You are given two classes $L, R$, each distributed by a univariate normal distribution:

$$
\rho_L(x) = \frac{1}{\sigma_L \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - \mu_L}{\sigma_L} \right)^2}
\log \rho_L(x) = -\frac{1}{2} \log \sigma_L^2 - \frac{1}{2} \log(2\pi) - \frac{1}{2} \left( \frac{x - \mu_L}{\sigma_L} \right)^2
$$

$$
\rho_R(x) = \frac{1}{\sigma_R \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - \mu_R}{\sigma_R} \right)^2}
\log \rho_R(x) = -\frac{1}{2} \log \sigma_R^2 - \frac{1}{2} \log(2\pi) - \frac{1}{2} \left( \frac{x - \mu_R}{\sigma_R} \right)^2
$$

with unequal priors:

$$
\log \left( \frac{\text{Prior Probability of } R}{\text{Prior Probability of } L} \right) = \log \left( \frac{\Pr(R)}{\Pr(L)} \right) = 1 - \frac{1}{2} \log 2 = \log \frac{\sqrt{e}}{\sqrt{2}} = 0.653426409678636
$$

For reference, here’s a formula for log of the ratio of the conditional distributions:

$$
\log \rho_R(x) - \log \rho_L(x) = \frac{1}{2} \left( \frac{1}{\sigma_L^2} - \frac{1}{\sigma_R^2} \right) x^2 + \left( \frac{\mu_R}{\sigma_R^2} - \frac{\mu_L}{\sigma_L^2} \right) x + \frac{1}{2} \left( \frac{\mu_R^2}{\sigma_R^2} - \frac{\mu_L^2}{\sigma_L^2} \right) + \frac{1}{2} \log \frac{\sigma_L^2}{\sigma_R^2}
$$

1. **Estimate parameters.** You are given the following training samples from class $L$: $-0.5, +0.5,$ and the following training samples from class $R$: $+1.5, +2.0, +2.5$. Compute the unbiased estimates for within-class means $\mu_L, \mu_R$ and variances $\sigma_L^2, \sigma_R^2$. Hint, the quantities $\mu_L, \mu_R, 1/\sigma_L^2, 1/\sigma_R^2$ should all be small integers.

2. **Learn Decision Fcn.** Determine the optimal decision region for choosing class $R$. **Hint:** the boundaries of the decision region are small integers. The polynomial determining those boundaries also has small integer coefficients.

3. **Test.** Normally this would be applied to a separate test set. Given you have so few points, what is the error rate on the training set?

4. If you have a calculator, find numerical value for the prior $\Pr(R)$. 