Problem solving and search

Chapter 3
Outline

♦ Problem-solving agents
♦ Problem types
♦ Problem formulation
♦ Example problems
♦ Basic search algorithms
Problem-solving agents

Restricted form of general agent:

```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept ) returns an action
    static: seq, an action sequence, initially empty
             state, some description of the current world state
             goal, a goal, initially null
             problem, a problem formulation

    state ← UPDATE-STATE( state, percept )
    if seq is empty then
        goal ← FORMULATE-GOAL( state )
        problem ← FORMULATE-PROBLEM( state, goal )
        seq ← SEARCH( problem )
        action ← RECOMMENDATION( seq, state )
        seq ← REMAINDER( seq, state )
    return action
```

Note: this is offline problem solving; solution executed “eyes closed.”
Online problem solving involves acting without complete knowledge.
Example: Romania

On holiday in Romania; currently in Arad. Flight leaves tomorrow from Bucharest

Formulate goal:
be in Bucharest

Formulate problem:
states: various cities
actions: drive between cities

Find solution:
sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest
Problem types

Deterministic, fully observable $\Rightarrow$ single-state problem
Agent knows exactly which state it will be in; solution is a sequence

Non-observable $\Rightarrow$ conformant problem
Agent may have no idea where it is; solution (if any) is a sequence

Nondeterministic and/or partially observable $\Rightarrow$ contingency problem
percepts provide new information about current state
solution is a contingent plan or a policy
often interleave search, execution

Unknown state space $\Rightarrow$ exploration problem ("online")
Example: vacuum world

Single-state, start in #5. Solution??
Example: vacuum world

Single-state, start in #5. Solution??
[Right, Suck]

Conformant, start in \{1, 2, 3, 4, 5, 6, 7, 8\}
e.g., Right goes to \{2, 4, 6, 8\}. Solution??
Example: vacuum world

Single-state, start in #5. Solution??
[Right, Suck]

Conformant, start in \{1, 2, 3, 4, 5, 6, 7, 8\}
e.g., Right goes to \{2, 4, 6, 8\}. Solution??
[Right, Suck, Left, Suck]

Contingency, start in #5
Murphy’s Law: Suck can dirty a clean carpet
Local sensing: dirt, location only.
Solution??
Example: vacuum world

Single-state, start in #5. Solution??
[Right, Suck]

Conformant, start in \{1, 2, 3, 4, 5, 6, 7, 8\}
e.g., Right goes to \{2, 4, 6, 8\}. Solution??
[Right, Suck, Left, Suck]

Contingency, start in #5
Murphy’s Law: Suck can dirty a clean carpet
Local sensing: dirt, location only.
Solution??
[Right, if dirt then Suck]
Single-state problem formulation

A problem is defined by four items:

**initial state** e.g., “at Arad”

**successor function** $S(x) = \text{set of action–state pairs}$
  
  e.g., $S(\text{Arad}) = \{⟨\text{Arad} \rightarrow \text{Zerind}, \text{Zerind}⟩, \ldots \}$

**goal test**, can be
  
  explicit, e.g., $x = \text{“at Bucharest”}$
  implicit, e.g., $\text{NoDirt}(x)$

**path cost** (additive)
  
  e.g., sum of distances, number of actions executed, etc.
  
  $c(x, a, y)$ is the **step cost**, assumed to be $\geq 0$

A solution is a sequence of actions leading from the initial state to a goal state
Selecting a state space

Real world is absurdly complex
⇒ state space must be **abstracted** for problem solving

(Abstract) state = set of real states

(Abstract) action = complex combination of real actions
e.g., “Arad → Zerind” represents a complex set of possible routes, detours, rest stops, etc.
For guaranteed realizability, **any** real state “in Arad” must get to **some** real state “in Zerind”

(Abstract) solution =
set of real paths that are solutions in the real world

Each abstract action should be “easier” than the original problem!
Example: vacuum world state space graph

- **states??**
- **actions??**
- **goal test??**
- **path cost??**
Example: vacuum world state space graph

- **states??**: integer dirt and robot locations (ignore dirt amounts etc.)
- **actions??**
- **goal test??**
- **path cost??**
Example: vacuum world state space graph

