**Non associativity in the presence of round-off.**

**Solution:** *This is done in a class demo and the diary should be posted. Here are the commands.*

```
n = 10000;
a = randn(n,1); b = randn(n,1); c = randn(n,1);
t = ((a+b)+c == a+(b+c));
sum(t)
```

*Right-hand side in 3rd line returns 1 for each instance when the two numbers are the same.*

**Find machine epsilon in matlab.**

**Solution:**

```
u = 1;
```
for i=0:999
    fprintf(1,' i = %d , u = %e \n',i,u)
    if (1.0 +u == 1.0) break, end
    u = u/2;
end
u = u*2

Proof of Lemma: If $|\delta_i| \leq u$ and $nu < 1$ then

$$\prod_{i=1}^{n}(1 + \delta_i) = 1 + \theta_n \text{ where } |\theta_n| \leq \frac{nu}{1 - nu}$$

Solution:

The proof is by induction on $n$.

1) Basis of induction. When $n = 1$ then the product reduces to $1 + \delta_i$ and so we can take
\( \theta_n = \delta_n \) and we know that \( |\delta_n| \leq u \) from the assumptions and so

\[
|\theta_n| \leq u \leq \frac{u}{1-u},
\]
as desired.

2) Induction step. Assume now that the result as stated is true for \( n \) and consider a product with \( n + 1 \) terms: \( \prod_{i=1}^{n+1} (1 + \delta_i) \). We can write this as \( (1 + \delta_{n+1})\prod_{i=1}^{n} (1 + \delta_i) \) and from the induction hypothesis we get:

\[
\prod_{i=1}^{n+1} (1 + \delta_i) = (1 + \theta_n)(1 + \delta_{n+1}) = 1 + \theta_n + \delta_{n+1} + \theta_n\delta_{n+1}
\]

with \( \theta_n \) satisfying the inequality \( \theta_n \leq \frac{nu}{(1-nu)} \). We call \( \theta_{n+1} \) the quantity \( \theta_{n+1} = \theta_n + \delta_{n+1} + \theta_n\delta_{n+1} \), and we have

\[
|\theta_{n+1}| = |\theta_n + \delta_{n+1} + \theta_n\delta_{n+1}|
\leq \frac{nu}{1-nu} + u + \frac{nu}{1-nu} \times u = \frac{nu + u(1-nu) + nu^2}{1-nu} = \frac{(n+1)u}{1-nu}
\leq \frac{(n+1)u}{1-(n+1)u}
\]
This establishes the result with $n$ replaced by $n + 1$ as wanted and completes the proof. □