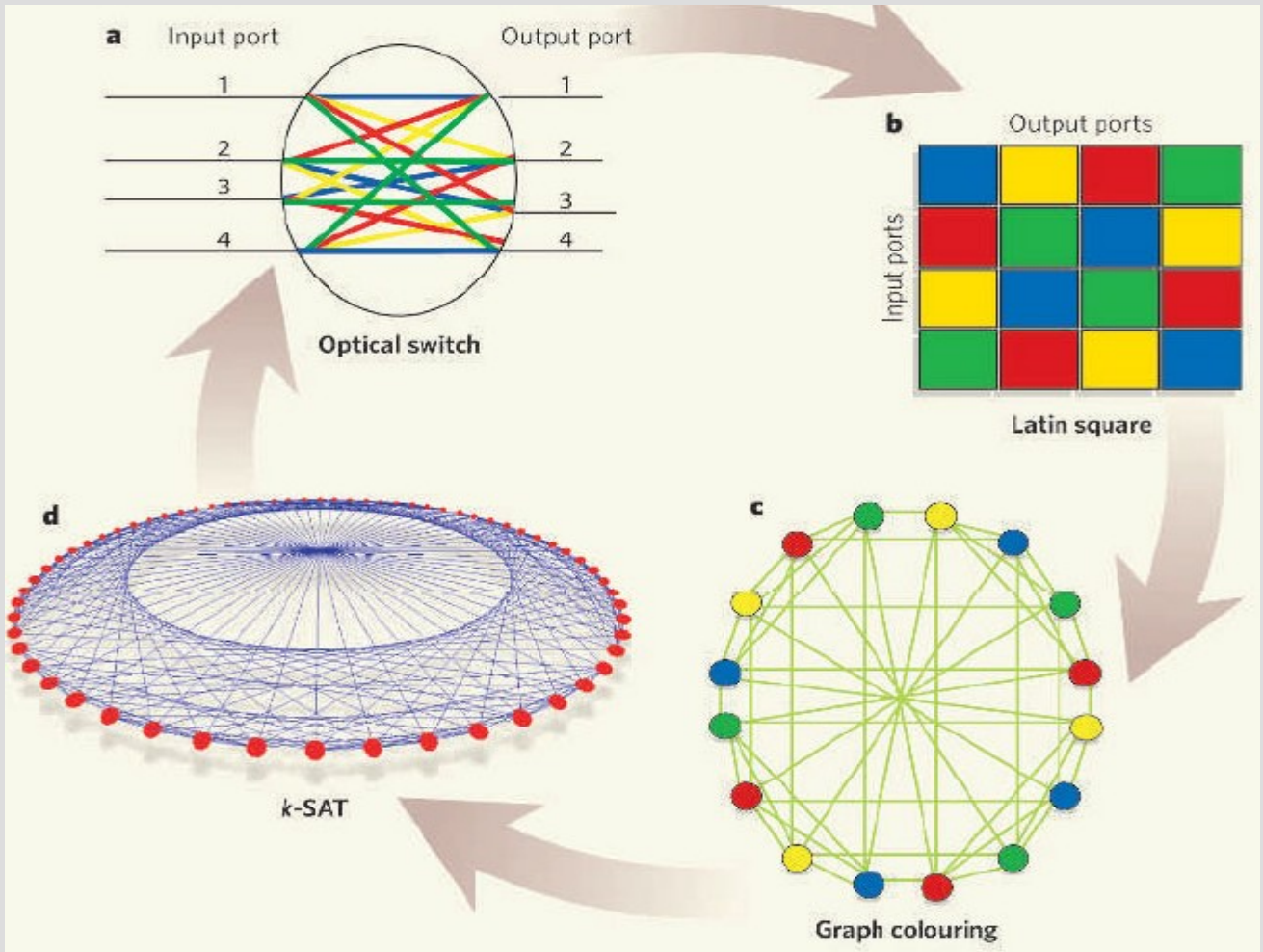
Constraint sat. prob. (Ch. 6)



Announcements

Writing 3 due next week

-Find papers for project

-scholar.google.com is your friend!