**states**: integer dirt and robot locations (ignore dirt amounts etc.)

**actions**: *Left*, *Right*, *Suck*, *NoOp*

**goal test**

**path cost**
Example: vacuum world state space graph

**states??**: integer dirt and robot locations (ignore dirt amounts etc.)

**actions??**: Left, Right, Suck, NoOp

**goal test??**: no dirt

**path cost??**
Example: vacuum world state space graph

states??: integer dirt and robot locations (ignore dirt amounts etc.)
actions??: Left, Right, Suck, NoOp
goal test??: no dirt
path cost??: 1 per action (0 for NoOp)
Example: The 8-puzzle

Start State

Goal State

states??
actions??
goal test??
path cost??
Example: The 8-puzzle

states??: integer locations of tiles (ignore intermediate positions)
actions??
goal test??
path cost??
Example: The 8-puzzle

Start State

Goal State

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??
path cost??
Example: The 8-puzzle

**states**: integer locations of tiles (ignore intermediate positions)

**actions**: move blank left, right, up, down (ignore unjamming etc.)

**goal test**: = goal state (given)

**path cost**
Example: The 8-puzzle

- **states**: integer locations of tiles (ignore intermediate positions)
- **actions**: move blank left, right, up, down (ignore unjamming etc.)
- **goal test**: = goal state (given)
- **path cost**: 1 per move

[Note: optimal solution of $n$-Puzzle family is NP-hard]
Example: robotic assembly

**states??**: real-valued coordinates of robot joint angles
parts of the object to be assembled

**actions??**: continuous motions of robot joints

**goal test??**: complete assembly *with no robot included!*

**path cost??**: time to execute
Tree search algorithms

Basic idea:
offline, simulated exploration of state space
by generating successors of already-explored states
(a.k.a. expanding states)

function Tree-Search(problem, strategy) returns a solution, or failure
initialize the search tree using the initial state of problem
loop do
  if there are no candidates for expansion then return failure
  choose a leaf node for expansion according to strategy
  if the node contains a goal state then return the corresponding solution
  else expand the node and add the resulting nodes to the search tree
end
Tree search example
Tree search example
Tree search example
Implementation: states vs. nodes

A state is a (representation of) a physical configuration
A node is a data structure constituting part of a search tree
includes parent, children, depth, path cost \( g(x) \)
States do not have parents, children, depth, or path cost!

The \textsc{Expand} function creates new nodes, filling in the various fields and using the \textsc{SuccessorFn} of the problem to create the corresponding states.
Implementation: general tree search
function **Tree-Search** *(problem, fringe)* returns a solution, or failure

```
fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
```

**loop do**

- if `fringe` is empty then return failure
- `node` ← Remove-Front(`fringe`)
- if Goal-Test(*problem*, State(*node*)) then return `node`
- `fringe` ← InsertAll(Expand(`node`, *problem*), `fringe`)

---

function **Expand** *(node, problem)* returns a set of nodes

```
successors ← the empty set
for each action, result in Successor-Fn(*problem*, State[*node]*) do
    s ← a new Node
    Parent-Node[*s*] ← *node*; Action[*s*] ← action; State[*s*] ← result
    Path-Cost[*s*] ← Path-Cost[*node*] + Step-Cost(State[*node*], action, result)
    Depth[*s*] ← Depth[*node*] + 1
    add *s* to successors
return successors
```
A strategy is defined by picking the **order of node expansion**

Strategies are evaluated along the following dimensions:
- **completeness**—does it always find a solution if one exists?
- **time complexity**—number of nodes generated/expanded
- **space complexity**—maximum number of nodes in memory
- **optimality**—does it always find a least-cost solution?

Time and space complexity are measured in terms of
- $b$—maximum branching factor of the search tree
- $d$—depth of the least-cost solution
- $m$—maximum depth of the state space (may be $\infty$)
Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search
Uniform-cost search
Depth-first search
Depth-limited search
Iterative deepening search
Breadth-first search

Expand shallowest unexpanded node

**Implementation:**

*fringe* is a FIFO queue, i.e., new successors go at end

```
       A
      /\  \\
    B   C
   /\   /\  \\
D  E  F  G
```
Breadth-first search

Expand shallowest unexpanded node

**Implementation:**

*fringe* is a FIFO queue, i.e., new successors go at end
Breadth-first search

Expand shallowest unexpanded node

Implementation:

*fringe* is a FIFO queue, i.e., new successors go at end
Breadth-first search

Expand shallowest unexpanded node

**Implementation:**

*fringe* is a FIFO queue, i.e., new successors go at end
Properties of breadth-first search

Complete??
Properties of breadth-first search

Complete: Yes (if $b$ is finite)

