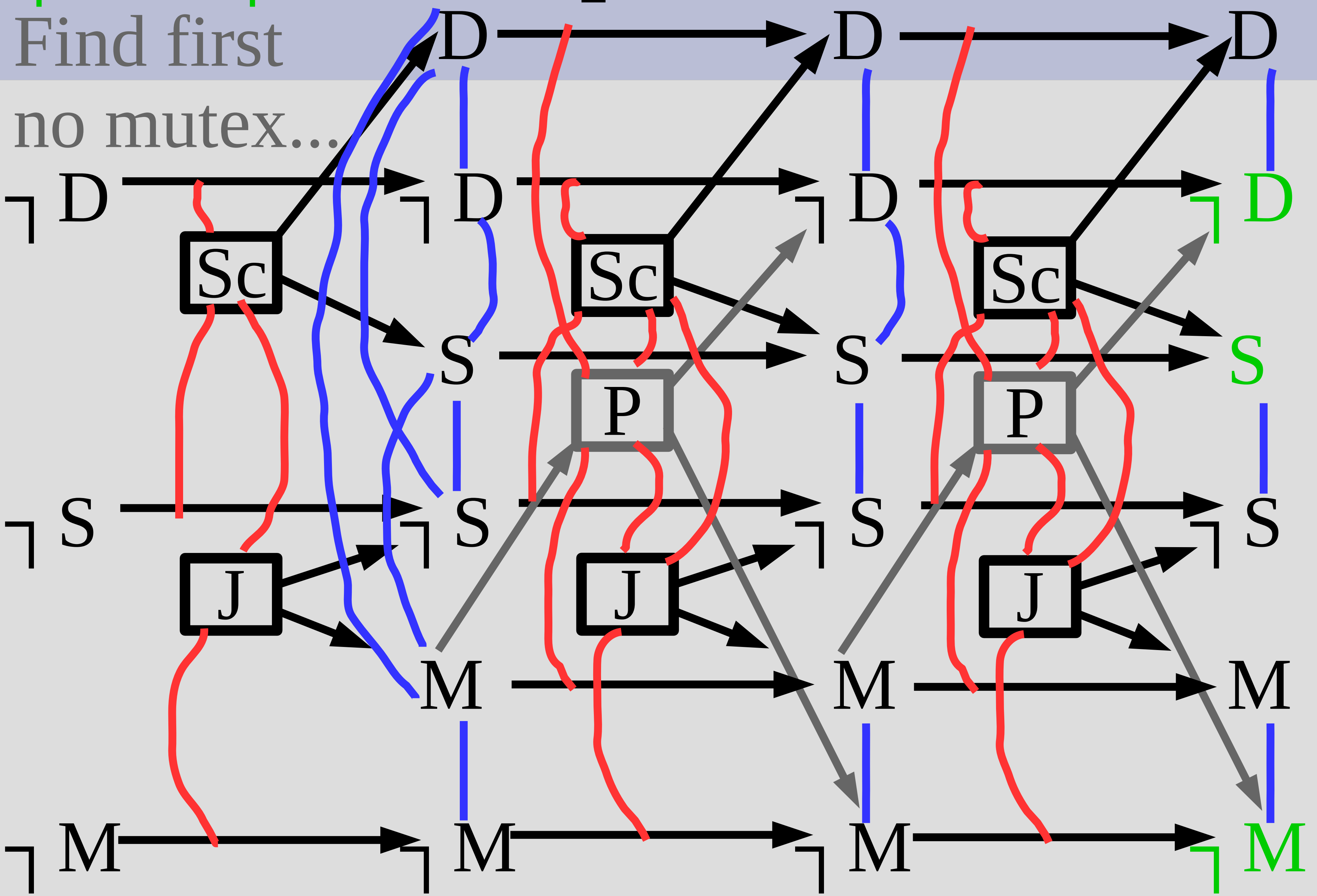
Graph Plan

Ask:

¬ D^S^¬ M

Find first

no mutex...