I suggest not looking for your problem
(e.g. if you want to do “poker” as your project, do not google search “poker research”)
... instead look at the technique used
(if you plan to solve “poker” using minimax, search for general “minimax” papers)

CSP

A constraint satisfaction problem is when there are a number of variables in a domain with some restrictions

These CSPs invovle 3 things:

- Variables (much like math ones, like “x”)
- Domains (what “values” are possible for each variable)
- Constraints (math constraints on variables, like “x<y”)

CSP

The goal of constraint satisfaction problems is to pick/assign values from the domain to variables that does not invalid constraints

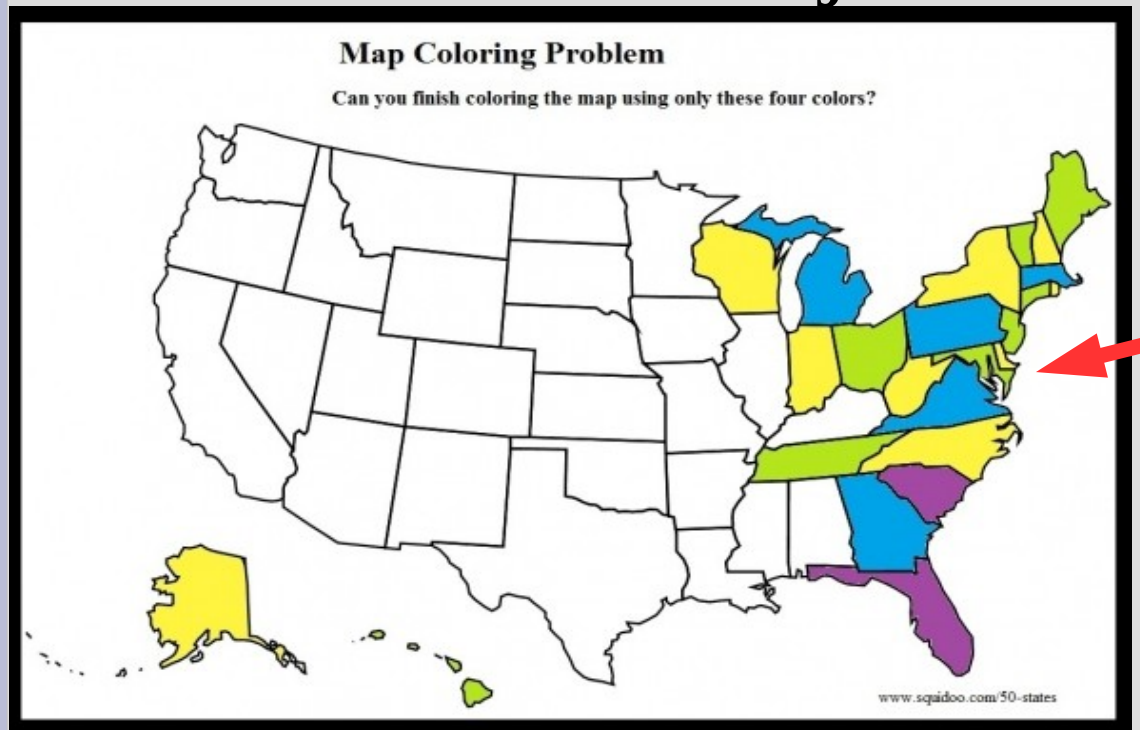
A consistent assignment of variables has no violated constraints

A complete assignment of variables has no unassigned variables

(A solution is complete and consistent)

CSP

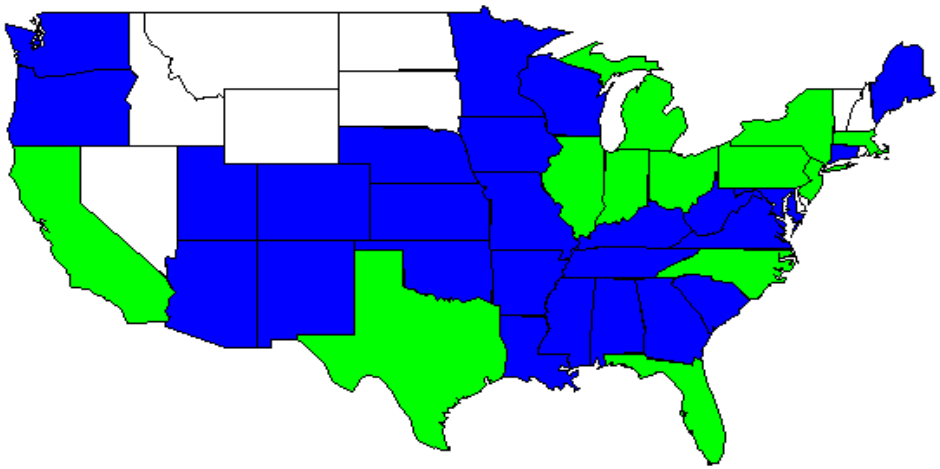
Map coloring is a famous CSP problem
Variables: each state/country
Domain: {yellow, blue, green, purple} (here)
Constraints: No adjacent variables same color