Time: ??
Properties of breadth-first search

**Complete?** Yes (if $b$ is finite)

**Time?** $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in $d$

**Space??**
Properties of breadth-first search

**Complete**? Yes (if \( b \) is finite)

**Time**? \( 1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1}) \), i.e., exp. in \( d \)

**Space**? \( O(b^{d+1}) \) (keeps every node in memory)

**Optimal**?
Properties of breadth-first search

**Complete?** Yes (if \( b \) is finite)

**Time?** \( 1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1}) \), i.e., exp. in \( d \)

**Space??** \( O(b^{d+1}) \) (keeps every node in memory)

**Optimal??** Yes (if cost = 1 per step); not optimal in general

**Space** is the big problem; can easily generate nodes at 100MB/sec so 24hrs = 8640GB.
Uniform-cost search

Expand least-cost unexpanded node

**Implementation:**

\[ \text{fringe} = \text{queue ordered by path cost, lowest first} \]

Equivalent to breadth-first if step costs all equal

**Complete**\(??\) Yes, if step cost \(\geq \epsilon\)

**Time**\(??\) \# of nodes with \(g \leq\) cost of optimal solution, \(O(b^{C^*/\epsilon})\)

where \(C^*\) is the cost of the optimal solution

**Space**\(??\) \# of nodes with \(g \leq\) cost of optimal solution, \(O(b^{C^*/\epsilon})\)

**Optimal**\(??\) Yes—nodes expanded in increasing order of \(g(n)\)
Depth-first search

Expand deepest unexpanded node

Implementation:

\[ \text{fringe} = \text{LIFO queue, i.e., put successors at front} \]
Depth-first search

Expand deepest unexpanded node

**Implementation:**

$fringe = \text{LIFO queue, i.e., put successors at front}$
Depth-first search

Expand deepest unexpanded node

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**Implementation:**

fringe = LIFO queue, i.e., put successors at front
Depth-first search

Expand deepest unexpanded node

**Implementation:**

\(fringe = \) LIFO queue, i.e., put successors at front
Depth-first search

Expand deepest unexpanded node

**Implementation:**

\[ \text{fringe} = \text{LIFO queue, i.e., put successors at front} \]
Depth-first search

Expand deepest unexpanded node

**Implementation:**

*fringe* = LIFO queue, i.e., put successors at front

![Tree diagram](image)
Depth-first search

Expand deepest unexpanded node

**Implementation:**

fringe = LIFO queue, i.e., put successors at front

Diagram:

```
  A
 / \
|   |   
B   C
 / \ / \  
|   |   |   
D   F   G
```

Fringe: L, M, N, O
Depth-first search

Expand deepest unexpanded node

**Implementation:**

*fringe* = LIFO queue, i.e., put successors at front
Depth-first search

Expand deepest unexpanded node

**Implementation:**

$fringe = \text{LIFO queue, i.e., put successors at front}$

---

$\begin{tikzpicture}
\node (A) at (0,0) {$A$};
\node (B) at (-2,-1) {$B$};
\node (C) at (2,-1) {$C$};
\node (D) at (-3,-2) {$D$};
\node (E) at (-1,-2) {$E$};
\node (F) at (1,-2) {$F$};
\node (G) at (3,-2) {$G$};
\node (H) at (-2,-3) {$H$};
\node (I) at (-1,-3) {$I$};
\node (J) at (0,-3) {$J$};
\node (K) at (1,-3) {$K$};
\node (L) at (2,-3) {$L$};
\node (M) at (-3,-4) {$M$};
\node (N) at (-2,-5) {$N$};
\node (O) at (0,-5) {$O$};

\draw (A) -- (B);
\draw (A) -- (C);
\draw (B) -- (D);
\draw (B) -- (E);
\draw (C) -- (F);
\draw (C) -- (G);
\draw (F) -- (M);
\draw (G) -- (N);
\draw (G) -- (O);
\end{tikzpicture}$
Properties of depth-first search

Complete??
Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
⇒ complete in finite spaces

Time??
Properties of depth-first search

**Complete** No: fails in infinite-depth spaces, spaces with loops
   Modify to avoid repeated states along path
   ⇒ complete in finite spaces

**Time** \( O(b^m) \): terrible if \( m \) is much larger than \( d \)
   but if solutions are dense, may be much faster than breadth-first

**Space**
Properties of depth-first search

**Complete**
No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
⇒ complete in finite spaces

**Time**
\[ O(b^m) \]: terrible if \( m \) is much larger than \( d \)
but if solutions are dense, may be much faster than breadth-first

