Instructions: Please review carefully the instructions given for Homework 1. They apply to this assignment, too.

Please hand in your answers to the following problems. Problem numbers, where indicated, are from the seventh edition of the Rosen text.

1. (7 points) On a popular sports talk show in the Twin Cities, the host makes the following argument ($P_1$–$P_4$ are the premises and $C$ is the conclusion):

$P_1$: If the Packers and Bears lose, then the Lions or Vikings (or both) make the playoffs.
$P_2$: If the Vikings or Bears (or both) win, then the Vikings will not get a high draft pick.
$P_3$: If the Vikings lose then the Packers lose.
$P_4$: The Vikings do not make the playoffs.
$C$: If the Lions do not make the playoffs, then the Vikings do not get a high draft pick.

Express the given statements using the propositional variables listed below. Then give a stepwise proof that the conclusion follows from the premises, justifying each step clearly. (See Examples 6 and 7 on pages 73–74, as well as the examples done in class.) Assume that there are no ties, so a team either wins or loses.

Let $p$ stand for “The Packers lose”, $b$ for “The Bears lose”, $v$ for “The Vikings lose”, $y$ for “The Vikings make the playoffs”, $d$ for “The Vikings get a high draft pick”, and $\ell$ for “The Lions make the playoffs”. Express each of $P_1$–$P_4$ and $C$ using these variables before beginning your proof.

Note: This problem does not involve predicates. Also, in doing your proof, you may find it helpful to include a portion of the conclusion as a premise. (Recall that this approach was covered in class and in Discussion.)

2. (6 points) p. 54, #20 c, e.

For simplicity, assume that the universe (or domain) consists of just the integers $-3, -1, 1$. (This will save you some writing :) Simplify the final expression as much as possible.

3. (4 points) p. 54, # 24 c, d.

State clearly the meaning of each propositional function you use.

4. (6 points) p. 55, #32 c, e

Assume that the universe is the set of all animals. State clearly the meaning of each propositional function you use. The English form of the negation should flow naturally.
5. (3 points) For each of the following universally-quantified statements, give a counterexample or state that one does not exist. In each case, consider separately the case where the universe is (i) the set of all reals and (ii) the set of all negative integers.

(a) \( \forall x(x^2 \neq x) \)
(c) \( \forall x(x^2 \neq 2) \)
(c) \( \forall x(|x| > 0) \)

6. (6 points) p. 65, #12, e, i, and o.

7. (3 points) p. 67, #24 a, b, and d.
State the English form as naturally as you can.
Homework Cover Page

*Please fill in and staple to the front of your homework*

Name (print):

Student ID #:

Homework #:

Discussion Section registered for (check one):

- [ ] Sec. 11 (4:40–5:30 p.m.)
- [ ] Sec. 12 (5:45–6:35 p.m.)