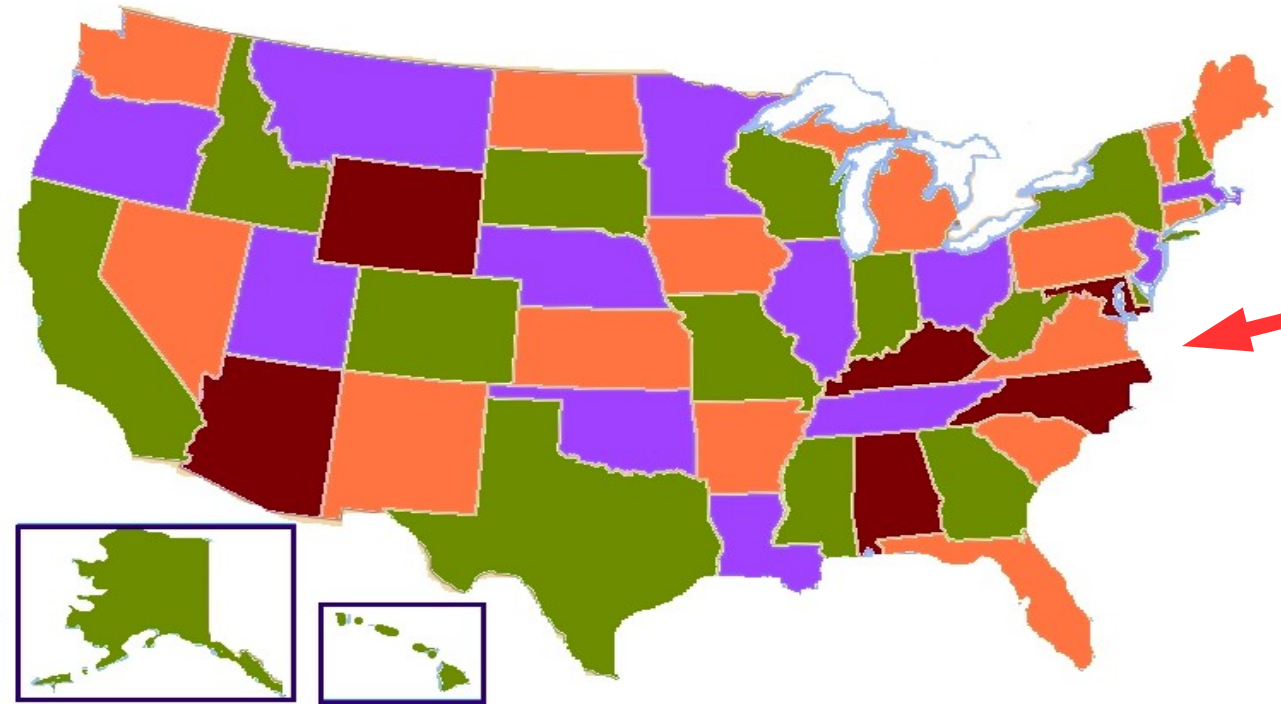


Consistent but partial

CSP

partial and
not consistent

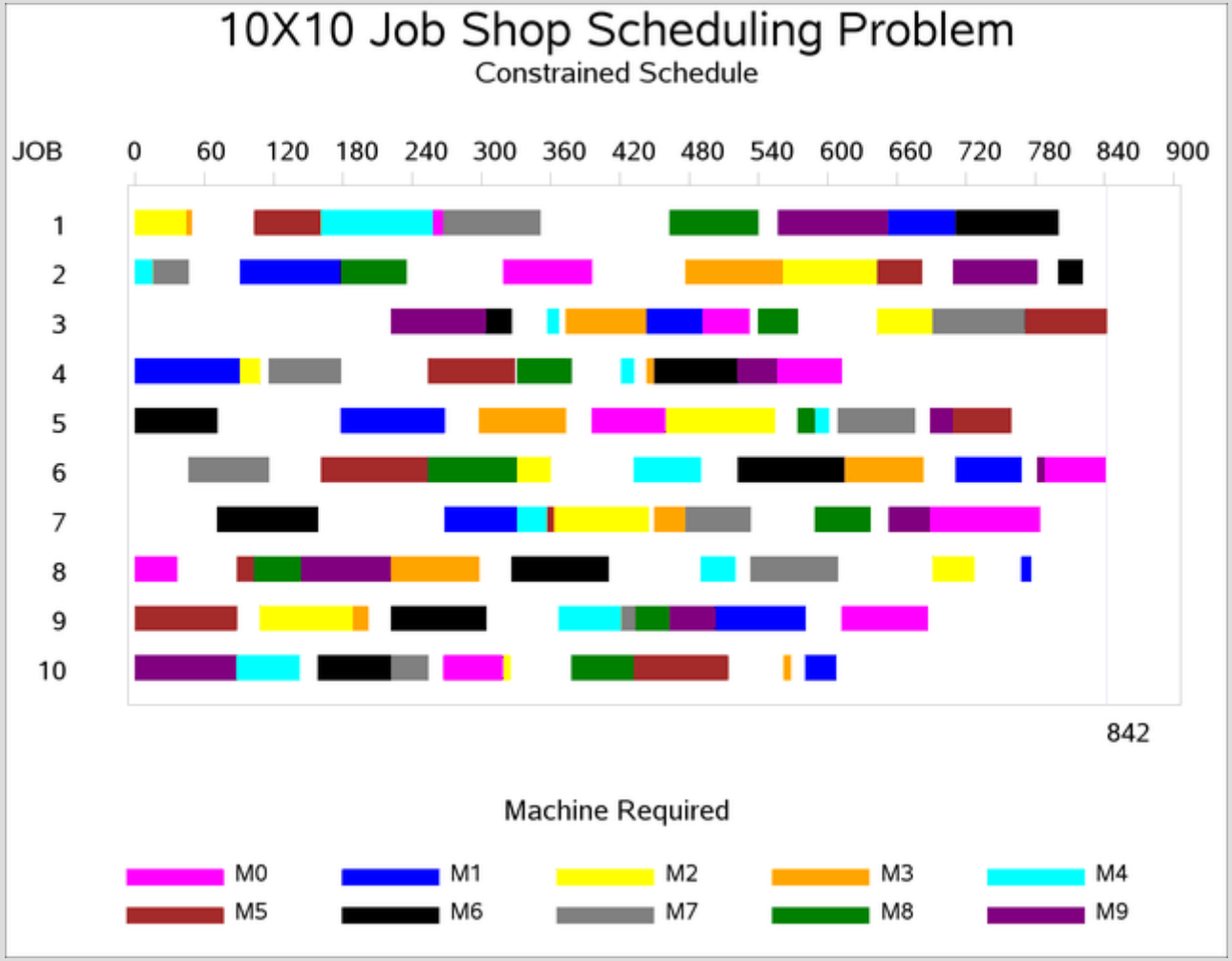




Consistent and
complete

CSP

Another common use of CSP is job scheduling



CSP

Suppose we have 3 jobs: J_1, J_2, J_3

If J_1 takes 20 time units to complete, J_2 takes 30 and J_3 takes 15 but J_1 must be done before J_3 (jobs cannot happen at the same time)

How to write this as a boolean expression?

CSP

Suppose we have 3 jobs: J_1, J_2, J_3

If J_1 takes 20 time units to complete, J_2 takes 30 and J_3 takes 15 <u>but</u> J_1 must be done before J_3

We can represent this as (**<u>and</u>** them together):

J_1 & J_2: ($J_1 + 20 \leq J_2$ **<u>or</u>** $J_2 + 30 \leq J_1$)

J_1 & J_3: ($J_1 + 20 \leq J_3$)

J_2 & J_3: ($J_2 + 30 \leq J_3$ **<u>or</u>** $J_3 + 15 \leq J_2$)

Types of constraints

A unary constraint is for a single variable (i.e. J_1 cannot start before time 5)

Binary constraints are between two variables (i.e. J_1 starts before J_2)

Constraints can involve more variables, such as in Sudoku all numbers on a row needs to be different: AllDiff(a11,a12,a13,a14, ...)

