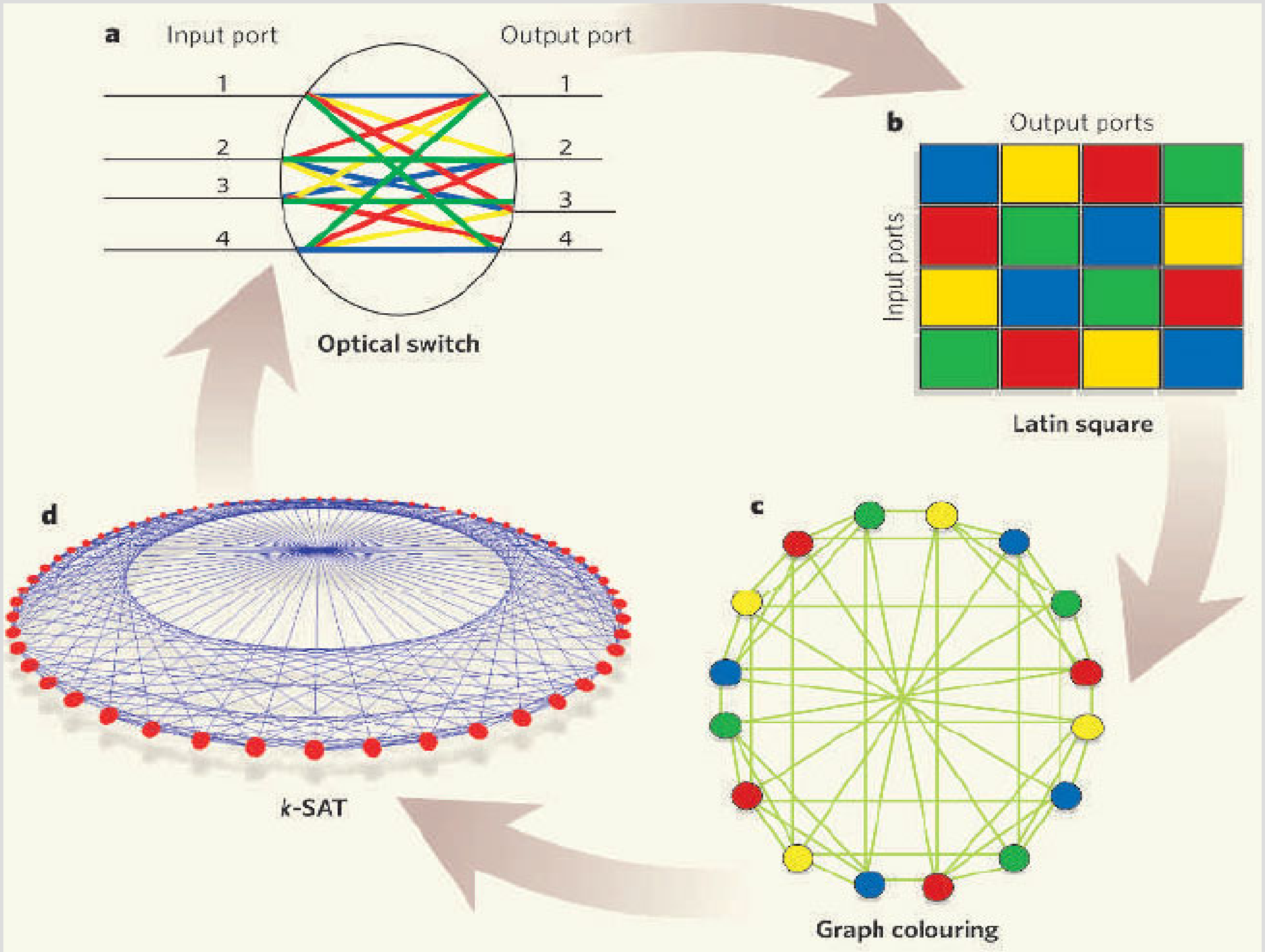
Constraint sat. prob. (Ch. 6)



Announcements

Writing 1 rewrites

-Check moodle for who graded you and email directly to them

Writing 2 due Sunday (more genetic algorithm stuff)

CSP

A <u>constraint satisfaction problem</u> is when there are a number of variables in a domain with some restrictions

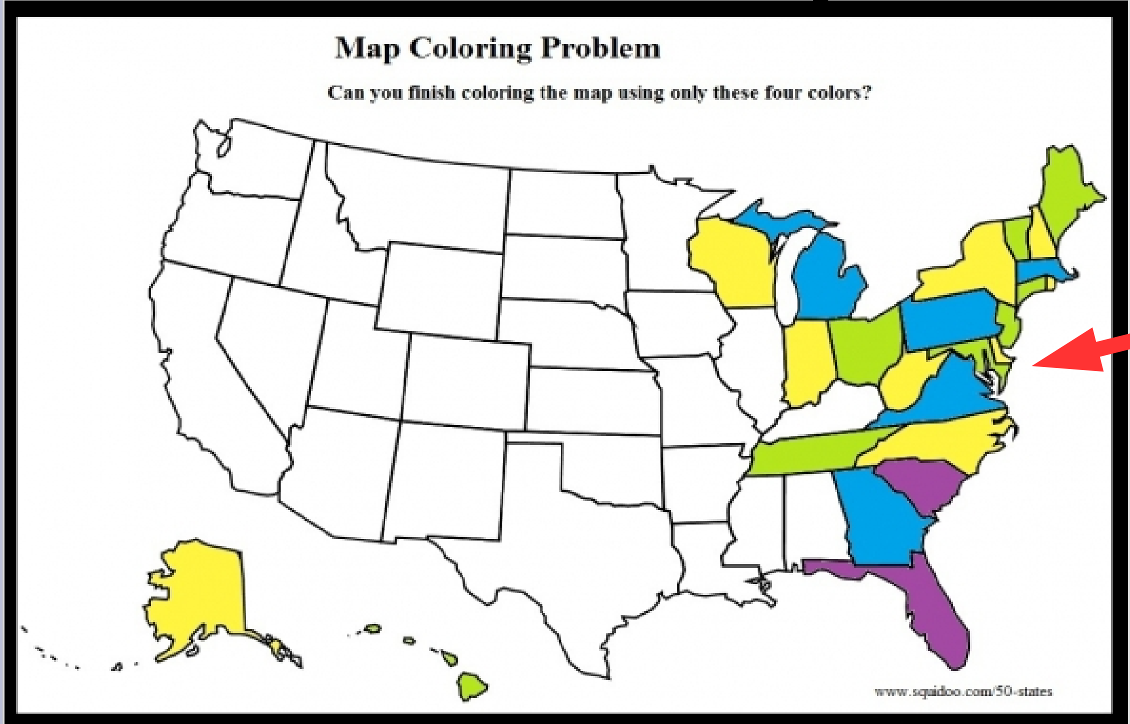
A <u>consistent</u> assignment of variables has no violated constraints