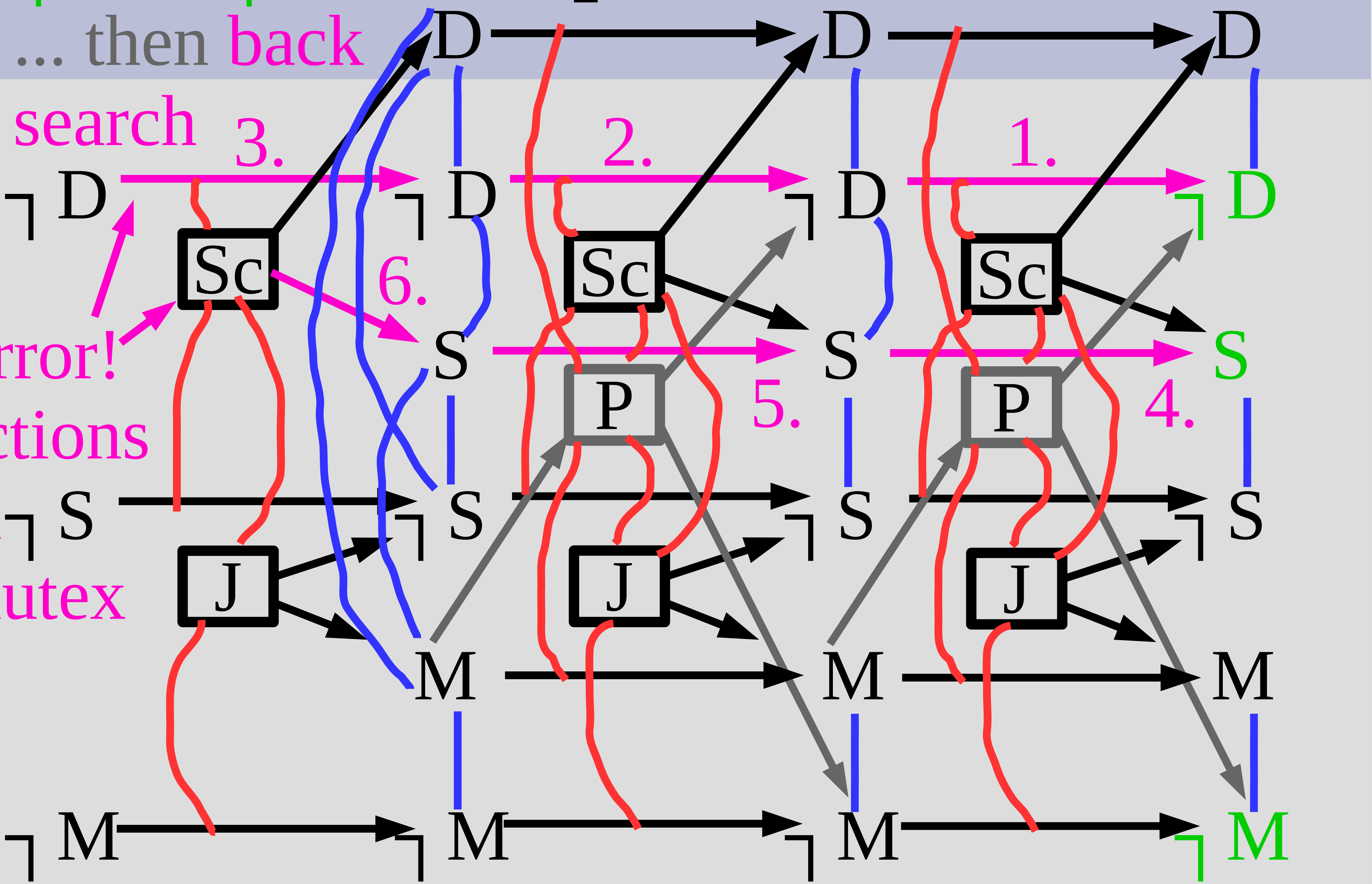
Graph Plan

Ask:

