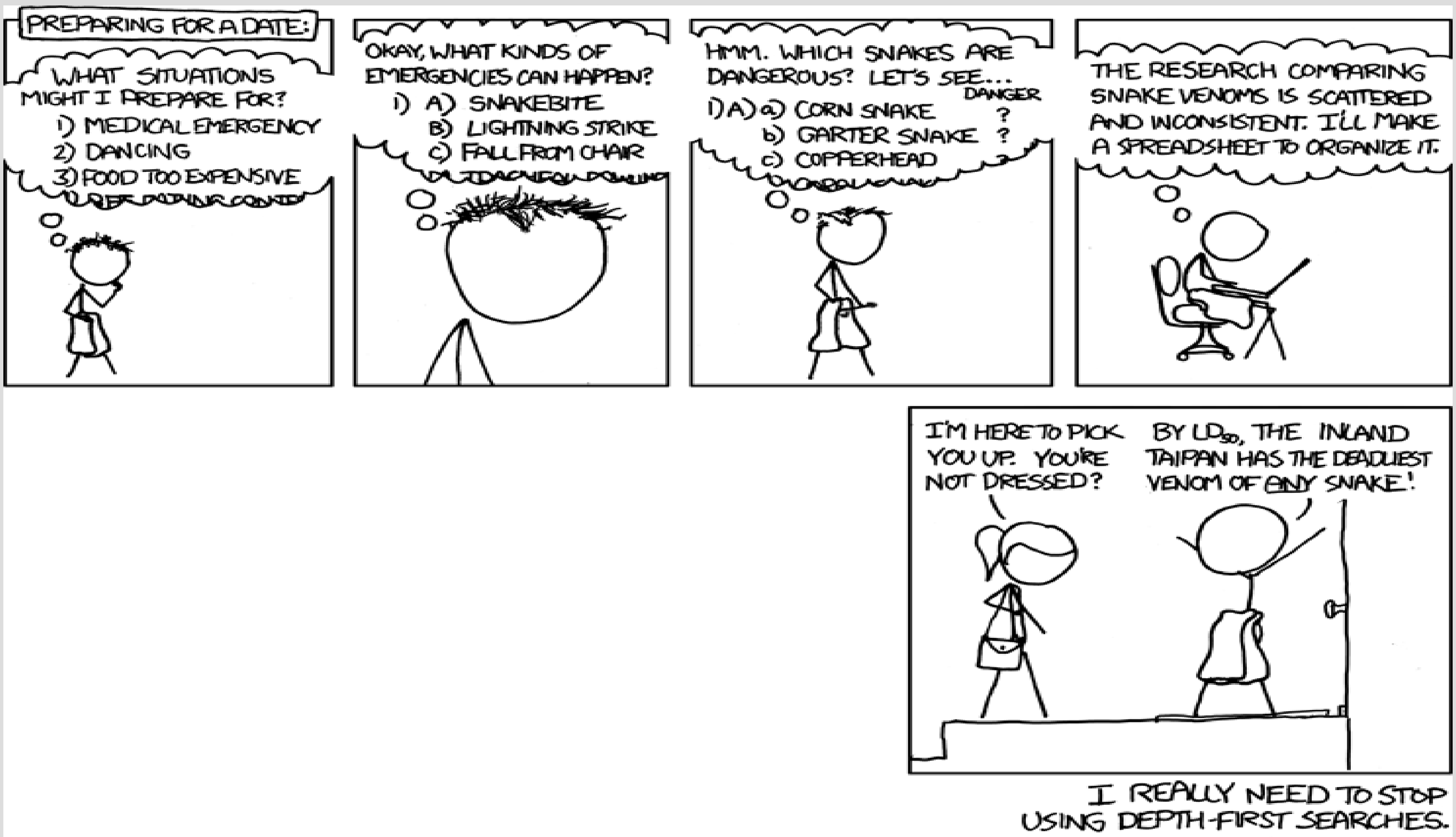
Uninformed Search (Ch. 3-3.4)



Announcements

Homework assigned

Due Sunday night (11:55pm)