A <u>complete</u> assignment of variables has no unassigned variables

(A solution is complete and consistent)

CSP

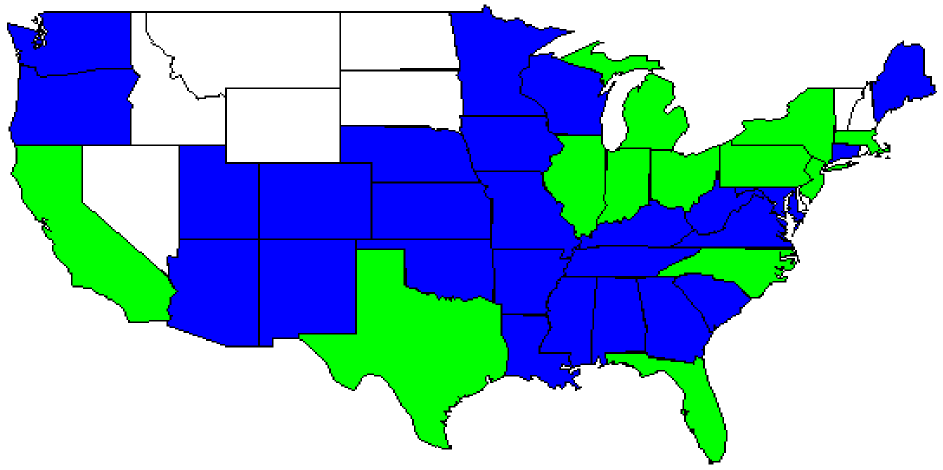
Map coloring is a famous CSP problem
Variables: each state/country
Domain: {yellow, blue, green, purple} (here)
Constraints: No adjacent variables same color

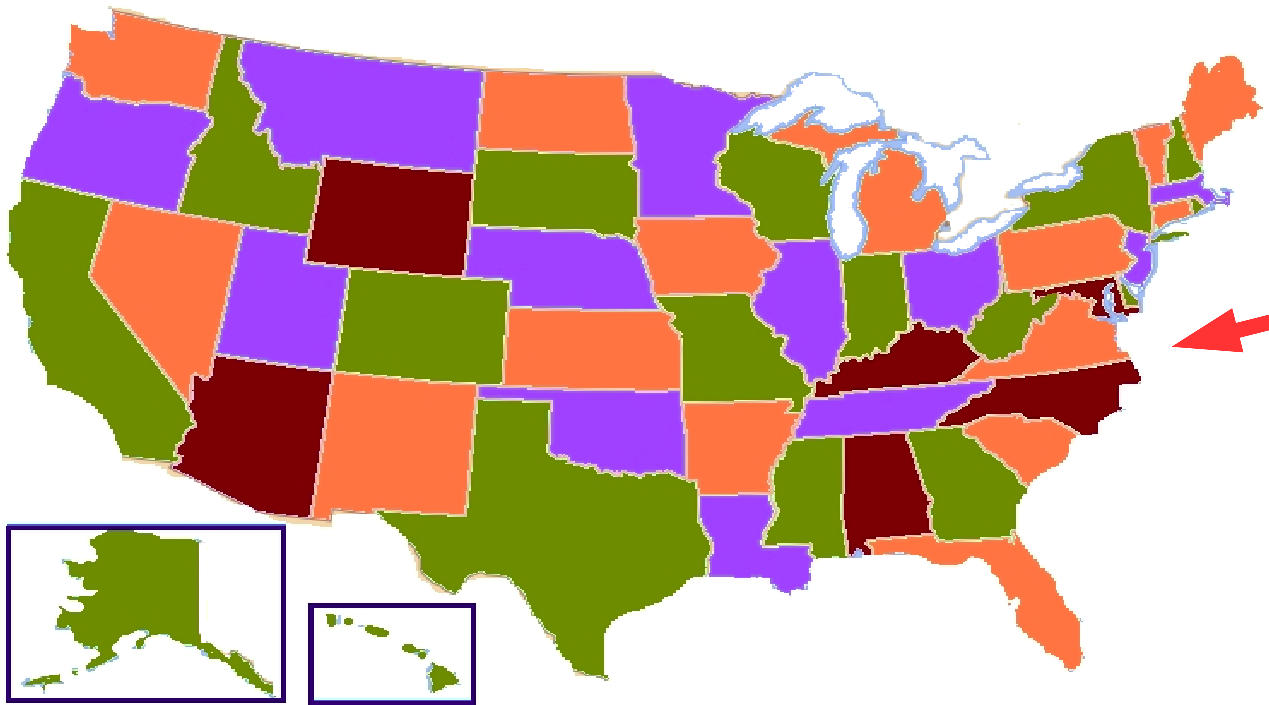

Map Coloring Problem
Can you finish coloring the map using only these four colors?
www.squidoo.com/50-states

