Problem 1. (20 points)
This question pertains to representing different aspects of the game “Tic-tac-toe” into propositional logic.

(1) List the propositions (e.g. like “A” in “A implies B”) you would use to represent a normal 3x3 tic-tac-toe board. For each proposition, clearly identify what Truth or Falsehood it is representing. (Note: If there is a pattern to them, you can use “...” if you think you have been clear enough for the graders to understand you.)

(2) Write a proposition using your answer from part (1) that encodes the winning condition for the “X” player. In other word, if the proposition you write here is true, then X-player has won.

(3) Write a proposition using your answer from part (1) that encodes the marks possible in the top-left box of the tic-tac-toe board. In other words, you need to represent what possible combinations of propositions is valid for the top-left box.

(4) Write a proposition using your answer from part (1) all possible “valid” boards (assuming “X” player moves first). Due to the size of this problem, restrict yourself to valid board in a 2x2 subset of the 3x3 board. The proposition you create tells whether the board is even possible or not. For example a set of 4 “X”s is not possible, as this means the X-player played 4 times in a row. This breaks the rules of tic-tac-toe.

Problem 2. (15 points)
Convert the following sentence into CNF (Conjunctive Normal Form). Show work for full credit.

\[ ((A \Rightarrow B) \lor (C \lor (\neg D \land \neg E))) \land ((F \lor G) \Rightarrow (\neg H \land I)) \]

Problem 3. (25 points)
Assume you have a KB defined in CNF as below:

\[ \neg A \lor B, \neg B \lor C, \neg C \lor D, \neg D \lor B, D \lor C \]

(1) Use resolution to determine if the following sentence is entailed from the KB:
\[ A \Rightarrow D \]

(2) Use resolution to determine if the following sentence is entailed from the KB:
\[ D \Rightarrow A \]

(3) Use resolution to determine if the following sentence is entailed from the KB:
\[ A \iff D \]
(4) Use resolution to determine if the following sentence is entailed from the KB:
\[ B \iff D \]

(5) Use resolution to determine if the following sentence is entailed from the KB:
\[ A \]

**Problem 4.** (30 points)
Convert the following paragraph into a sufficiently expressive set of First order logic sentences.

“All planets orbit the sun. These planets can be classified as either a rocky planet or a gas giant. Earth is a rocky planet. The Moon orbits the Earth. The moon is not a planet or asteroid. Jupiter is the largest planet. Additionally asteroids orbit the sun in either the Asteroid belt or the Kuiper belt. Planets and asteroids are distinct subsets. All planets are larger than asteroids. Ceres is no longer classified as a planet, but as an asteroid.”

**Programming (python/lisp):**
This time we will look at the constraint satisfaction code in:
/root/csp.py
/root/tests/test_csp.py

**Problem 5.** (15 points)
The “Zebra” problem is a classic constraint satisfaction problem defined as a set of 5: houses, people (of nationality), pets, cigars, drinks and colors. House 1 is on the far left with house 5 on the far right (house 3 is in the middle). Each house (numbered) has a single person (nationality), pet, cigar, drink and color associated with the house. No two houses share the same of any property (for example, each house has its own unique pet). The rules are:

- There are five houses.
- The Englishman lives in the red house.
- The Spaniard owns the dog.
- Coffee is drunk in the green house.
- The Ukrainian drinks tea.
- The green house is immediately to the right of the ivory house.
- The Old Gold smoker owns snails.
- Kools are smoked in the yellow house.
- Milk is drunk in the middle house.
- The Norwegian lives in the first house.
- The man who smokes Chesterfields lives in the house next to the man with the fox.
- Kools are smoked in the house next to the house where the horse is kept.
- The Lucky Strike smoker drinks orange juice.
- The Japanese smokes Parliaments.
- The Norwegian lives next to the blue house.
The question is then: Who owns the Zebra? And who drinks water?

(5/15 points) This classic problem is what is already put in as the Zebra problem. Run the backtracking_search() on this problem and report the answer to the two questions above. (It is fine to use the default parameters for backtracking_search().)

(10/15 points) Modify the problem to match the Zebra problem below (with names, sports, transportation and lawns) and report: Who has flowers in their yard? Who likes to watch Starcraft2?

- There are five houses.
- Albert likes baseball.
- Dietfried has a lawn with cleanly cut grass.
- Virgilijus enjoys watching rugby.
- Gallchobhar gets to work by walking.
- Bricius lives next to Virgilijus.
- The owner of the house with an Astroturf lawn likes baseball.
- The house with flowers in the lawn is between the house with trees and the house with rocks in the lawns.
- The person who likes baseball goes to work on a motorcycle.
- The 2nd house owners like to watch debates.
- The owner of the 4th house goes to work on a bicycle.
- The person who likes soccer takes a bus to work.
- The house with trees on the lawn to the right of the house with an owner who drives a car to work.
- Gallchobhar lives to the right of the person who likes Rugby.