Types of constraints

All constraints can be reduced to just binary and unary constraints, but may require making variables to store temporary information

Our goal is to use the constraints to narrow the domains of possible values of variables

k-consistency is a measurement of how well the domains satisfy different degrees of the constraints (larger ‘k’ = satisfy more complex)

Types of constraints

K-consistency is:
For any consistent sets size (k-1), there exists a valid value for any other variable (not in set)

1-consistency: All values in the domain satisfy the variable's unary constraints
2-consistency: All binary values are in domain
3-consistency: Given consistent 2 variables, there is a value for a third variable(i.e. if {A,B} is consistent, then exists C s.t. {A,C}&{B,C})

Types of constraints

For example, 1-consistent means you can pick 0 consistent variables (if you pick nothing it is always consistent) then any assignment to a new variable is also consistent

This boils down to saying you can pick any valid pick of a single variable in isolation

In other words, you satisfy the unary constraints

Types of constraints

2-consistent means you pick a valid value from the domain for one variable and see if there is any valid assignment for a second var

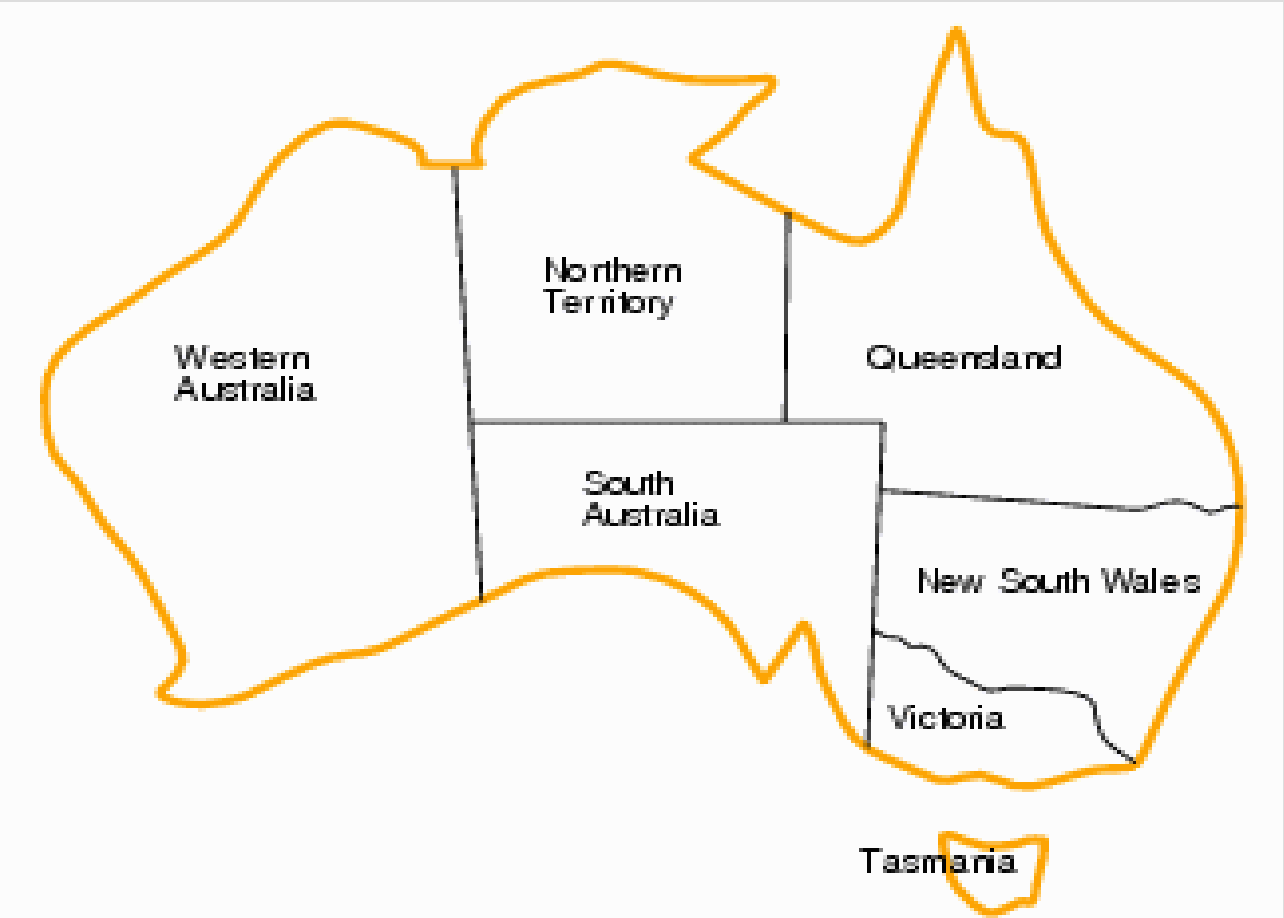
3-consistent means you pick a valid pair of values for 2 variables and see if there is any valid assignment for a third variable