Consistent
but partial

CSP

partial and
not consistent

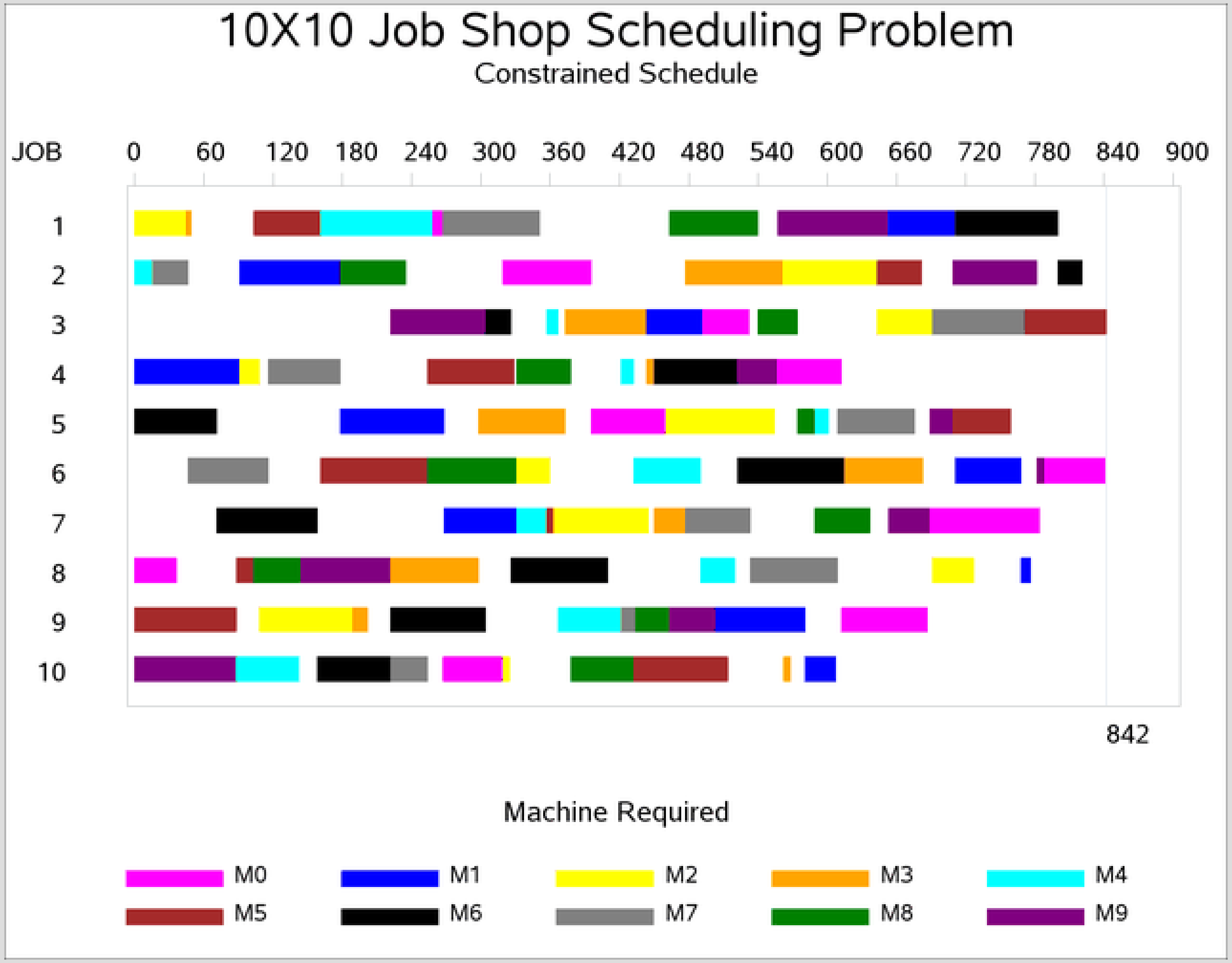




Consistent and
complete

CSP

Another common use of CSP is job scheduling



CSP

Suppose we have 3 jobs: J_1, J_2, J_3

If J_1 takes 20 time units to complete, J_2 takes 30 and J_3 takes 15 <u>but</u> J_1 must be done before J_3

How to write this as a boolean expression?

CSP

Suppose we have 3 jobs: J_1, J_2, J_3

If J_1 takes 20 time units to complete, J_2 takes 30 and J_3 takes 15 <u>but</u> J_1 must be done before J_3

We can represent this as (<u>**and**</u> them together):

J_1 & J_2: ($J_1 + 20 \leq J_2$ <u>**or**</u> $J_2 + 30 \leq J_1$)

J_1 & J_3: ($J_1 + 20 \leq J_3$)

J_2 & J_3: ($J_2 + 30 \leq J_3$ <u>**or**</u> $J_3 + 15 \leq J_2$)

Types of constraints

A unary constraint is for a single variable (i.e. J_1 cannot start before time 5)

Binary constraints are between two variables (i.e. J_1 starts before J_2)

All constraints can be broken down into using only binary and unary

Types of constraints

K-consistency is:
For any consistent sets size (k-1), there exists a valid value for any other variable (not in set)

1-consistency: All values in the domain satisfy the variable's unary constraints
2-consistency: All binary values are in domain
3-consistency: Given consistent 2 variables, there is a value for a third variable(i.e. if {A,B} is consistent, then exists C s.t. {A,C}&{B,C})

Types of constraints

For example, 1-consistent means you can pick 0 consistent variables (if you pick nothing it is always consistent) then any assignment to a new variable is also consistent

This boils down to saying you can pick any valid pick of a single variable in isolation

In other words, you satisfy the unary constraints

Types of constraints

2-consistent means you pick a valid value from the domain for one variable and see if there is <u>any</u> valid assignment for a second var

