Game theory (Ch. 17.5)
Announcements

Midterm Tuesday!
Find best strategy

How does this compare on PD?

Player 1: \( p = \text{prob confess} \)

P2 Confesses: \(-8p + 0(1-p)\)

P2 Lies: \(-10p + (-1)(1-p)\)

Cross at negative \( p \), but red line is better (confess)
What is Nash for this game?
What is Pareto optimum?

Chicken

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<tr>
<th></th>
<th>S</th>
<th>C</th>
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<tbody>
<tr>
<td>S</td>
<td>-10, -10</td>
<td>1, -1</td>
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<tr>
<td>C</td>
<td>-1, 1</td>
<td>0, 0</td>
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Game of Chicken
To find Nash, assume we (blue) play S probability p, C prob 1-p

Column 1 (red=S): \( p \times (-10) + (1-p) \times 1 \)
Column 2 (red=C): \( p \times (-1) + (1-p) \times 0 \)

Intersection: \(-11p + 1 = -p\), \( p = 1/10 \)

Conclusion: should always go straight \(1/10\) and chicken \(9/10\) the time
We can see that 10% straight makes the opponent not care what strategy they use:

(Red numbers)
100% straight: \((1/10)*(-10) + (9/10)*(1) = -0.1\)
100% chicken: \((1/10)*(-1) + (9/10)*(0) = -0.1\)
50% straight: \((0.5)*[(1/10)*(-10) + (9/10)*(1)] + (0.5)*[(1/10)*(-1) + (9/10)*(0)] = (0.5)*[-0.1] + (0.5)*[-0.1] = -0.1\)
The opponent does not care about action, but you still do (never considered our values)

Your rewards, opponent 100% straight:

\[(0.1) \times (-10) + (0.9) \times (-1) = -1.9\]

Your rewards, opponent 100% curve:

\[(0.1) \times (1) + (0.9) \times (0) = 0.1\]

The opponent also needs to play at your value intersection to achieve Nash Chicken.
Pareto optimum?
All points except (-10,10)

Can think about this as taking a string from the top right and bringing it down & left

Stop when string going straight left and down
Repeated games

In repeated games, things are complicated

For example, in the basic PD, there is no benefit to “lying”

However, if you play this game multiple times, it would be beneficial to try and cooperate and stay in the [lie, lie] strategy
Repeated games

One way to do this is the tit-for-tat strategy:
1. Play a cooperative move first turn
2. Play the type of move the opponent last played every turn after (i.e. answer competitive moves with a competitive one)

This ensure that no strategy can “take advantage” of this and it is able to reach cooperative outcomes
Repeated games

Two “hard” topics (if you are interested) are:

1. We have been talking about how to find best responses, but it is very hard to take advantage if an opponent is playing a sub-optimal strategy

2. How to “learn” or “convince” the opponent to play cooperatively if there is an option that benefits both (yet dominated)
Repeated games

http://ncase.me/trust/