If you are unable to find a valid assignment for the last variable, it is not consistent

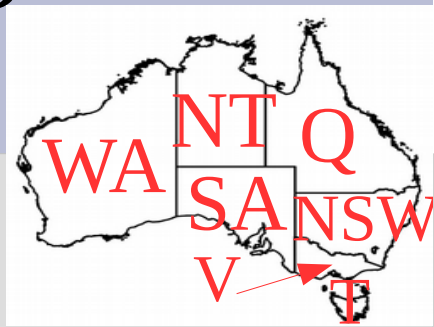
Types of constraints

Rules: 1. Tasmania cannot be red
2.Neighboring providences cannot share colors

2 Colors:
red
green



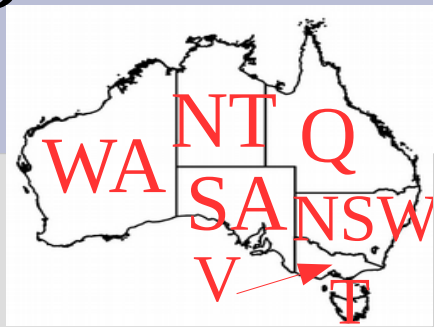
Types of constraints



WA = {red, green}
NT = {red, green}
Q = {red, green}
SA = {red, green}
NSW = {red, green}
V = {red, green}
T = {red, green}

Not 1-consistent as we need T to not be red (i.e. rule #2 eliminates T=red)

Types of constraints

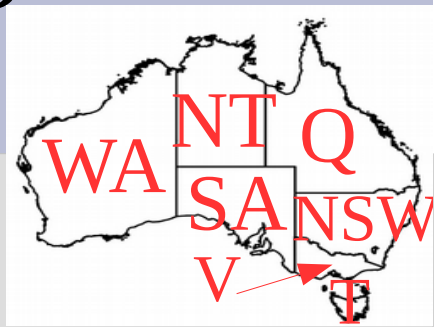


WA = NT = Q = SA = NSW = V
= {red, green}
T = {green}

1-consistent now

Also 2-consistent, for example:
Pick WA as “set k-1”, then try to pick NT...
If WA=green, then we can make NT=red
if WA=red, NT=green (true for all pairs)

Types of constraints



WA = NT = Q = SA = NSW = V
= {red, green}
T = {green}

Not 3-consistent!

Pick (WA, SA) and add NT... If NT=green, will not work with either: (WA=red,SA=green) or (WA=green,SA=red)... NT=red also will not work, so NT's domain is empty and not 3-cons.

Types of constraints

Try to do k-consistency for this job problem
(Domains for all: {1, 2, 3, 4, 5, 6, 7, 8...})
Jobs cannot overlap
J3 takes 3 time units
J2 takes 2 time units
J1 takes 1 time unit
J1 must happen before J3
J2 cannot happen at time 1
All jobs must finish by time 7
(i.e. you can start J2 at time 5 but not at time 6)

Applying constraints

We can repeatedly apply our constraint rules to shrink the domain of variables (we just shrunk NT's domain to nothing)

This reduces the size of the domain, making it easier to check:

- If the domain size is zero, there are no solutions for this problem
- If the domain size is one, this variable must take on that value (the only one in domain)

Applying constraints

AC-3 checks all 2-consistency constraints:

1. Add all binary constraints to queue
2. Pick a binary constraint (X_i, Y_j) from queue
3. If x in domain(X_i) and no consistent y in domain(Y_j), then remove x from domain(X_i)
4. If you removed in step 3, update all other binary constraints involving X_i (i.e. (X_i, X_k))
5. Goto step 2 until queue empty

Applying constraints

Some problems can be solved by applying constraint restrictions (such as sudoku)
(i.e. the size of domain is one after reduction)

Harder problems this is insufficient and we will need to search to find a solution

Which is what we will do... next