¬ D^S^¬ M

... then back search

Error! actions in mutex



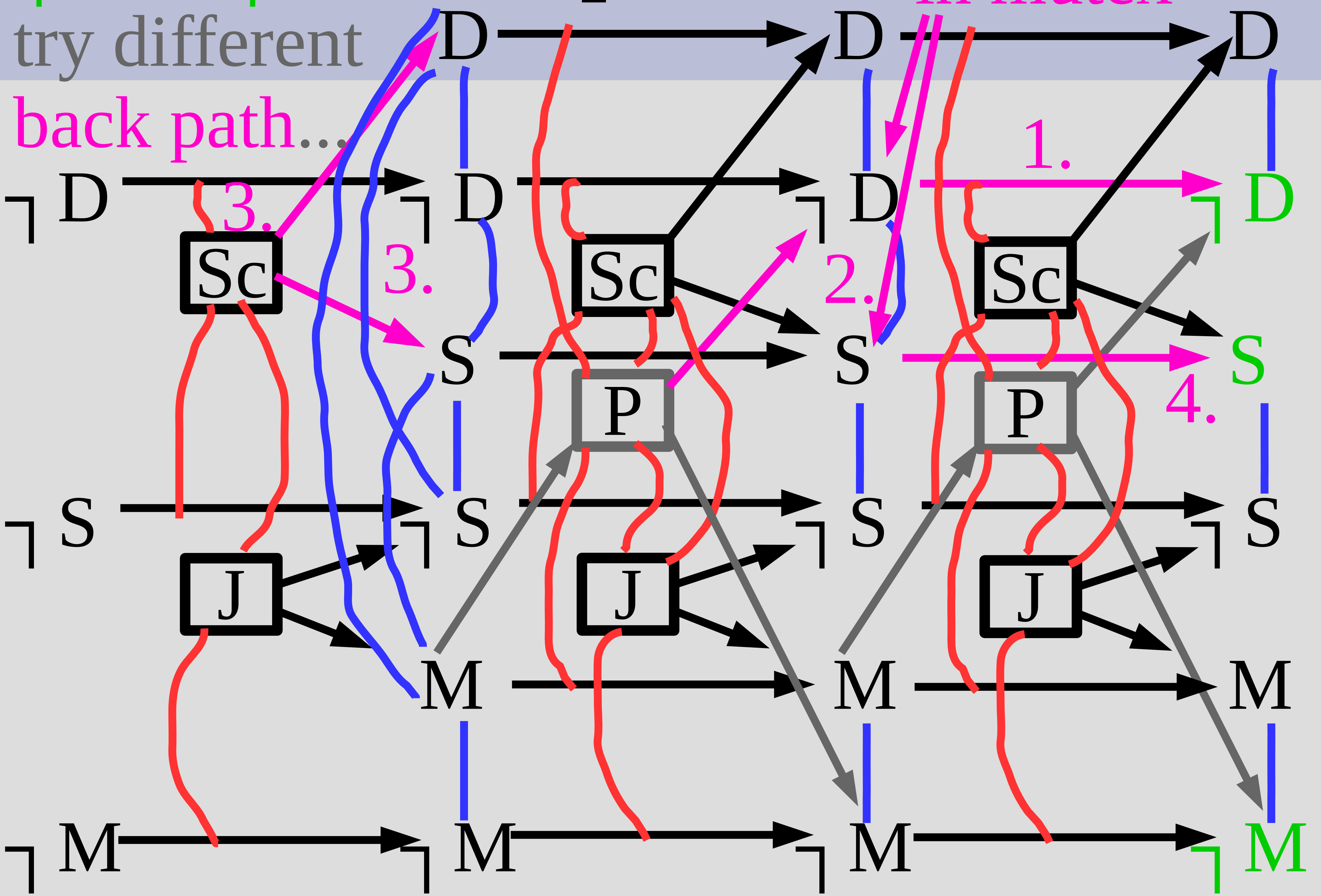
Graph Plan

Ask:

¬ D^S^¬ M

try different

back path...



