Spring 16: CSci 2011E—Discrete Structures of Computer Science  
(Evening)

30 points  Homework 4  
Out Wed., 2/10  
Due Wed., 2/17

INSTRUCTIONS: Please review carefully the instructions given for Homework 1. They apply to this assignment, too.

Please hand in your answers to the following problems. Problem numbers, where indicated, are from the seventh edition of the Rosen text.

1. (5 points) Consider the quantified statements \( L : \exists x(P(x) \lor Q(x)) \) and \( R : \exists xP(x) \lor \exists xQ(x) \). Establish that \( L \) and \( R \) are logically equivalent by showing separately that \( L \to R \) and \( R \to L \).

2. (4 points) Consider the quantified statements \( L : \forall x(P(x) \lor Q(x)) \) and \( R : \forall xP(x) \lor \forall xQ(x) \). Establish that \( L \) and \( R \) are not logically equivalent by specifying an appropriate universe for \( x \) and meanings for \( P(x) \) and \( Q(x) \) and showing that \( L \) and \( R \) have different truth values for this choice.

3. (4 points) p. 79, # 10 c, d. For each part, state the inference rules, if any, that apply. Work directly with the English sentences instead of converting them to formal quantified statements.

4. (4 points) p. 79, # 16 a, c. Explain your reasoning in each case. Work directly with the English sentences instead of converting them to formal quantified statements.

5. (4 points) p. 80, #28.

6. (6 points) Translate the following premises and conclusion to quantified statementsThen show in stepwise fashion (with justification for each step) that the conclusion follows from the premises. Let the domain consist of all things in the world. Let \( D(x) \) stand for \( x \) is a dessert, \( H(x) \) for \( x \) is a dish, \( N(x) \) for \( x \) is nice, \( W(x) \) for \( x \) is wholesome.

\( P_1 \): All desserts are nice
\( P_2 \): Some dish is a dessert
\( P_3 \): No nice things are wholesome
\( C \): Some dish is not wholesome

1The original version is attributed to Lewis Carroll of Alice in Wonderland fame! This version differs slightly from the original in that a couple of statements have been modified slightly to make them less ambiguous.
7. (3 points) The following “proof” attempts to show that premises $P_1$ and $P_2$ imply conclusion $C$. However, it contains a flaw. Identify the step that contains the flaw and explain what is wrong.

$P_1$: $\exists x(P(x) \land Q(x))$, $P_2$: $\exists x(Q(x) \rightarrow R(x))$, $C$: $\exists x(P(x) \land R(x))$

1. $\exists x(P(x) \land Q(x))$  Premise
2. $\exists x(Q(x) \rightarrow R(x))$  Premise
3. $P(c) \land Q(c)$  1, Existential Instantiation
4. $P(c)$  3, Simplification
5. $Q(c)$  3, Simplification
6. $Q(c) \rightarrow R(c)$  2, Existential Instantiation
7. $R(c)$  5, 6, Modus Ponens
8. $P(c) \land R(c)$  4, 7, Conjunction
9. $\exists x(P(x) \land R(x))$  8, Existential Generalization

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Homework Cover Page

Please fill in and staple to the front of your homework

Name (print):__________________________

Student ID #:______________________

Homework #:____________________

Discussion Section registered for (check one):

○ Sec. 11 (4:40–5:30 p.m.)

○ Sec. 12 (5:45–6:35 p.m.)