Constraint sat. prob. (Ch. 6)
Announcements

Writing 2 due Wednesday
Find best strategy

Clarification: I am an idiot

Last time I explained how to find what action you should pick for the worst-case scenario

However, this is not the Nash equilibrium (which I think I indicated it was)

To find the Nash, you flip it: assume opponent is playing \( p \times \text{[action#1]} + (1-p) \times \text{[action#2]} \)
Find best strategy

On the intersection for their rewards, they will have no preference to switch strategies

If both players do this you are at a Nash...

The only difference to what I introduced last time is that we use the opponent's values instead of our own to decide our strategy
Find best strategy

Best strategy in this game?

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<thead>
<tr>
<th></th>
<th>S</th>
<th>C</th>
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<tbody>
<tr>
<td>S</td>
<td>-10, -10</td>
<td>1, -1</td>
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<tr>
<td>C</td>
<td>-1, 1</td>
<td>0, 0</td>
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</tbody>
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Game of Chicken
Find best strategy

Last time we were blue and parameterized our rewards

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Column 1 (blue): \( p*(-10) + (1-p)*(-1) \)
Column 2 (blue): \( p*(1) + (1-p)*(0) \)

Intersection: \(-9*p - 1 = p, p = -1/10\)

Conclusion: should always chicken (best in the worst-cast scenario)
Nash equilibrium

This time we are blue and parameterized their rewards

Column 1 (red): \( p*(-10) + (1-p)*(1) \)
Column 2 (red): \( p*(-1) + (1-p)*(0) \)

Intersection: \(-11*p + 1 = -p\), \( p = 1/10 \)

Conclusion: should always go straight 1/10 and chicken 9/10 the time
We can see that 10% straight makes the opponent not care what strategy they use:

(Red numbers)
100% straight: \((1/10)*(-10) + (9/10)*(1) = -0.1\)
100% curve: \((1/10)*(-1) + (9/10)*(0) = -0.1\)
50% straight: \(0.5*[(1/10)*(-10) + (9/10)*(1)] + (0.5)*[(1/10)*(-1) + (9/10)*(0)]\)
\[= (0.5)*[-0.1] + (0.5)*[-0.1] = -0.1\]
Nash equilibrium

The opponent does not care about action, but you still do (never considered our values)

Your rewards, opponent 100% straight:
\[(0.1)\times(-10) + (0.9)\times(-1) = -1.9\]

Your rewards, opponent 100% curve:
\[(0.1)\times(1) + (0.9)\times(0) = 0.1\]

The opponent also needs to play at your value intersection to achieve Nash
Repeated games

In repeated games, things are complicated.

For example, in the basic PD, there is no benefit to “lying”.

However, if you play this game multiple times, it would be beneficial to try and cooperate and stay in the [lie, lie] strategy.

\[
\begin{array}{c|cc}
\text{PRISONER 1} & \text{Confess} & \text{Lie} \\
\hline
\text{Confess} & -8, -8 & 0, -10 \\
\text{Lie} & -10, 0 & -1, -1 \\
\end{array}
\]
Repeated games

One way to do this is the tit-for-tat strategy:
1. Play a cooperative move first turn
2. Play the type of move the opponent last played every turn after (i.e. answer competitive moves with a competitive one)

This ensure that no strategy can “take advantage” of this and it is able to reach cooperative outcomes
Repeated games

Two “hard” topics (if you are interested) are:

1. We have been talking about how to find best responses, but it is very hard to take advantage if an opponent is playing a sub-optimal strategy.

2. How to “learn” or “convince” the opponent to play cooperatively if there is an option that benefits both (yet dominated).
Switching gears!
A constraint satisfaction problem is when there are a number of variables in a domain with some restrictions

A consistent assignment of variables has no violated constraints

A complete assignment of variables has no unassigned variables

(A solution is complete and consistent)
CSP

Map coloring is a famous CSP problem
Variables: each state/country
Domain: {yellow, blue, green, purple} (here)
Constraints: No adjacent variables same color

Consistent but partial
CSP

partial and not consistent

Consistent and complete
Another common use of CSP is job scheduling.
Suppose we have 3 jobs: $J_1$, $J_2$, $J_3$

If $J_1$ takes 20 time units to complete, $J_2$ takes 30 and $J_3$ takes 15 but $J_1$ must be done before $J_3$

We can represent this as (and them together): 

$J_1 \& J_2$: $(J_1 + 20 \leq J_2 \text{ or } J_2 + 30 \leq J_1)$

$J_1 \& J_3$: $(J_1 + 20 \leq J_3)$

$J_2 \& J_3$: $(J_2 + 30 \leq J_3 \text{ or } J_3 + 15 \leq J_2)$
Types of constraints

A unary constraint is for a single variable (i.e. $J_1$ cannot start before time 5)

Binary constraints are between two variables (i.e. $J_1$ starts before $J_2$)

All constraints can be broken down into using only binary and unary
Types of constraints

K-consistency is:
for all [consistent sets size k-1] there exists another variable that can be consistent

1-consistency: All values in the domain satisfy the variable's unary constraints

2-consistency: All binary values are in domain

3-consistency: Given consistent binary values, there is a value for a third variable (i.e. if \{A,B\} is consistent, then exists C s.t. \{A,C\} & \{B,C\})
Types of constraints

Rules: 1. Tasmania cannot be red
2. Neighboring providences cannot share colors

Colors:
red
green
Types of constraints

WA = \{ \text{red, green} \}
NT = \{ \text{red, green} \}
Q = \{ \text{red, green} \}
SA = \{ \text{red, green} \}
NSW = \{ \text{red, green} \}
V = \{ \text{red, green} \}
T = \{ \text{red, green} \}

Not 1-consistent as we need T to not be red
Types of constraints

\[
\begin{align*}
WA &= NT = Q = SA = NSW = V \\
&= \{\text{red, green}\} \\
T &= \{\text{blue, green}\}
\end{align*}
\]

1-consistent now

Also 2-consistent, for example:
Pick \( WA = \{\text{green}\} \), this can be consistent with \( NT \) by picking \( NT = \{\text{red}\} \) (true for all pairs as you can just pick the opposite color)
Types of constraints

WA = NT = Q = SA = NSW = V
= \{\text{red, blue, green}\}
T = \{\text{blue, green}\}

\textbf{Not} 3-consistent!

Pick (WA=\{\text{red}\}, SA=\{\text{green}\}) and add NT...
There is no color in \{\text{red, green}\} to make NT consistent with our WA and SA pick, thus it is not 3-consistent (and cannot color with 2)
Applying constraints

We can repeatedly apply our constraint rules to shrink the domain of variables (we just shrunk NT's domain to nothing)

This reduces the size of the domain, making it easier to check:
- If the domain size is zero, there are no solutions for this problem
- If the domain size is one, this variable must take on that value (the only one in domain)
Applying constraints

AC-3 checks all 2-consistency constraints:

1. Add all binary constraints to queue
2. Pick a binary constraint \((X_i, Y_j)\) from queue
3. If \(x\) in domain\((X_i)\) and no consistent \(y\) in domain\((Y_j)\), then remove \(x\) from domain\((X_i)\)
4. If you removed in step 3, update all other binary constraints involving \(X_i\) (i.e. \((X_i, X_k)\))
5. Goto step 2 until queue empty
Applying constraints

Some problems can be solved by applying constraint restrictions (such as sudoku) (i.e. the size of domain is one after reduction)

Harder problems this is insufficient and we will need to search to find a solution

Which is Wednesday's topic!