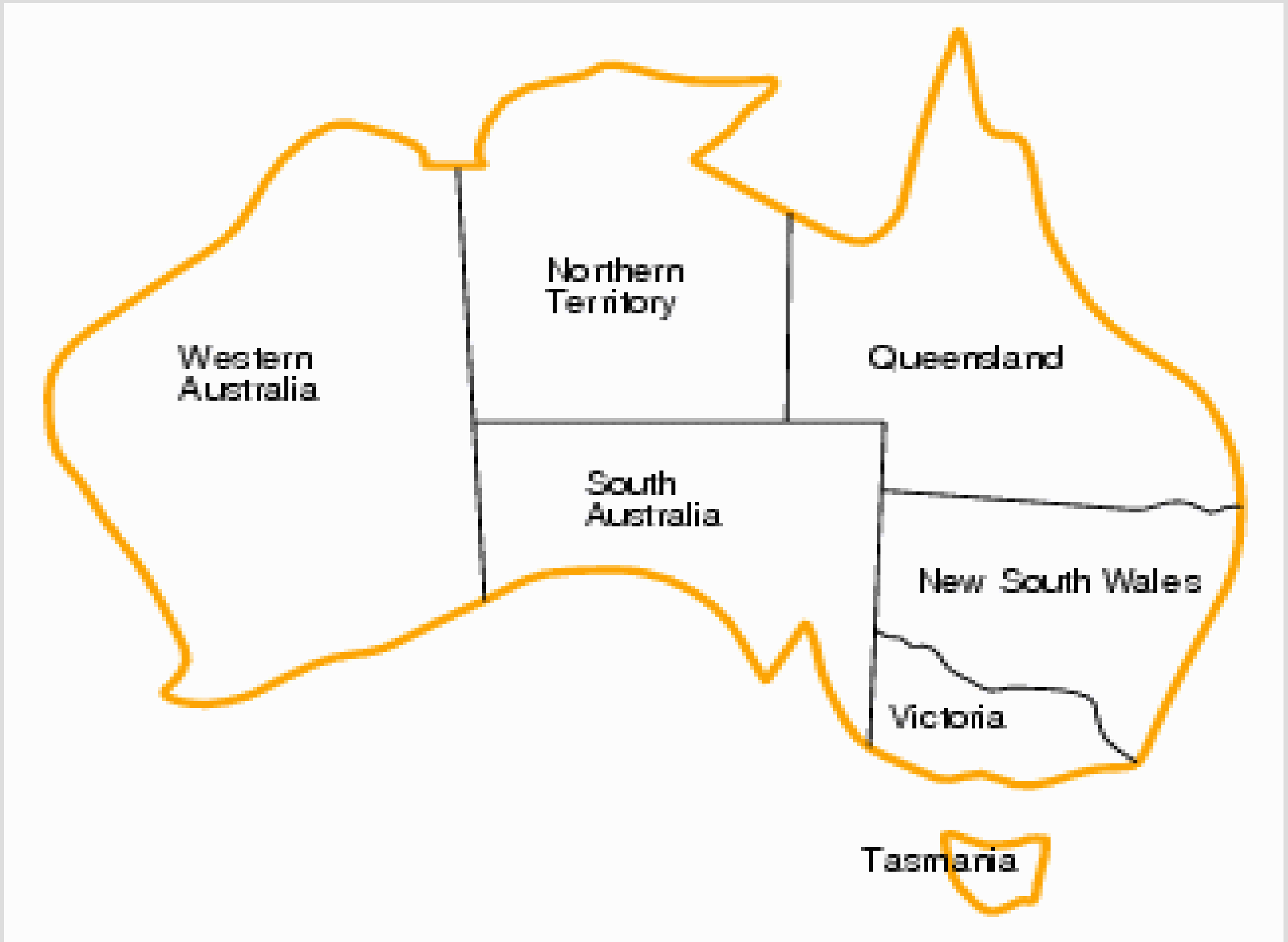
3-consistent means you pick a valid pair of values for 2 variables and see if there is <u>any</u> valid assignment for a third variable

If you are unable to find a valid assignment for the last variable, it is not consistent

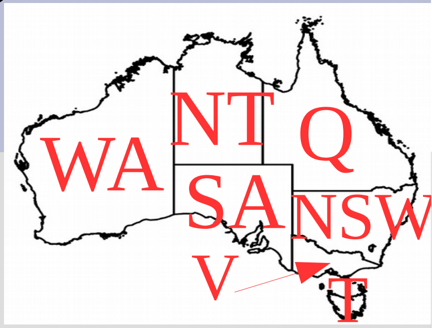
Types of constraints

Rules: 1. Tasmania cannot be red
2.Neighboring providences cannot share colors

2 Colors:
red
green



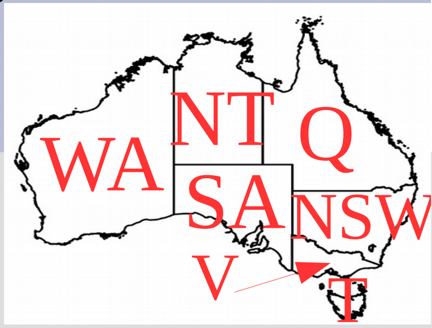
Types of constraints



WA = {red, green}
NT = {red, green}
Q = {red, green}
SA = {red, green}
NSW = {red, green}
V = {red, green}
T = {red, green}

Not 1-consistent as we need T to not be red
(i.e. rule #2 eliminates T=red)

Types of constraints

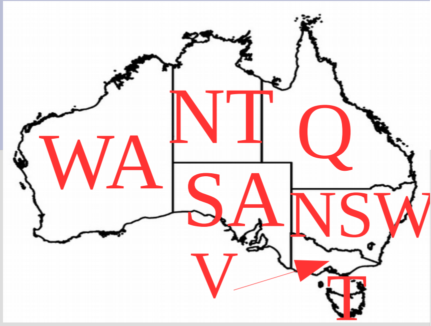


WA = NT = Q = SA = NSW = V
= {red, green}
T = {green}

1-consistent now

Also 2-consistent, for example:
Pick WA as "set k-1", then try to pick NT...
If WA=green, then we can make NT=red
if WA=red, NT=green (true for all pairs)

Types of constraints



WA = NT = Q = SA = NSW = V
= {red, green}
T = {green}

Not 3-consistent!

Pick (WA, SA) and add NT... If NT=green, will not work with either: (WA=red,SA=green) or (WA=green,SA=red)... NT=red also will not work, so NT's domain is empty and not 3-cons.

Types of constraints

Try to do this problem:

3 jobs: J1, J2 and J3

J3 takes 3 time units

J2 takes 2 time units

J1 takes 1 time unit

J1 must happen before J3

J2 cannot happen at time 0 or 1

All jobs must finish by time 7

(i.e. you can start J2 at time 5 but not time 6)

Applying constraints

We can repeatedly apply our constraint rules to shrink the domain of variables (we just shrunk NT's domain to nothing)

This reduces the size of the domain, making it easier to check:

- If the domain size is zero, there are no solutions for this problem
- If the domain size is one, this variable must take on that value (the only one in domain)

Applying constraints

AC-3 checks all 2-consistency constraints:

1. Add all binary constraints to queue
2. Pick a binary constraint (X_i, Y_j) from queue
3. If x in domain(X_i) and no consistent y in domain(Y_j), then remove x from domain(X_i)
4. If you removed in step 3, update all other binary constraints involving X_i (i.e. (X_i, X_k))
5. Goto step 2 until queue empty