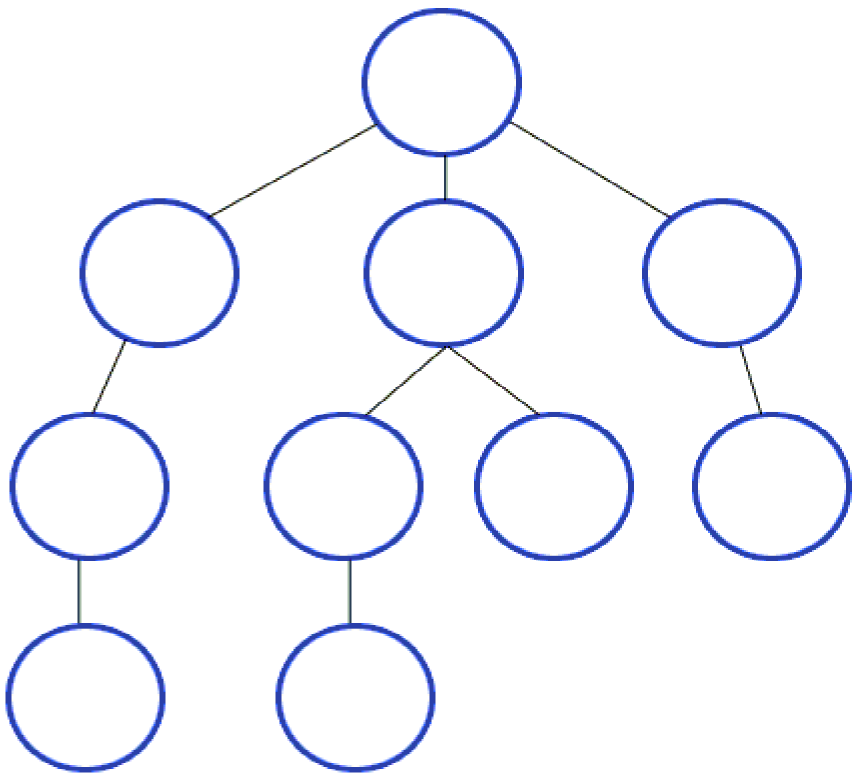
Search algorithm

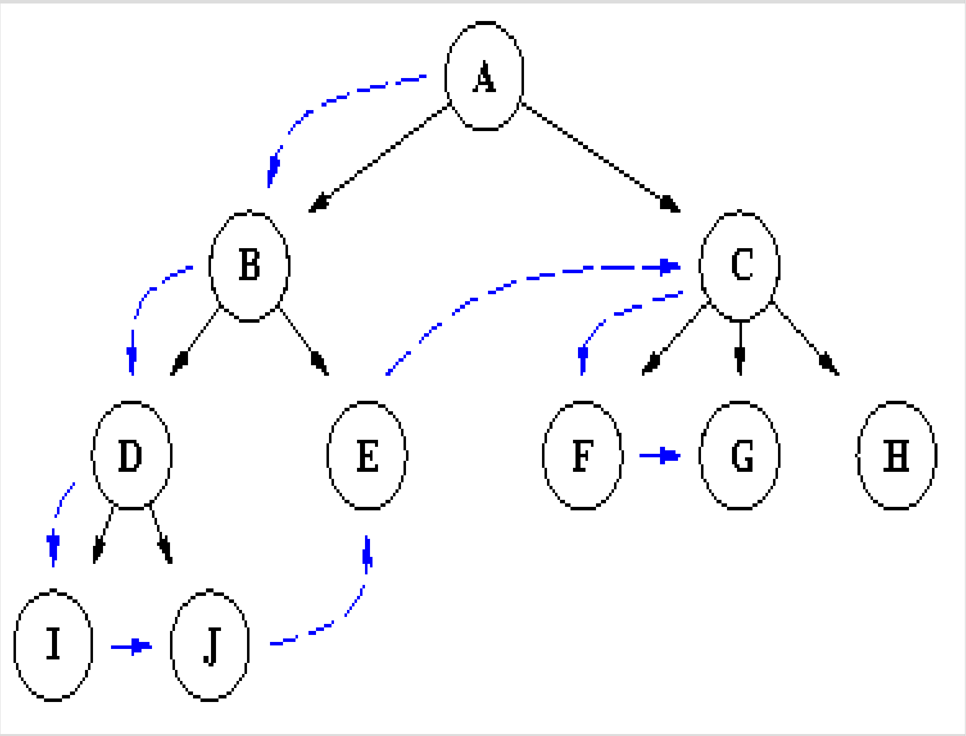
Generic search algorithm:

```
function tree-search(root-node)
  fringe ← successors(root-node)
  explored ← empty
  while ( notempty(fringe) )
       {node ← remove-first(fringe)
          state ← state(node)
          if goal-test(state) return solution(node)
          explored ← insert(node,explored)
          fringe ← insert-all(successors(node),fringe,  if node not in explored)
       }
  return failure
end tree-search
```

Depth first search

DFS is same as BFS except with a FILO (or LIFO) instead of a FIFO queue





Depth first search

Metrics:

1. Might not terminate (not correct) (e.g. in vacuum world, if first expand is action L)
2. Non-optimal (just... no)
3. Time complexity = $O(b^d)$
4. Space complexity = $O(b*d)$

Only way this is better than BFS is the space complexity...

Depth limited search

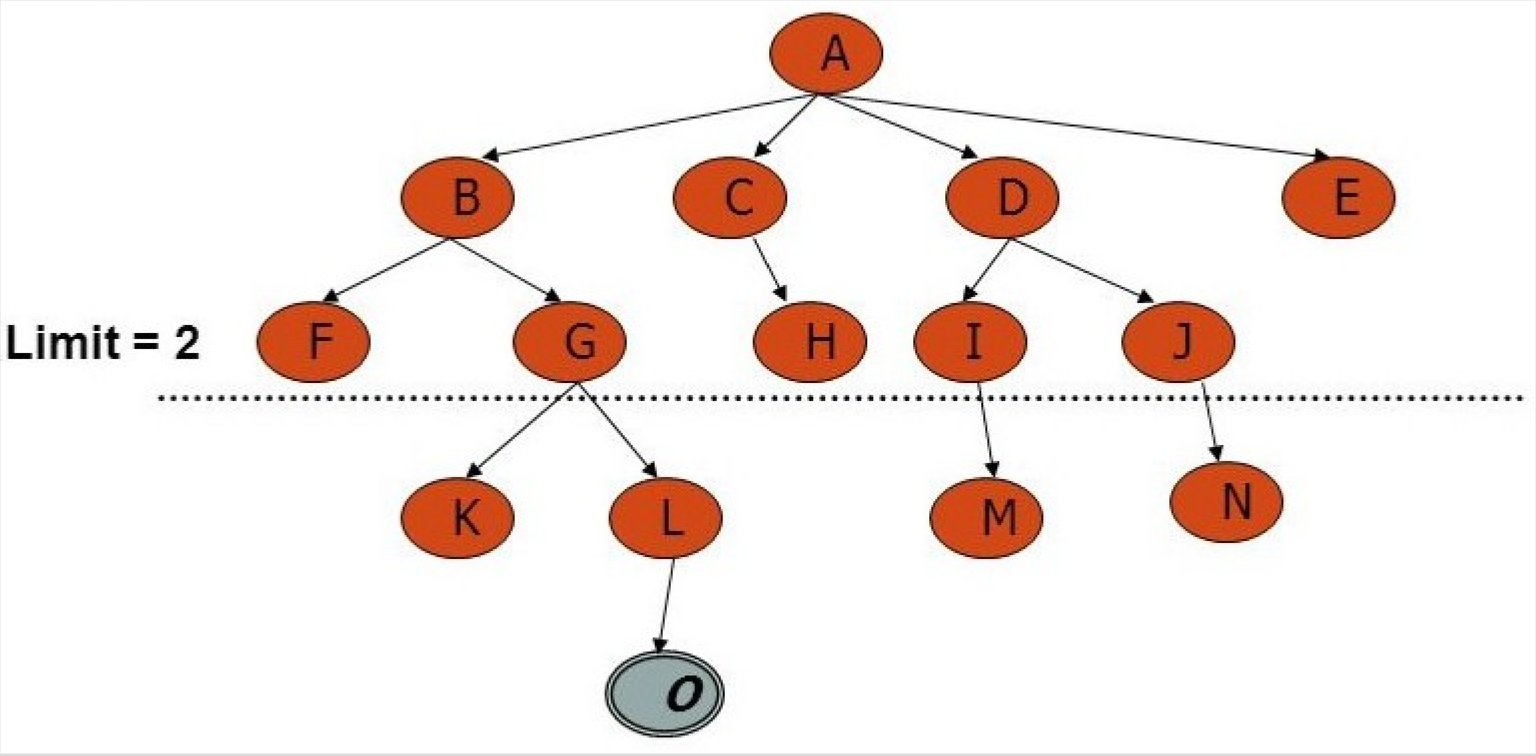
DFS by itself is not great, but it has two (very) useful modifications

Depth limited search runs normal DFS, but if it is at a specified depth limit, you cannot have children (i.e. take another action)

Typically with a little more knowledge, you can create a reasonable limit and makes the algorithm correct

Depth limited search

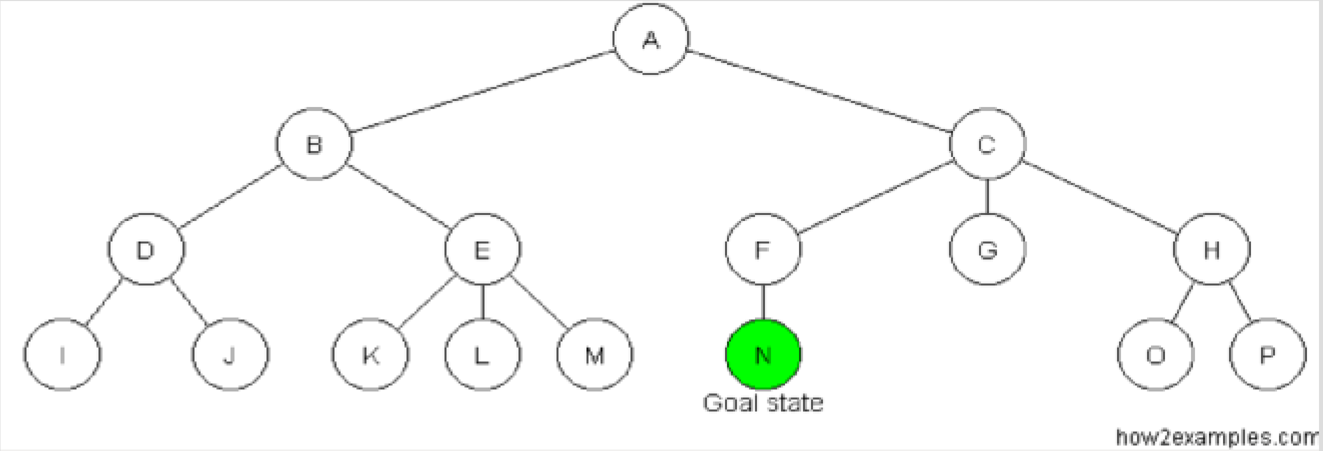
However, if you pick the depth limit before d, you will not find a solution (not correct, but will terminate)



