1. We have seen in class that when $A$ is symmetric, the so-called ‘progressive’ or ’direct’ version of FOM gives an algorithm that can be written as

1. Update LU factorization of $H_m$, $H_m = L_m U_m$

2. $p_m = u_{mm}^{-1} [v_m - u_{m-1,m} p_{m-1}]$

3. $x_m = x_{m-1} + \zeta_m p_m$

4. $\beta_{m+1} v_{m+1} = A v_m - \alpha v_m - \beta_m v_{m-1}$ (Lanczos step)

A. (seen in class) Explain why we can replace line 4 by

4. $r_m = r_{m-1} - \zeta_m A p_{m-1}$

B. Prove that $(A p_i, p_j) = 0$ for $i \neq j$.

C. Obtain the Conjugate Gradient algorithm by rescaling $p_m$ so that $p_m = r_m + \mu_m p_{m-1}$ and then obtain the needed scalars by imposing the right orthogonality conditions.