Applying constraints

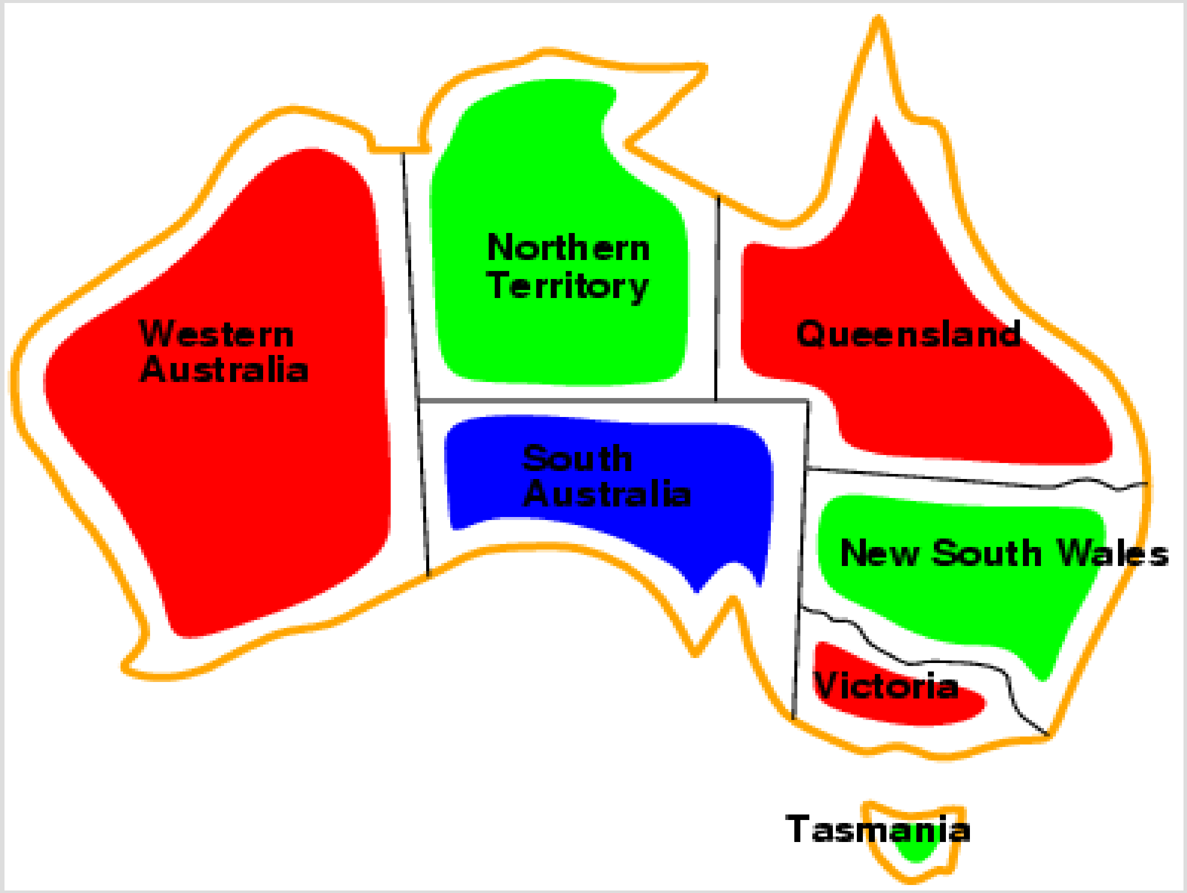
Some problems can be solved by applying constraint restrictions (such as sudoku)
(i.e. the size of domain is one after reduction)

Harder problems this is insufficient and we will need to search to find a solution

Which is what we will do... now

CSP vs. search

Let us go back to Australia coloring:



How can you color using search techniques?

CSP vs. search

We can use an incremental approach:

State = currently colored provinces (and their color choices)

Action = add a new color to any province that does not conflict with the constraints

Goal: To find a state where all provinces are colored

CSP vs. search

Is there a problem?

CSP vs. search

Is there a problem?

Let d = domain size (number of colorings),
n = number of variables (provinces)

The number of leaves are n! * d^n

However, there are only d^n possible states
in the CSP so there must be a lot of duplicate
leaves (not including mid-tree parts)

CSP vs. search

CSP assumes one thing general search does not: the order of actions does not matter

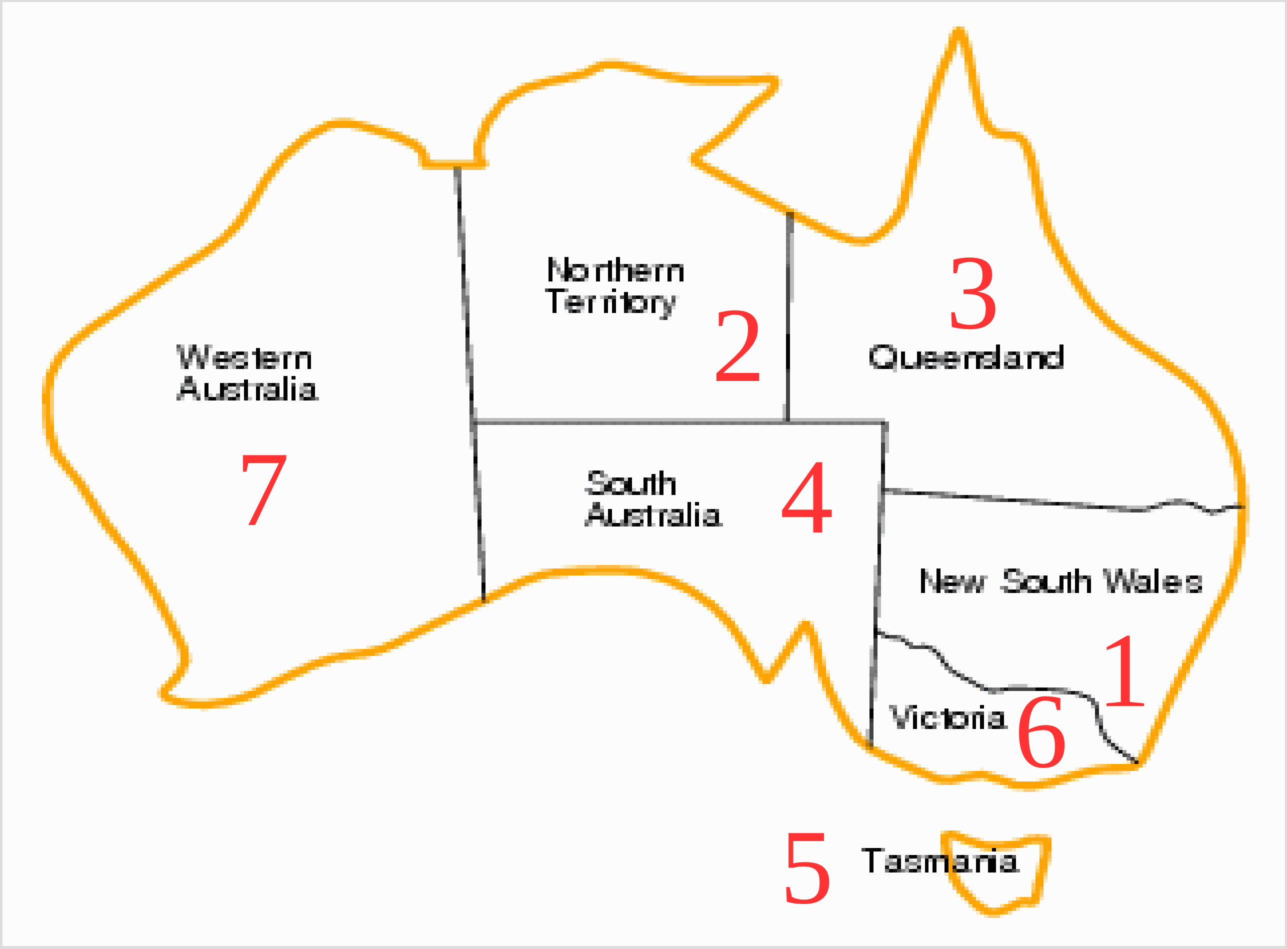
In CSP, we can assign a value to a variable at any time and in any order without changing the problem (all we care about is the end state)