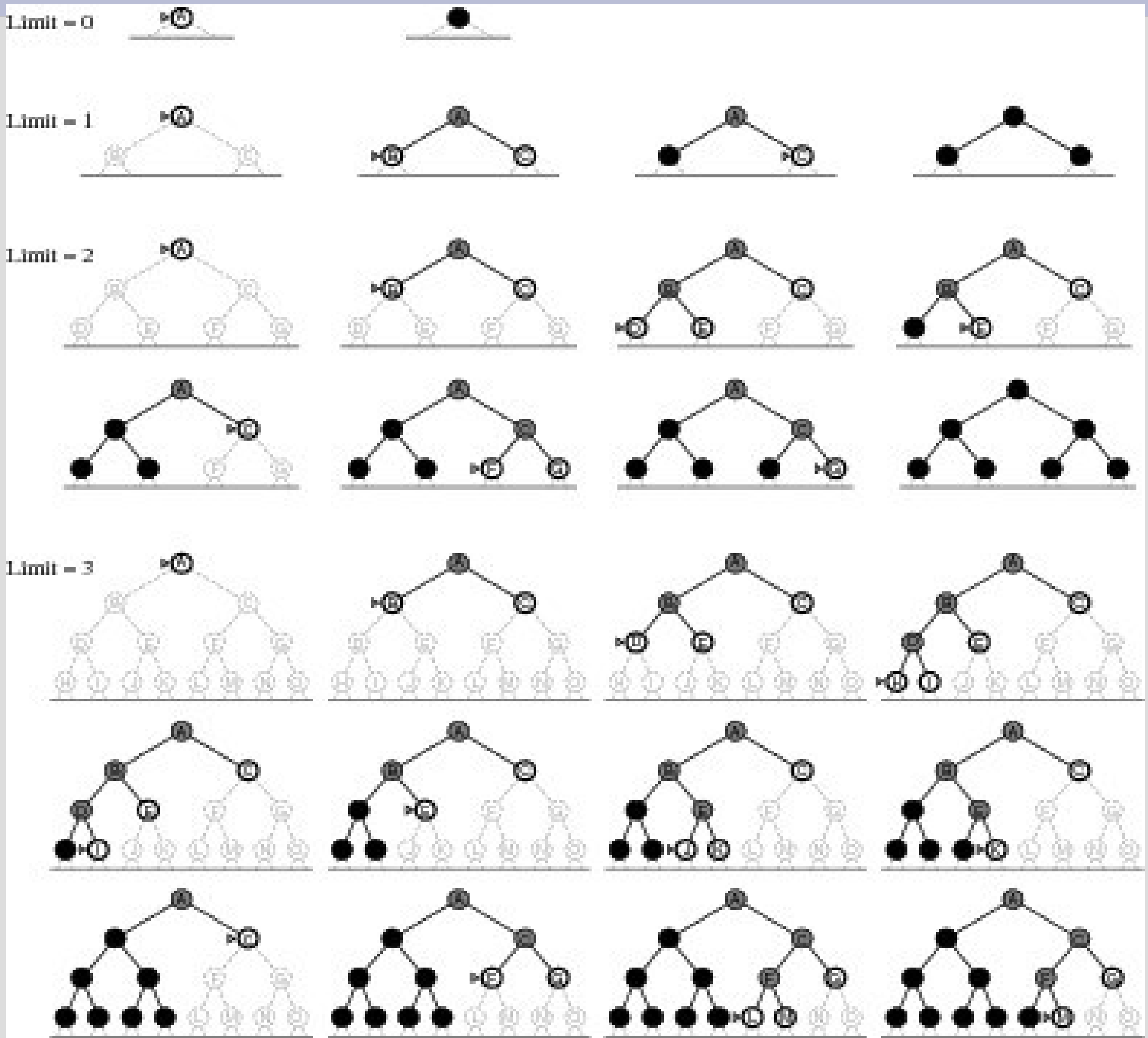
Iterative deepening DFS

Probably the most useful uninformed search is iterative deepening DFS

This search performs depth limited search with maximum depth 1, then maximum depth 2, then 3... until it finds a solution



Iterative deepening DFS



Iterative deepening DFS

The first few states do get re-checked multiple times in IDS, however it is not too many

When you find the solution at depth d, depth 1 is expanded d times (at most b of them)

The second depth are expanded d-1 times (at most b^2 of them)

Thus $d \cdot b + (d-1) \cdot b^2 + ... + 1 \cdot b^d = O(b^d)$

Iterative deepening DFS

Metrics:
1. Complete
2. Non-optimal (unless uniform cost)
3. $O(b^d)$
4. O(bd)

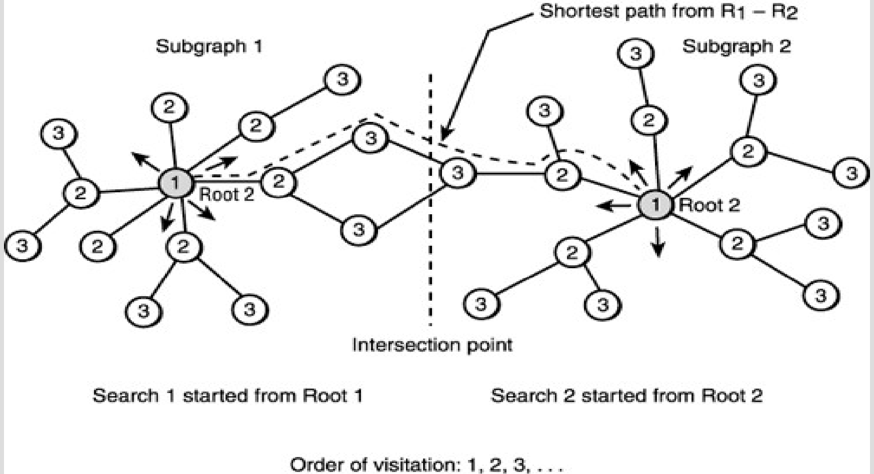
Thus IDS is better in every way than BFS (asymptotically)

Best uninformed we will talk about

Bidirectional search

Bidirectional search starts from both the goal and start (using BFS) until the trees meet

This is better as $2*(b^{d/2}) < b^d$
(the space is much worse than IDS, so only applicable to small problems)