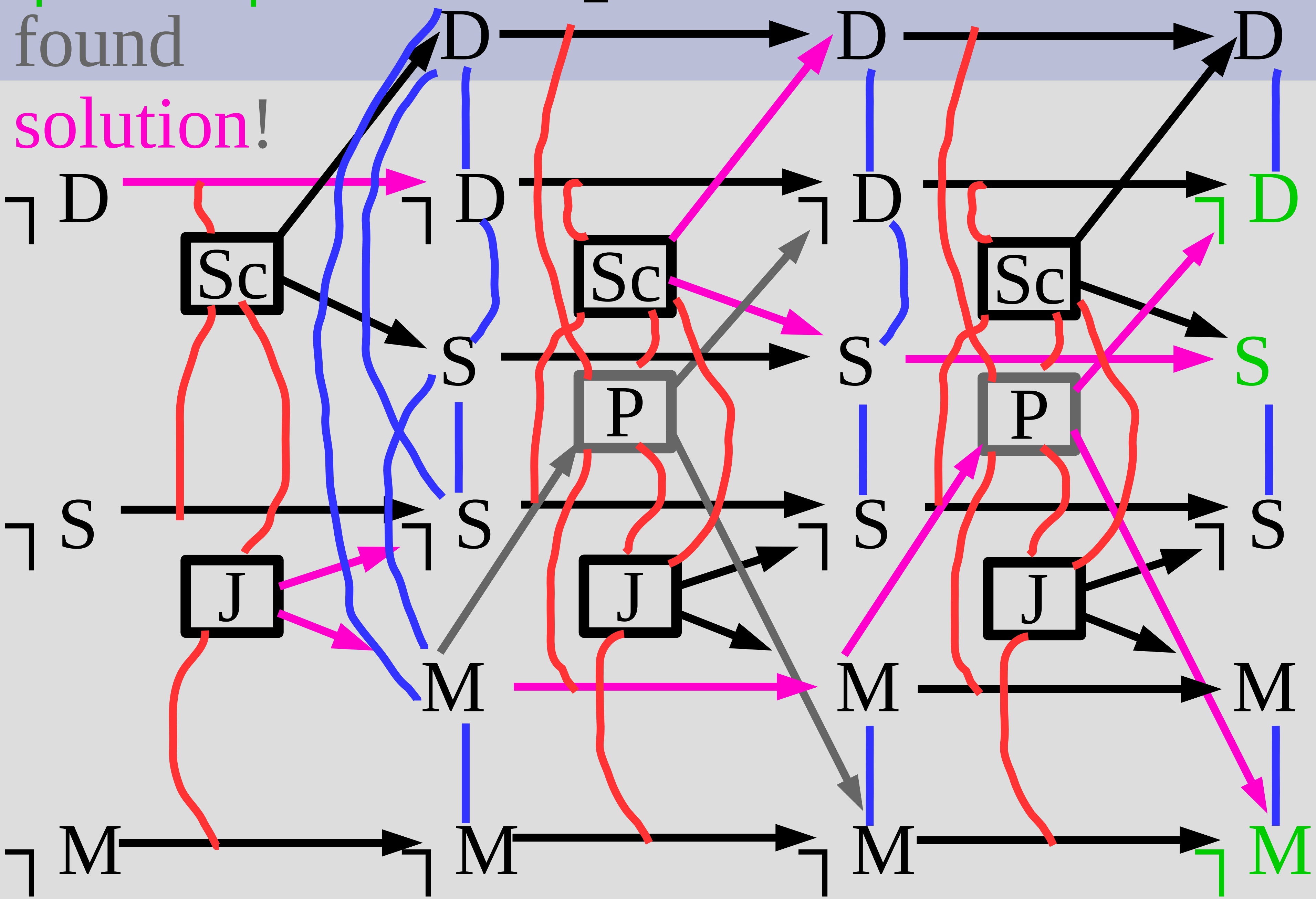
Ask:

¬ D^S^¬ M

found

solution!

Graph Plan



Finding a solution

Formally, the algorithm is:

```
graph = initial
noGoods = empty table (hash)
for level = 0 to infinity
    if all goal pairs not in mutex
        solution = DFS with noGoods
        if success, return paths
    if graph & noGoods converged, return fail
    graaph = expand graph
```

Mutexes

You try it!