So all we need to do is limit our search to one variable per depth, and we will have a match with CSP of d^n leaves (all combinations)

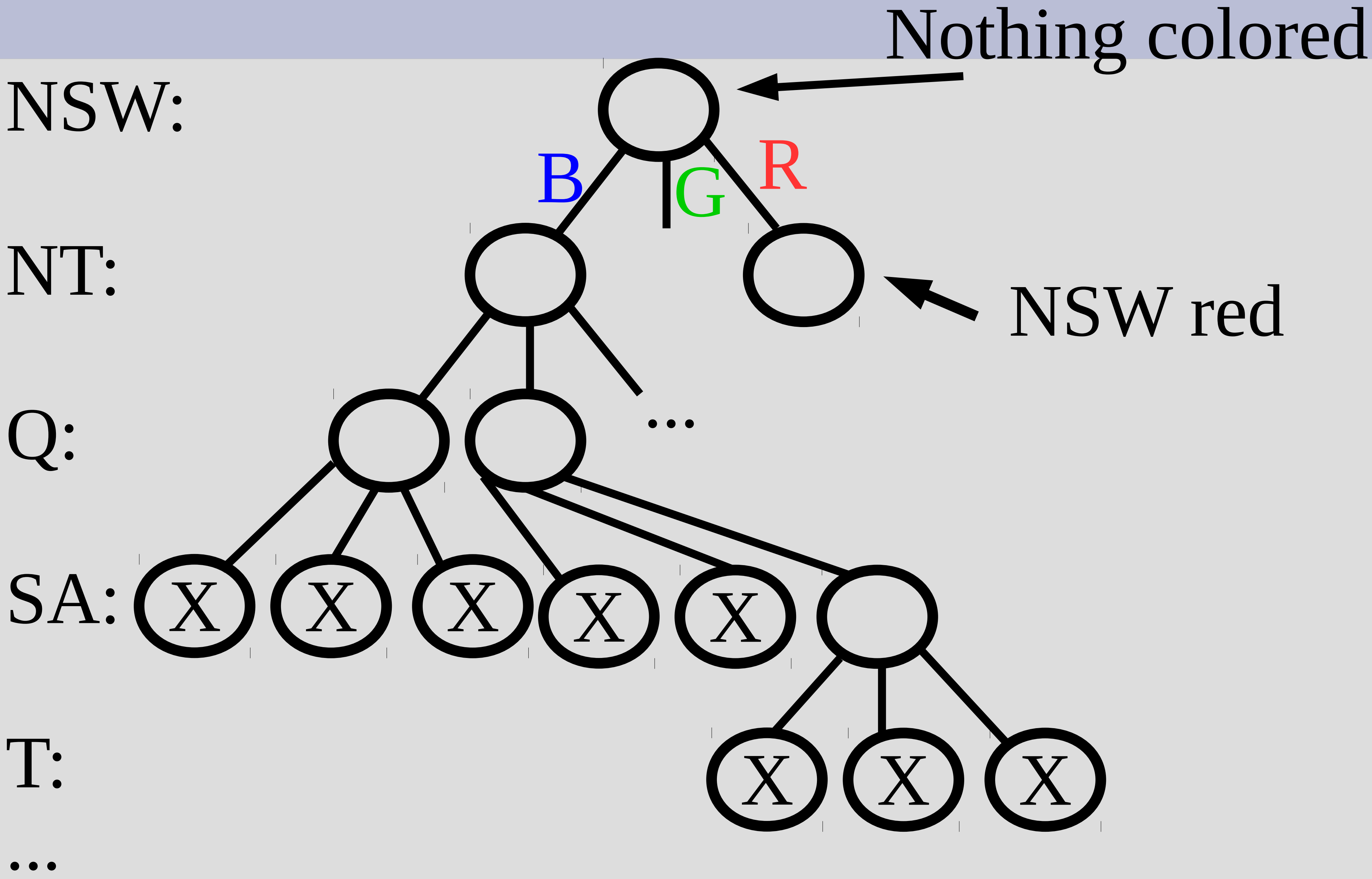
CSP vs. search

Let's apply CSP modified DFS on Australia:
(assign values&variables in alphabetical order)

1st: blue
2nd: green
3rd: red



CSP vs. search



CSP vs. search

STOP PICKING BLUE EVERY TIME!!!!

THIRD PARTY FACE PALM
For when there is so much fail.... you need that extra bit of outside help..