Summary of algorithms
Fig. 3.21, p. 91

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening DLS	Bidirectional (if applicable)
Complete?	Yes[a]	Yes[a,b]	No	No	Yes[a]	Yes[a,d]
Time	$O(b^d)$	$O(b^{\lfloor 1+C^*/\varepsilon \rfloor})$	$O(b^m)$	$O(b^l)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{\lfloor 1+C^*/\varepsilon \rfloor})$	$O(bm)$	$O(bl)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes[c]	Yes	No	No	Yes[c]	Yes[c,d]

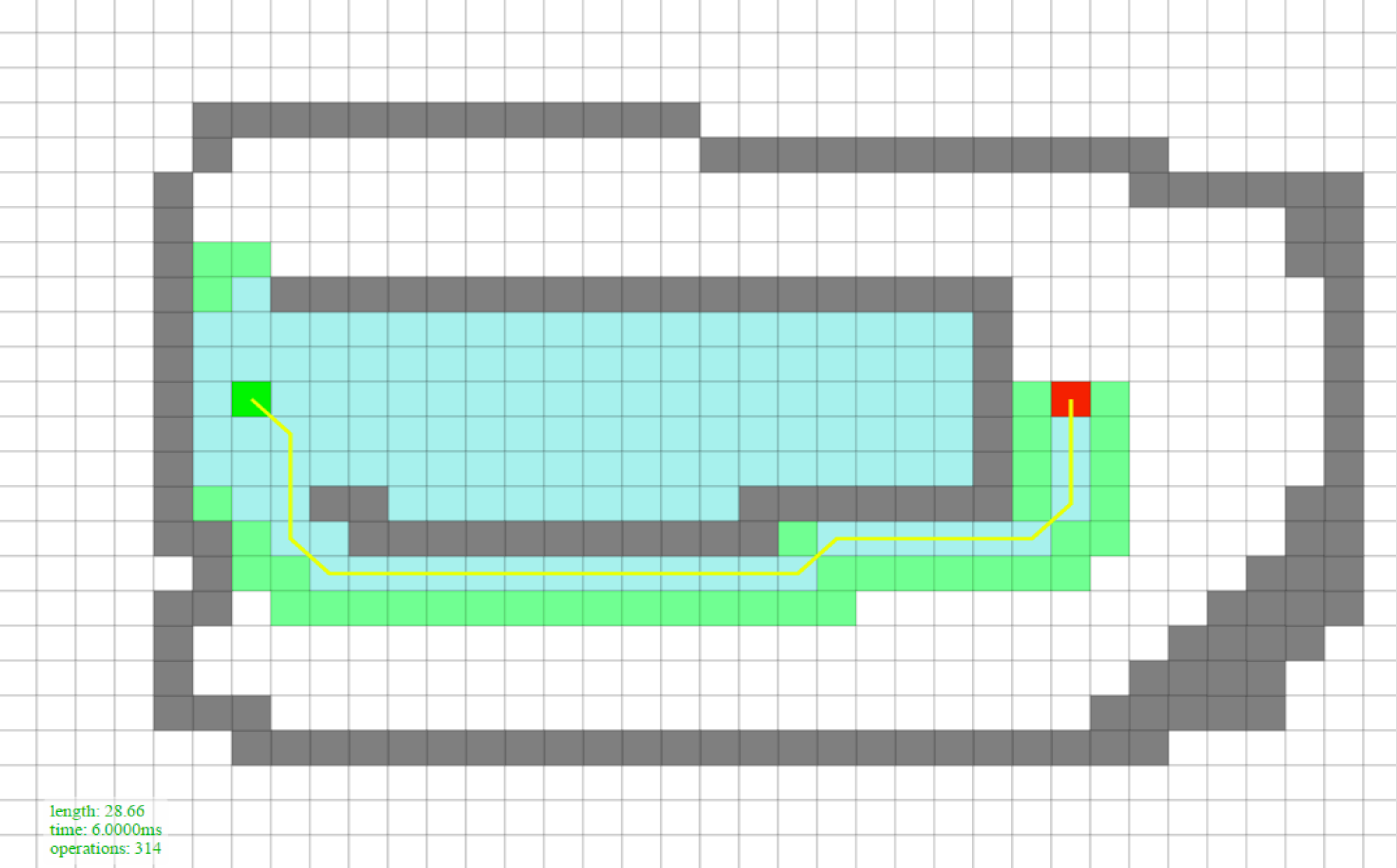
l = depth limit

Generally the preferred uninformed search strategy

There are a number of footnotes, caveats, and assumptions.
See Fig. 3.21, p. 91.

[a] complete if b is finite
[b] complete if step costs $\geq \varepsilon > 0$
[c] optimal if step costs are all identical
(also if path cost non-decreasing function of depth only)
[d] if both directions use breadth-first search
(also if both directions use uniform-cost search with step costs $\geq \varepsilon > 0$)

Informed Search (Ch. 3.5-3.6)



Informed search

In uninformed search, we only had the node information (parent, children, cost of actions)

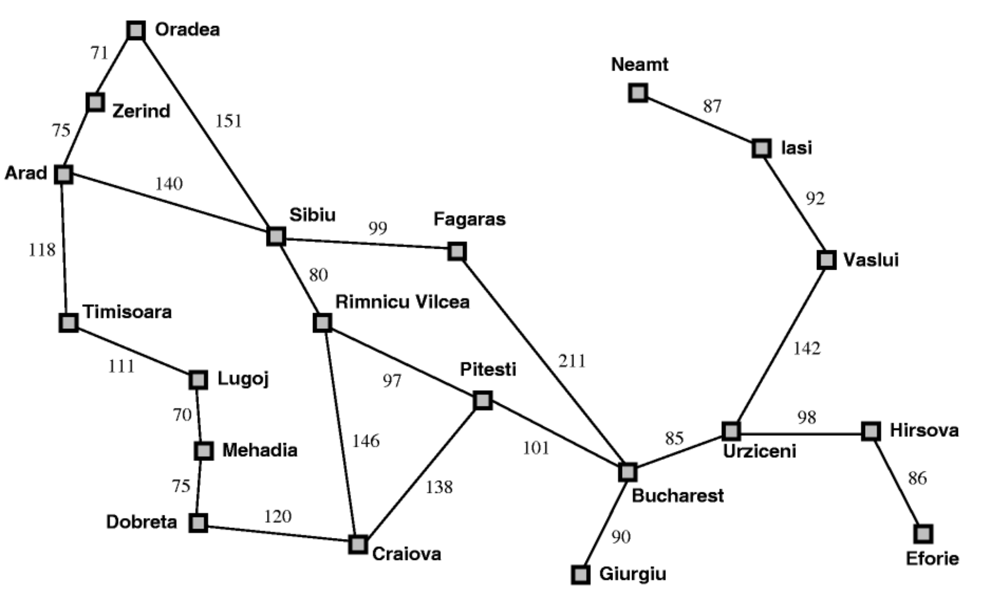
Now we will assume there is some additional information, we will call a heuristic that estimates the distance to the goal

Previously, we had no idea how close we were to goal, simply how far we had gone already

Greedy best-first search

To introduce heuristics, let us look at the tree version of <u>greedy best-first search</u>

This search will simply repeatedly select the child with the lowest heuristic(cost to goal est.)



