Quiz 11 Solution

1. Two fair dice are thrown. What is the conditional probability that the total score is 7 given that the score on the first die is 4? Justify your answer using the definition of conditional probability.

**Solution:** Let $E$ be the event that the total score is 7 and let $F$ be the event that the first die is 4. Then the event $E \cap F$ is that the first die is 4 and the second die is 3. Therefore, we have

$$p(F) = \frac{1}{6} \quad \text{and} \quad p(E \cap F) = \left(\frac{1}{6}\right)^2 = \frac{1}{36}.$$ 

Based on the definition of the conditional probability,

$$p(E|F) = \frac{p(E \cap F)}{p(F)} = \frac{1}{6}.$$ 

2. A process generates bitstrings of length 3 randomly where each such string is equally likely to appear. Let the event $E$ be the event of generating a bitstring of length 3 that has at least 2 ones in it, and let the event $F$ be the event of generating a bitstring of length 3 that has an odd number of ones in it. Are $E$ and $F$ independent? Justify your answer.

**Solution:** Generating a bitstring with length 3 makes a sample space with $2^3 = 8$ different outcomes.

For the event $E$, there are 2 different cases:

1. There are 2 ones and 1 zero. The number of outcomes is \( \binom{3}{2} = 3 \).
2. There are 3 ones. The number of outcomes is 1.

Thus, $p(E) = \frac{1}{2}$.

For the event $F$, there are also 2 different cases:

1. There are 1 one in it. The number of outcomes is \( \binom{3}{1} = 3 \).
2. There are 3 ones in it. The number of outcomes is 1.

Thus, $p(F) = \frac{4}{8} = \frac{1}{2}$.

For the event $E \cap F$, the only case is there are 3 ones in it and $p(E \cap F) = \frac{1}{8}$.

Therefore,

$$p(E \cap F) \neq p(E)p(F),$$

and $E$ and $F$ are not independent.