CSP backtracking

However, this is still hope for searching (called backtracking search (it backups up at conflict))

We will improve it by...
1. The order we pick variables
2. The order we pick values for variables
3. Mix search with inference
4. Smarter backtracking

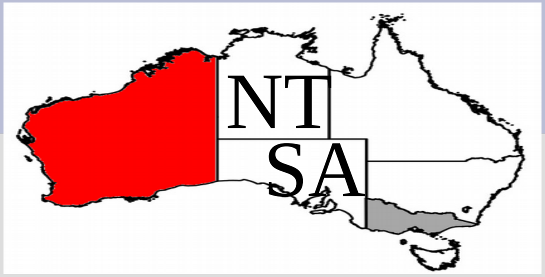
1. What variable?

When picking the variables, we want to the variable with the smallest domain (the most restricted variable)

The best-case is that there is only one value in the domain to remain consistent

By picking the most constrained variables, we fail faster and are able to prune more of the tree

1. What variable?

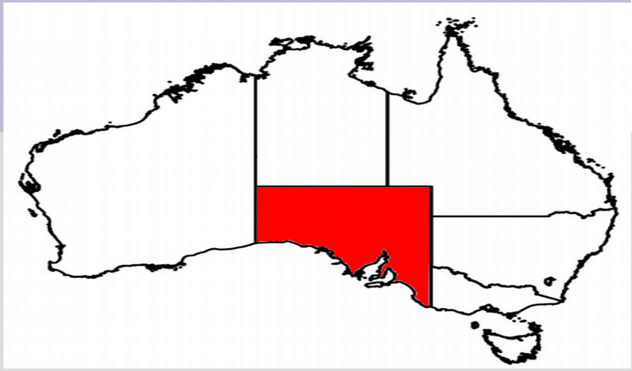


Suppose we pick {WA = red}, it would be silly to try and color V next

Instead we should try to color NT or SA, as these only have 2 possible colorings, while the rest have 3

This will immediately let the computer know that it cannot color NT or SA red (prune these branches right way)

1. What variable?



But we can do even better!

If there is a tie for possible values to take, we pick the variable with the most connections

This ensures that other nodes are more restricted to again prune earlier

For example, we should color SA first as it connects to 5 other provinces

2. What value?

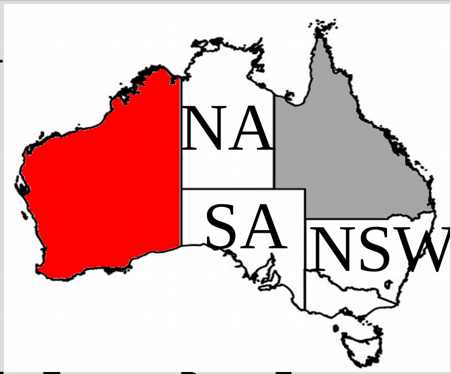
After we picked a variable to look at,
we must assign a value

Here we want to do the opposite: choose the value which constrains the neighbors the <u>least</u>

This is "putting your best foot forward" or trying your best to find a goal (while failing fast helps pruning, we do actually want to find a goal not prune as much as possible)

2. What value?

For example, if we color {WA = red} then pick Q next



Our options for Q are {red, green or blue}, but picking {green or blue} limit NT & SA to only one valid color and NSW to 2

If we pick {Q=red}, then NT, SA & NSW all have 2 valid possibilities (and this happens to be on a solution path)

1. & 2.

An analogy to 1&2 is: “trying our best (2) to solve the weakest link (1)”

By tackling the weakest link first, it will be easier for less constrained nodes to adapt/ pick up the slack

However, we do want to try and solve the problem, not find the quickest way to fail (i.e. always picking blue... ... >.<)

3. Mix search & inference?

We described how AC-3 can use inference to reduce the domain size

Inference does not need to run in isolation; it works better to assign a value then apply inference to prune before even searching

This works well in combination with 1 as uses the domain size to choose the variable and 3 shrinks domain sizes to be consistent

3. Mix search & inference?

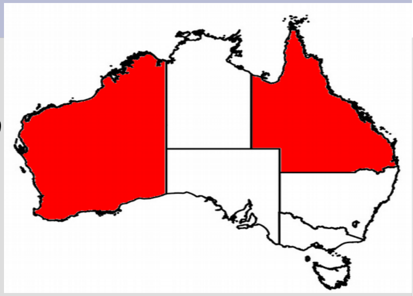
This is somewhat similar to providing
a heuristic for our original search

Inference lets us know an estimation of what
colors are left and can be done efficiently

We can use this estimate to guide our search
more directly towards the goal

3. Mix search & inference?

In the previous example: {WA = red}, then color Q



We want to choose {Q = red} to allow the most choices for NT and SA

Without inference we will not know about this restriction and just have assign and realize this constraint when we create a conflict

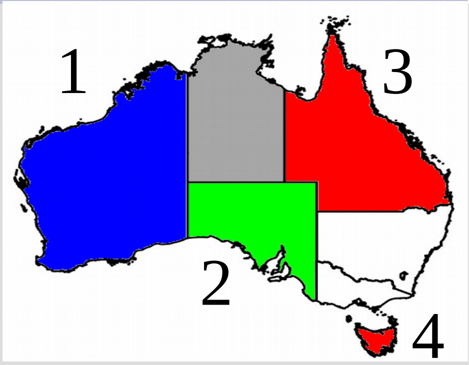
4. Smart backtracking

Instead of moving our search back up a single layer of the tree and picking from there...