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy best-first search

This finds the path: Arad -> Sibiu -> Fagaras -> Bucharest

However, this greedy approach is not optimal, as that is the path: Arad -> Sibiu -> Rimmicu Vilcea -> Pitesti -> Bucharest

In fact, it is not guaranteed to converge (if a path reaches a dead-end, it will loop infinitely)

A*

We can combine the distance traveled and the estimate to the goal, which is called A* (a star)

The method goes: (red is for "graphs")

initialize explored={}, fringe={[start,f(start)]}

1. Choose C = argmin(f-cost) in fringe
2. Add or update C's children to fringe, with associated f-value, remove C from fringe
3. Add C to explored
4. Repeat 1. until C == goal or fringe empty

A*

f(node) = g(node) + h(node) ← heuristic (estimate to-goal distance)

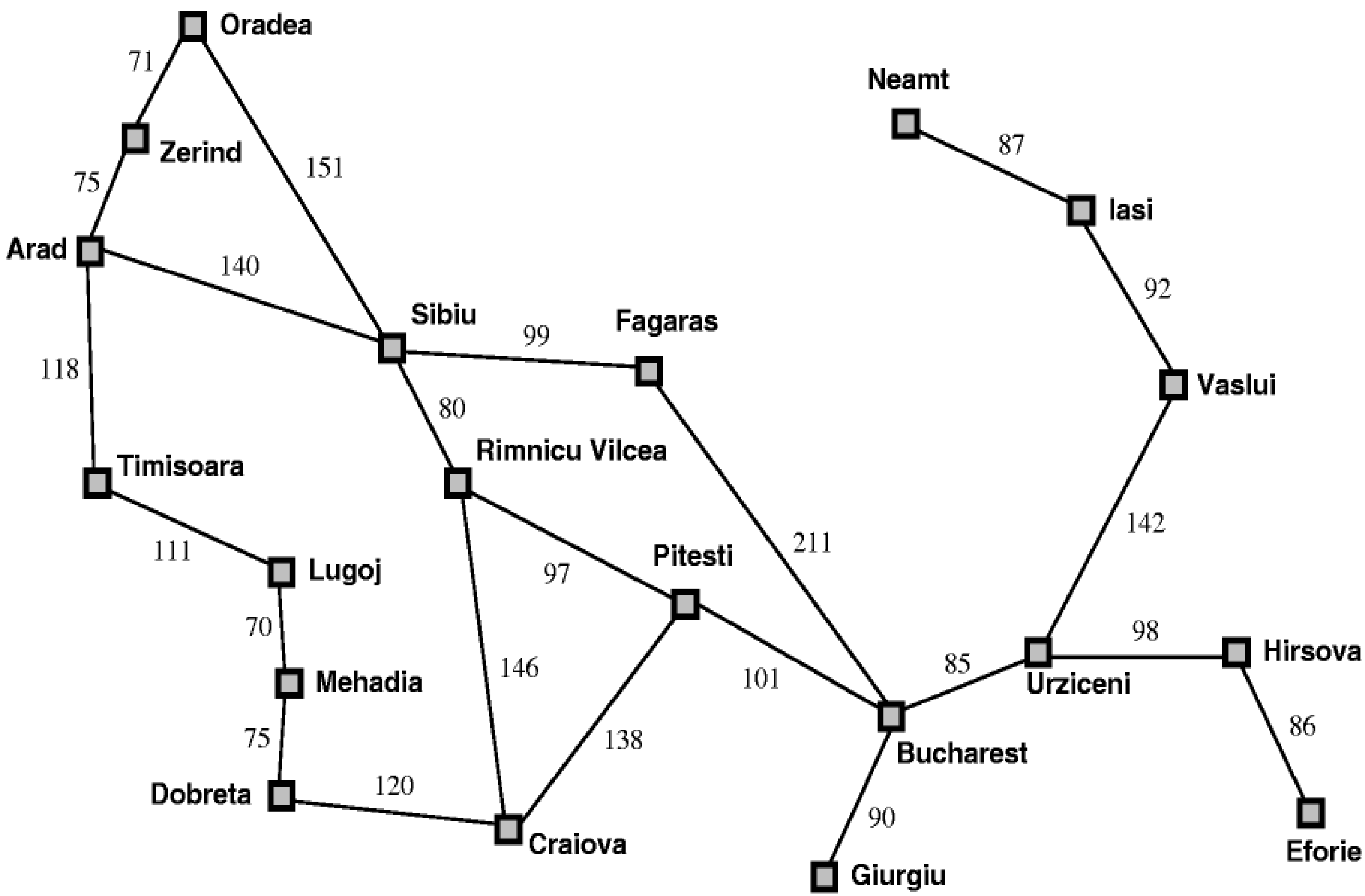
total cost estimate

distance gone (traveled) so far

We will talk more about what heuristics are good or should be used later

Priority queues can be used to efficiently store
and insert states and their f-values into the

A*



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A*

Step: Fringe (argmin)

0: [Arad, 366]

1: [Zerind, 75+374],[Sibu, 140+253],[Timisoara, 118+329]

1: [Zerind, 449], [Sibu, 393], [Timisoara, 447]

2: [Fagaras, 140+99+176], [Rimmicu Vilcea, 140+80+193], [Zerind, 449], [Timisoara, 447]

2: [Fagaras, 415], [Rimmicu Vilcea, 413], [Zerind, 449], [Timisoara, 447]

3: [Craiova, 140+80+146+160], [Pitesti, 140+80+97+100], [Fagaras, 415], [Zerind, 449], [Timisoara, 447]

3: [Craiova, 526], [Pitesti, 417], [Fagaras, 415], [Zerind, 449], [Timisoara, 447]

4: ... on next slide

A*

4: [Bucharest, 140+99+211+0], [Craiova, 526], [Pitesti, 417], [Zerind, 449], [Timisoara, 447]

4: [Bucharest, 450], [Craiova, 526], [Pitesti, 417], [Zerind, 449], [Timisoara, 447]

5: [Craiova from Pitesti, 140+80+97+138+160], [Bucharest from Pitesti, 140+80+97+101+0], [Bucharest from Fagaras, 450], [Timisoara, 447], [Craiova from Rimmicu Vilcea, 526], [Zerind, 449]

5: [Craiova from Pitesti, 615], [Bucharest from Pitesti, 418], [Bucharest from Fagaras, 450], [Timisoara, 447], [Craiova from Rimmicu Vilcea, 526], [Zerind, 449]

