1. Let $A \in \mathbb{R}^{n \times n}$. Show that for any $x \in \mathbb{R}^n$ we have

$$(Ax, x) = \frac{1}{2}((A + A^T)x, x).$$

Let $S = (A + A^T)/2$. Complete the following statements:

- $A$ is SPD iff ________
- $A$ is PSD iff ________

2. (Continuation) If $A$ is PSD and $\alpha > 0$ then is $A + \alpha I$ SPD?

**Note:** It can be shown that if $(Ax, x)$ is real for all complex vectors $x$ then $A$ must be Hermitian.