1. Consider the ‘affine plane’ consisting of all vectors in $\mathbb{R}^3$ that are of the form

$$\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

(a) Referring to what you did in HW1, use matlab to plot the surface of the points for $\alpha = [-2 : 5]$ and $\beta = [-2 : 5]$ (Note that $\alpha = [1 : 8] - 3$; and $\beta = [1 : 8] - 3$). [Hint: you can simply run a double loop that generates the coordinates $x(i,j), y(i,j), z(i,j)$ for $i, j = 1 : 8$ ...]

(b) Using plot3 plot the line that joins the 2 points [3, -1, -1]; and [3, -2, 0]. From looking at the plot, which of these 2 points belong(s) to the plane?

(c) Verify you answer the the question (b) theoretically, i.e., determine if each of the given points belongs to the plane.

2. (Magic squares). Associate a variable with each of the 9 cells of a $3 \times 3$ array as follows:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$x_2$</td>
<td>$x_3$</td>
</tr>
<tr>
<td>$x_4$</td>
<td>$x_5$</td>
<td>$x_6$</td>
</tr>
<tr>
<td>$x_7$</td>
<td>$x_8$</td>
<td>$x_9$</td>
</tr>
</tbody>
</table>

It is required to find values for all the variables satisfying the magical property that the sum of the variables in every row, column, and the two main diagonals is the same, here 15. Write the resulting system of linear equations. Find the general solution to this system. You may use rref from matlab to perform the calculations.

Show that *any* solution has the property that $x_i + x_j = 10$ when the indices $i$ and $j$ sum up to 10. Show one particular solution for which all values are nonnegative integers and which is different from the trivial solution of all 5’s.

3. This is related to the computations used in Global Positioning Systems. This is really an oversimplification to the problem. A very nice Wikipedia article is available to tell you the whole story. You need to know that the calculations involved in GPS take into account things like time corrections, and even relativity effects.

A simplified version of the calculations which excludes time and which works in 2-D space is as follows. Suppose you can calculate distances between 3 points (representing satellites) $A, B, C$. These distances can be obtained by getting the time it takes for a signal to travel from points $A, B, C$ and your location - say at point $X$. See figure for an illustration,
Distances $r_a, r_b, r_c$ from $A = (a_1, a_2), B = (b_1, b_2),$ and $C = (c_1, c_2)$ are known so:

\[
\begin{align*}
(x - a_1)^2 + (y - a_2)^2 &= r_a^2 \\
(x - b_1)^2 + (y - b_2)^2 &= r_b^2 \\
(x - c_1)^2 + (y - c_2)^2 &= r_c^2
\end{align*}
\]

or

\[
\begin{align*}
x^2 - 2a_1x + a_1^2 + y^2 - 2a_2y + a_2^2 &= r_a^2 \\
x^2 - 2b_1x + b_1^2 + y^2 - 2b_2y + b_2^2 &= r_b^2 \\
x^2 - 2c_1x + c_1^2 + y^2 - 2c_2y + c_2^2 &= r_c^2
\end{align*}
\]

Note that this system of equations is quadratic (nonlinear). Also, in principle we need only two equations to solve this system because there are two unknowns. Two circles intersect at two points and after we solve [we get the two points] we can figure out which of the two solutions is valid (closest to the current guess of the location of $X$). If we have 3 measurements then it is easier.

(a) Show a $2 \times 2$ linear system obtained from subtracting equation 2 from equation 1 and equation 3.

(b) Assuming that the positions of $A, B, C$ are $A = (-6, 100), B = (22, -4)$ and $C = (26, 68)$ and that the distances are $r_a = 82, r_b = 26,$ and $r_c = 50$ calculate your position $(x, y)$.

4. (matlab) In this exercise you will develop a script to determine the standard row-echelon form of a matrix. Your starting point will be the code `gaussp.m` seen in class earlier in the semester (posted). As was seen earlier in class, the only differences with `gaussp.m` script are: 1) No need for a right-hand side $b$; 2) The row and column dimensions are now different ($m$ rows, $n$ columns); 3) the row number (called $k$ in `gaussp`) evolves differently from the column number $l$. You will need to test if a sub-column is zero and if it is zero (or very close to zero) your algorithm needs to skip to the next column (i.e., increment $l$ by one). If all sub-columns are zero then stop (completion). Some help will be provided in class.

\[
\text{if (MaxColEntry < tolA) ...’skip to next column’}
\]

here `MaxColEntry` is the maximum absolute value of entries in the subcolumn being considered, and `tolA` is the tolerance $\text{norm}(A,1)*1.e-10$ computed at the very beginning of the script.
Your `rechln.m` script should also return an array `jb` which contains the indices of column pivots - just like `rref` does.

You need to run your script on the matrix called `A1` provided in `MatHW2.mat` save file (available from class home page – read with the command `load('MatHW2')`).

Hand in: print out of your row echelon script `rechln.m` and the diary of the execution for the data set provided.

5. A particle moves in a small triangular grid of 6 points with certain probabilities of moving from one point to another given by the following matrix:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1/2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1/4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1/4</td>
<td>0</td>
<td>1/4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/4</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The table is to be read column-wise: if a particle is node $j$ then in the next step it will be in columns 1, 2, ..., 6 with probabilities $a_{1j}, a_{2j}, \ldots, a_{6j}$. Assume the particle is in node 1 initially. Do a few iterations (moves) to determine the particle be in nodes 1, 2, ..., 6 after 1, 2, 4 steps. What is the probability that the particle be at nodes 1, 2, ..., 6 after a very long time?

**Extra credit** (5pts) Find these probabilities by solving a homogeneous system.