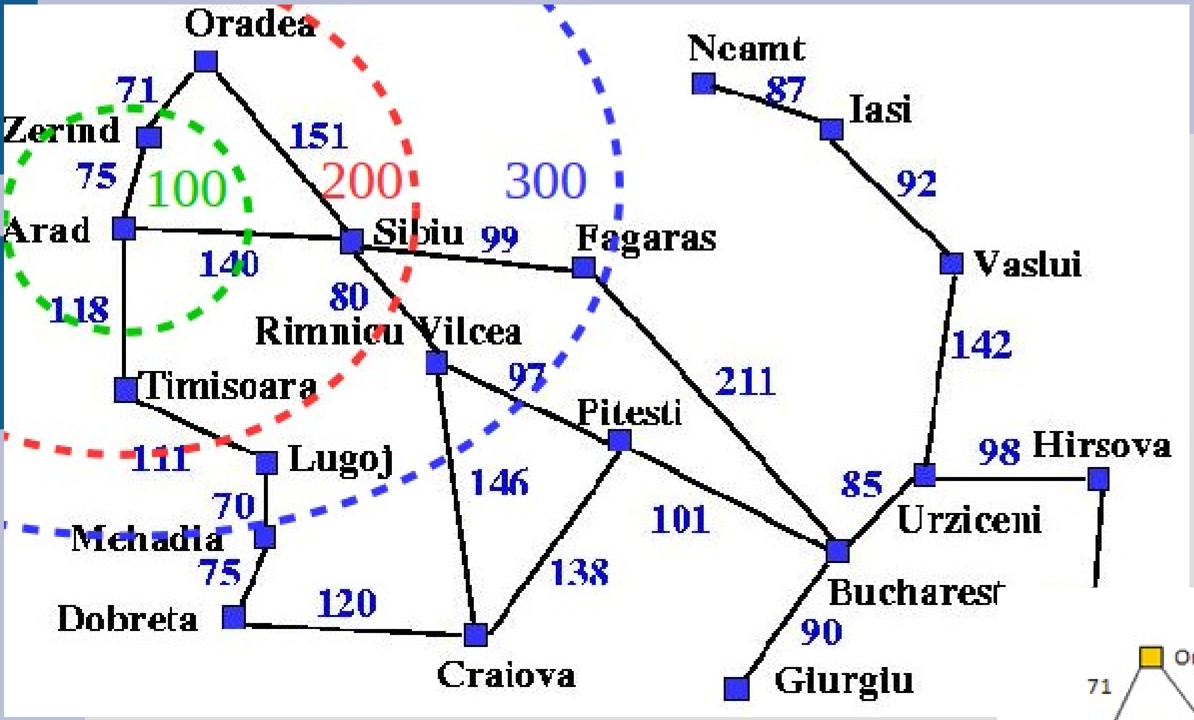
A*

You can choose multiple heuristics (more later) but good ones skew the search to the goal

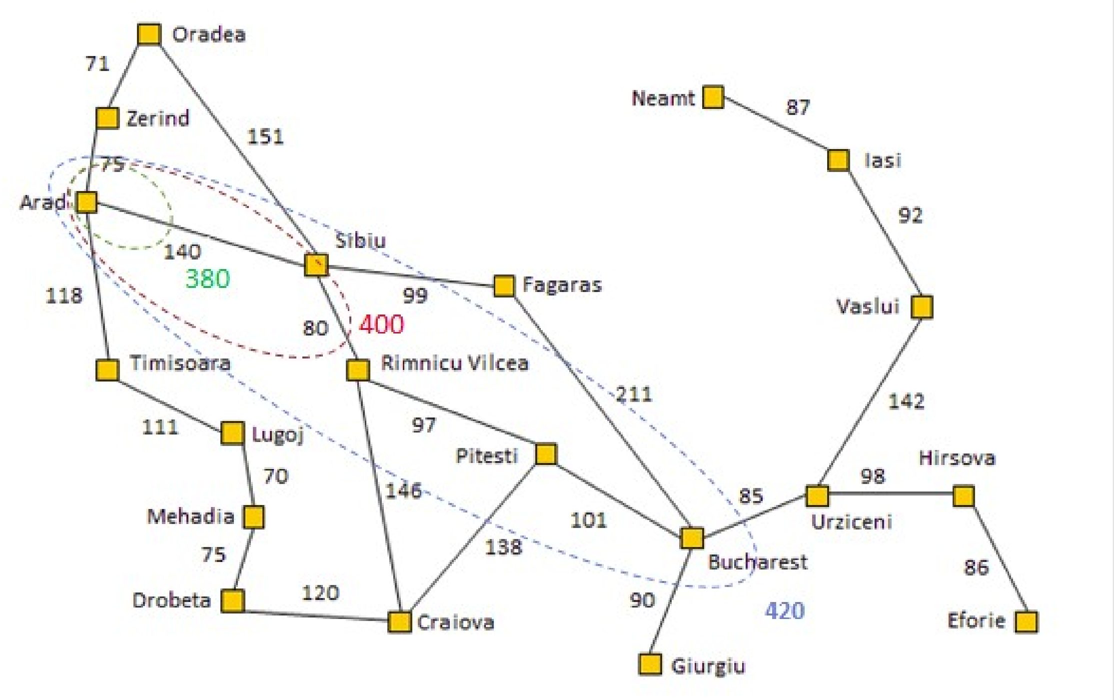
You can think circles based on f-cost:
- if h(node) = 0, f-cost are circles
- if h(node) = very good, f-cost long and thin ellipse

This can also be though of as topographical maps (in a sense)

A*



h(node) = 0
(bad heuristic,
no
goal guidance)



h(node) =
straight
line distance

A*

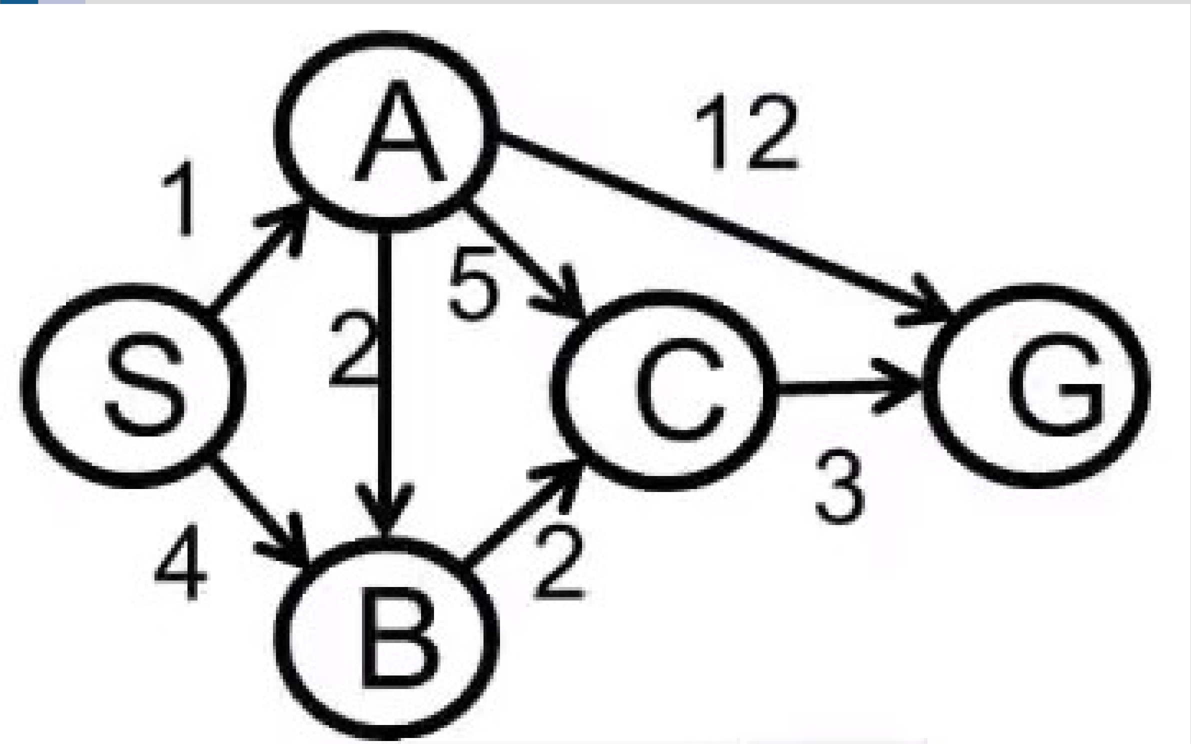
Good heuristics can remove "bad" sections of the search space that will not be on any optimal solution (called pruning)

A* is optimal and in fact, no optimal algorithm could expand less nodes (optimally efficient)

However, the time and memory cost is still exponential (memory tighter constraint)

A*

You do it!