We could backup to the first node above the conflict that was actually involved in the conflict

This avoids in-between nodes which did not participate in the conflict

4. Smart backtracking



Suppose we assigned (in this order):
{WA = B, SA = G, Q = R, T = R}
then pick NT

NT has all three colors neighboring it, so a conflict is reached

In normally, we would backtrack and try to change T (i.e. 4), but this was actually not involved in the conflict (1, 2 & 3 were)

Complete-state CSP

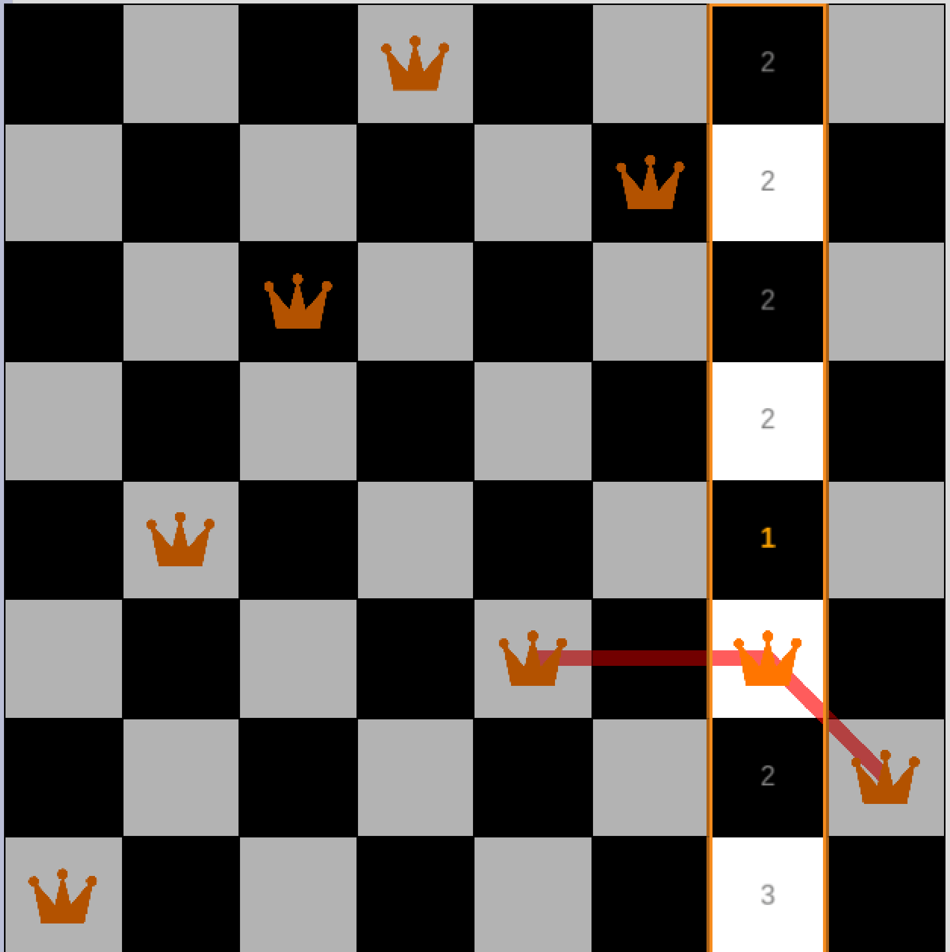
So far we have been looking at incremental search (adding one value at a time)

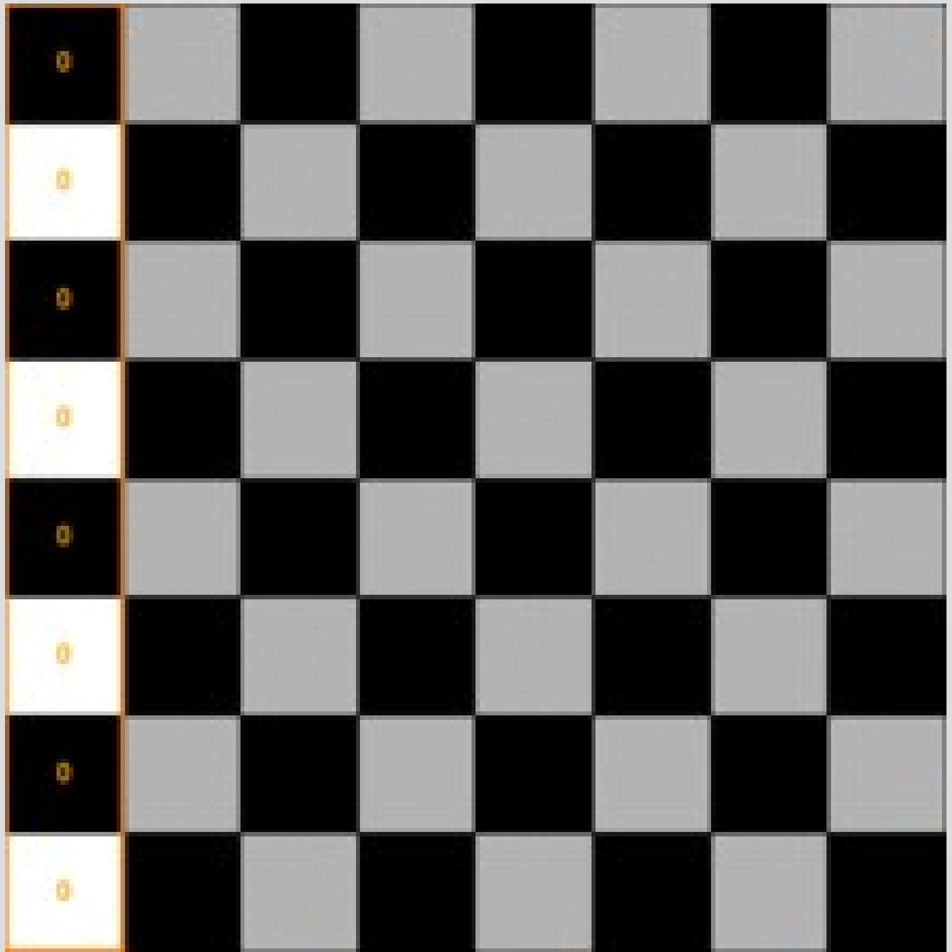
Complete-state searches are also possible in CSPs and can be quite effective

A popular method is to find the min-conflict, where you pick a random variable and update the choice to be one that creates the least number of conflicts

Complete-state CSP

This works incredibly well for the n-queens problem (partially due to dense solutions)





Complete-state CSP

As with most local searches (hill-climbing), this method has issues with plateaus

This can be mitigated by avoiding recently assigned variables (forces more exploration)

You can also apply weights to constraints and update them based on how often they are violated (to estimate which constraints are more restrictive than others)

Complete-state CSP

Local search does not have “locally optimal” solution our general search does

As we have a CSP, the “local optimal” may occur, but if it is not 0 then we know we are not satisfied (unless we searched the whole space and find no goal)

This is almost as if we had an almost perfect heuristic built in to the problem!