**Space**
\[ O(bm) \], i.e., linear space!

**Optimal**
Properties of depth-first search

**Complete**? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
⇒ complete in finite spaces

**Time**? $O(b^m)$: terrible if $m$ is much larger than $d$
but if solutions are dense, may be much faster than breadth-first

**Space**? $O(bm)$, i.e., linear space!

**Optimal**? No
Depth-limited search

= depth-first search with depth limit \( l \),
i.e., nodes at depth \( l \) have no successors

**Recursive implementation:**

```python
function Depth-Limited-Search(problem, limit) returns soln/fail/cutoff
    Recursive-DLS(Make-Node(Initial-State[problem]), problem, limit)

function Recursive-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if Goal-Test(problem, State[node]) then return node
    else if Depth[node] = limit then return cutoff
    else for each successor in Expand(node, problem) do
        result ← Recursive-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```
function Iterative-Deepening-Search(problem) returns a solution
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← Depth-Limited-Search(problem, depth)
        if result ≠ cutoff then return result
    end
Iterative deepening search $l = 0$
Iterative deepening search $l = 1$

Limit = 1
Iterative deepening search $l = 2$

Limit = 2
Iterative deepening search $l = 3$

Limit = 3
Properties of iterative deepening search

Complete??
Properties of iterative deepening search

Complete?? Yes

Time??
Properties of iterative deepening search

Complete? Yes

Time? \((d + 1)b^0 + db^1 + (d - 1)b^2 + \ldots + b^d = O(b^d)\)

Space??
Properties of iterative deepening search

**Complete**? Yes

**Time**? \((d + 1)b^0 + dB^1 + (d - 1)b^2 + \ldots + b^d = O(b^d)\)

**Space**? \(O(bd)\)

**Optimal**??
Properties of iterative deepening search

**Complete??** Yes

**Time??** \((d + 1)b^0 + db^1 + (d - 1)b^2 + \ldots + b^d = O(b^d)\)

**Space??** \(O(bd)\)

**Optimal??** Yes, if step cost = 1

Can be modified to explore uniform-cost tree

Numerical comparison for \(b = 10\) and \(d = 5\), solution at far right leaf:

- \(N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450\)
- \(N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100\)

IDS does better because other nodes at depth \(d\) are not expanded

BFS can be modified to apply goal test when a node is *generated*
## Summary of algorithms

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Breadth-First</th>
<th>Uniform-Cost</th>
<th>Depth-First</th>
<th>Depth-Limited</th>
<th>Iterative Deepening</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete?</td>
<td>Yes*</td>
<td>Yes*</td>
<td>No</td>
<td>Yes, if ( l \geq d )</td>
<td>Yes</td>
</tr>
<tr>
<td>Time</td>
<td>( b^{d+1} )</td>
<td>( b \left[ \frac{C^*}{\epsilon} \right] )</td>
<td>( b^m )</td>
<td>( b^l )</td>
<td>( b^d )</td>
</tr>
<tr>
<td>Space</td>
<td>( b^{d+1} )</td>
<td>( b \left[ \frac{C^*}{\epsilon} \right] )</td>
<td>( bm )</td>
<td>( bl )</td>
<td>( bd )</td>
</tr>
<tr>
<td>Optimal?</td>
<td>Yes*</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes*</td>
</tr>
</tbody>
</table>
Repeated states

Failure to detect repeated states can turn a linear problem into an exponential one!

A
B
C
D

A
B
C
C
B
C
C
function **GRAPH-SEARCH**(problem, fringe) returns a solution, or failure

\[
\begin{align*}
\text{closed} & \leftarrow \text{an empty set} \\
\text{fringe} & \leftarrow \text{INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)} \\
\text{loop do} & \\
& \quad \text{if fringe is empty then return failure} \\
& \quad \text{node} \leftarrow \text{REMOVE-FRONT(fringe)} \\
& \quad \text{if GOAL-TEST(problem, STATE[node]) then return node} \\
& \quad \text{if STATE[node] is not in closed then} \\
& \quad \quad \text{add STATE[node] to closed} \\
& \quad \quad \text{fringe} \leftarrow \text{INSERT-ALL(EXPAND(node, problem), fringe)} \\
\text{end}
\end{align*}
\]
Summary

Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored.

Variety of uninformed search strategies.

Iterative deepening search uses only linear space and not much more time than other uninformed algorithms.

Graph search can be exponentially more efficient than tree search.