State	H
S	7
A	6
B	2
C	1
G	0

Arrows show children (easier for you)

(see: https://www.youtube.com/watch?v=sAoBeujec74)

Iterative deepening A*

You can combine iterative deepening with A*

Idea:

1. Run DFS in IDS, but instead of using depth as cutoff, use f-cost
2. If search fails to find goal, increase f-cost to next smallest seen value

Pros: Efficient on memory

Cons: Large (LARGE) amount of re-searching

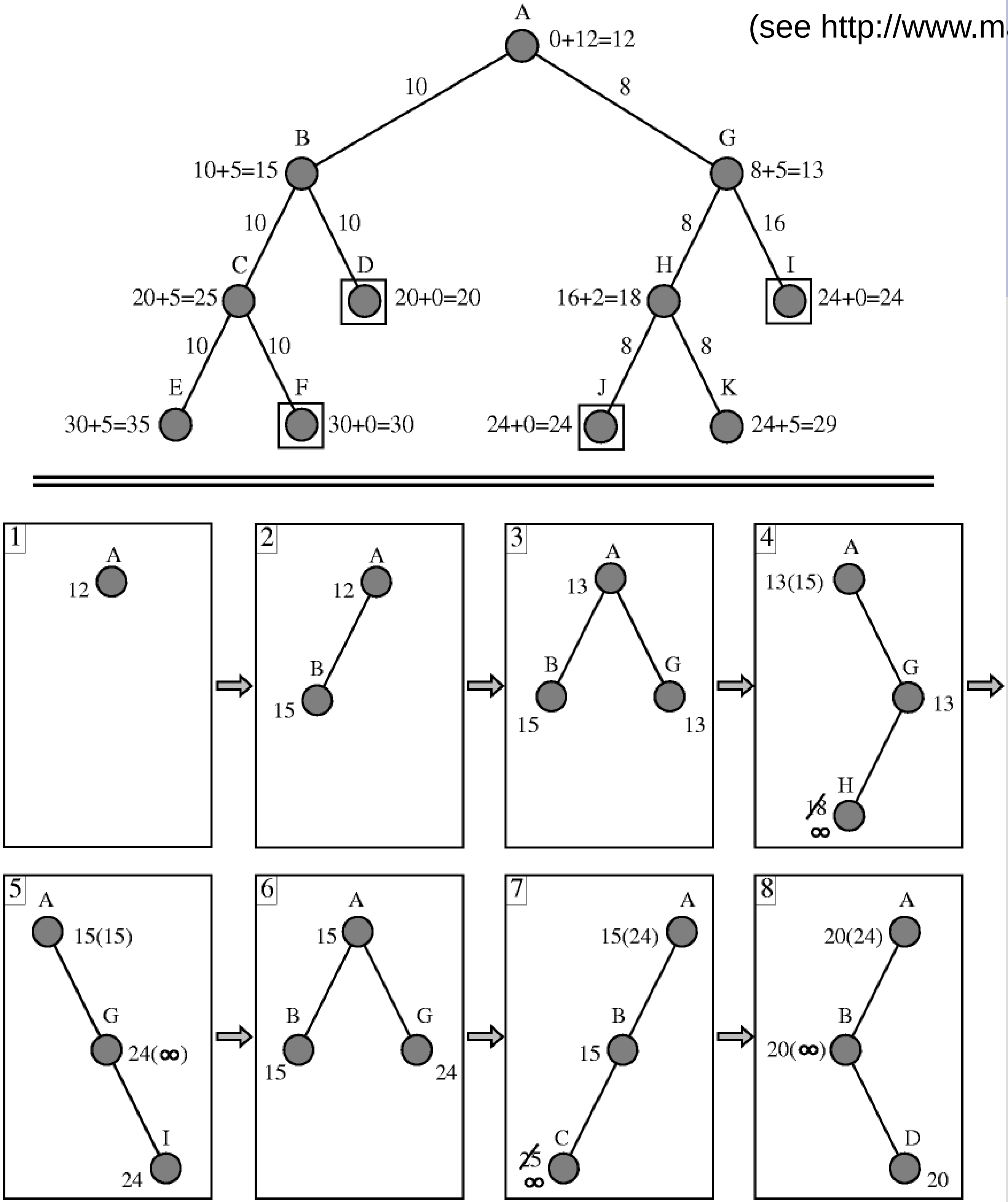
SMA*

One fairly straight-forward modification to A* is simplified memory-bounded A* (SMA*)

Idea:

1. Run A* normally until out of memory
2. Let C = argmax(f-cost) in fringe
3. Collapse C's parent into the min cost of all its children (remove these children from fringe, adding parent instead)
4. Goto 1.

(see http://www.massey.ac.nz/~mjjohnso/notes/59302/l04.ht



Here assume you
can only hold
at most 3
nodes
in memory

SMA*

SMA* is nice as it (like A*) find the optimal solution while keeping re-searching low (given your memory size)

IDA* only keeps a single number in memory, and thus re-searches many times (inefficient use of memory)

Typically there is some time to memory trade-off

Heuristics

However, for A* to be optimal the heuristic h(node) needs to be...

For trees: <u>admissible</u> which means:

  $h(node) \leq$ optimal path from h to goal

  (i.e. h(node) is an underestimate of cost)

For graphs: <u>consistent</u> which means:

  $h(node) \leq cost(node\ to\ child) + h(child)$

  (i.e. triangle inequality holds true)

  (i.e. along any path, f-cost increases)

Heuristics

In our example, the h(node) was the straight line distance from node to goal

This is an underestimate as phyiscal roads cannot be shorter than this
(it also satisfies the triangle inequality)

Thus this heuristic is admissible
(and consistent)

Heuristics

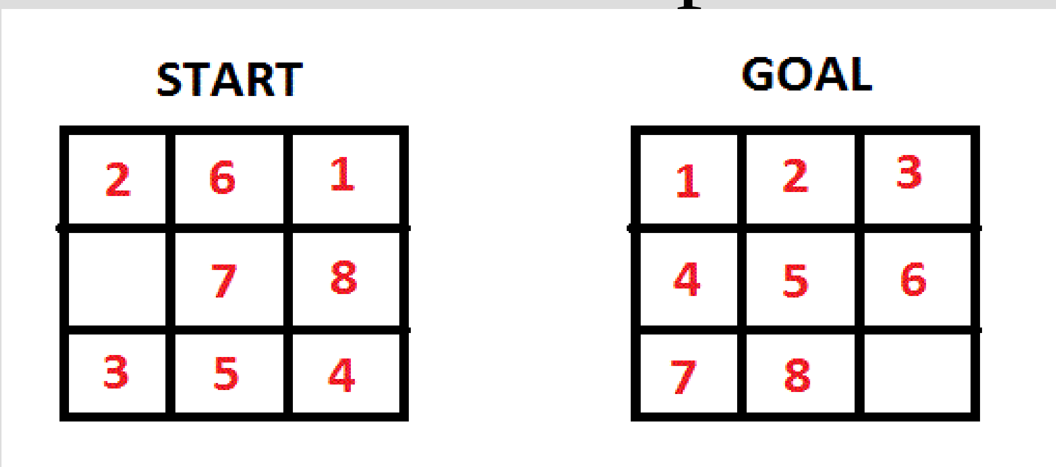
The straight line cost works for distances in the physical world, do any others exist?

One way to make heuristics is to relax the problem (i.e. simplify in a useful way)

The optimal path cost in the relaxed problem can be a heuristic for the original problem (i.e. if we were not constrained to driving on roads, we could take the straight line path)

Heuristics

Let us look at 8-puzzle heuristics:



The rules of the game are:

- You can swap any square with the blank

Relaxed rules:

1. Teleport any square to any destination
2. Move any square 1 space (overlapping ok)

Heuristics

1. Teleport any square to any destination
Optimal path cost is the number of mismatched squares (blank included)

2. Move any square 1 space (overlapping ok)
Optimal path cost is Manhattan distance for each square to goal summed up

Which ones is better? (Note: these optimal solutions in relaxed need to be computed fast)

Heuristics

h1 = mismatch count
h2 = number to goal difference sum

	Search Cost			Effective Branching Factor		
d	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Heuristics

The real branching factor in the 8-puzzle:

- 2 if in a corner
- 3 if on a side
- 4 if in the center
- (Thus larger "8-puzzles" tend to 4)

Effective branching factor is assuming the tree is exponential:

Let N be number of nodes in memory, then effective branching factor ≈ $N^{(1/(d+1))}$

Heuristics

h2 has a better branching factor than h1, and this is not a coincidence...

h2(node) $\geq$ h1(node) for all nodes, thus we say h2 dominates h1 (and will thus perform better)

If there are multiple non-dominating heuristics: h1, h2... Then h* = max(h1, h2, ...) will dominate h1, h2, ... and will also be admissible /consistent if h1, h2 ... are as well

Heuristics

If larger is better, why do we not just set $h(node) = 9001$?

Heuristics

If larger is better, why do we not just set h(node) = 9001?

This would (probably) not be admissible...

If h(node) = 0, then you are doing the uninformed uniform cost search

If h(node) = optimal_cost(node to goal) then will ONLY explore nodes on an optimal path

Heuristics

You cannot add two heuristics ($h^* = h1 + h2$), unless there is no overlap (i.e. h1 cost is independent of h2 cost)

For example, in the 8-puzzles:

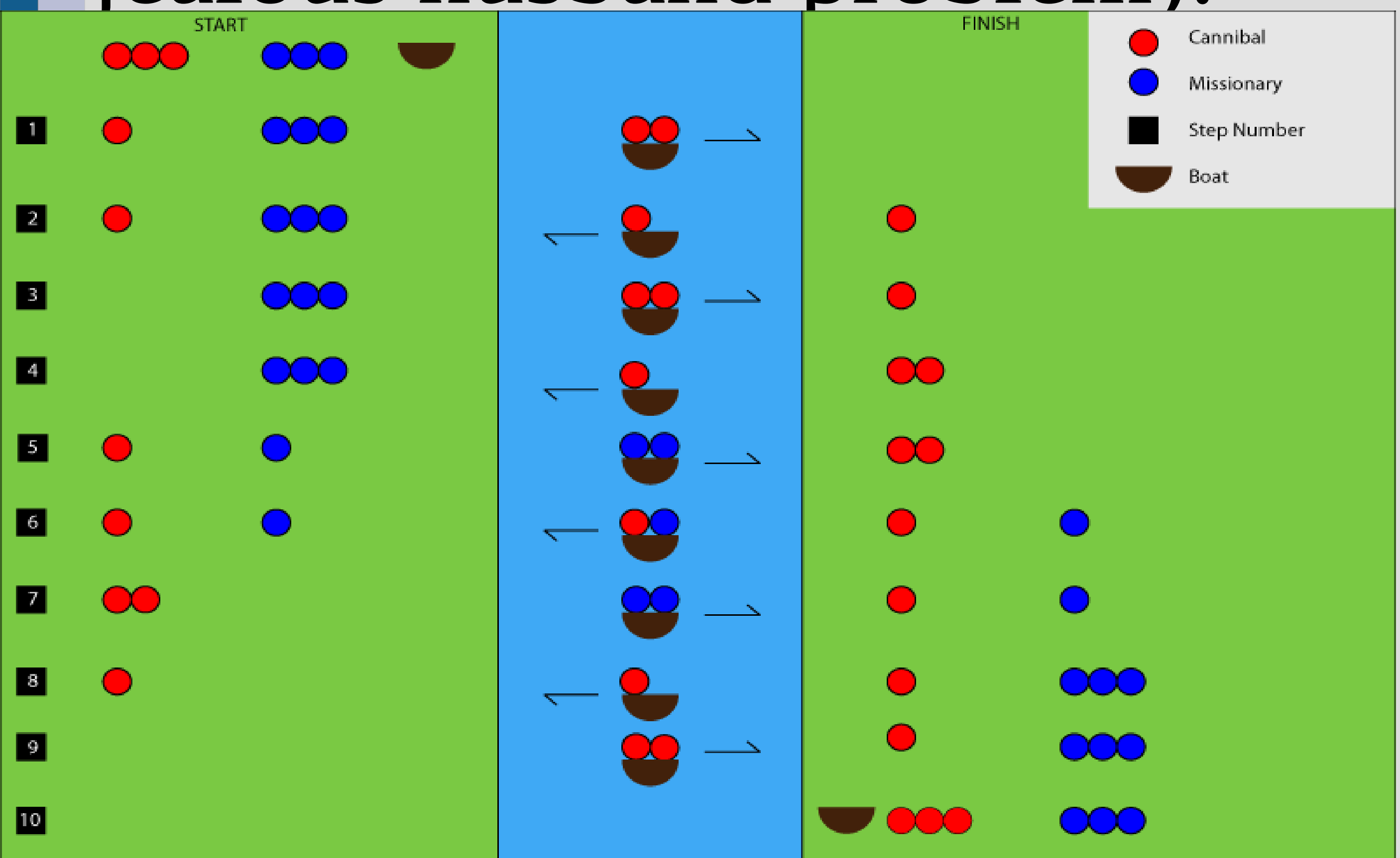
h3: number of 1, 2, 3, 4 that are misplaced
h4: number of 5, 6, 7, 8 that are misplaced

There is no overlap, and in fact:

$h3 + h4 = h1$ (as defined earlier)

Heuristics

Cannibals & missionaries problem (also jealous husband problem):



Goal:fewest steps to swap

Rules:

1. One either bank, cannot have m < c

2. 2 ppl in boa

3. 3m & 3c

Heuristics

What relaxation did you use? (sample)

Make a heuristic for this problem

Is the heuristic admissible/consistent?

Heuristics

What relaxation did you use? (sample)

- Remove rule 1 (m > c on both banks)

Make a heuristic for this problem

- h1 = [num people wrong bank]/2 (boat cap.)

Is the heuristic admissible/consistent?

- YES! ('cause I say so)