Consider the polynomial $p(x) = x^3 + 199.99x^2 + 9998x - 100$. Suppose the goal is to find a root near $x = -100$. Using Newton’s method in Matlab in single precision we calculate the root to be $x_c = -99.9746475$. When we plug this into the polynomial (again in single precision) we find the calculated value of $p(x_c)$ to be $fl(p(x_c)) = -0.0253525$. How does one decide if this answer is accurate or the best one can do in single precision?

1. Let $x_t$ denote the true exact root. Suppose $x_c - x_t \approx 3 \times 10^{-2}$. What is the absolute error in the computed solution $x_c$?

2. What is the relative error in the computed solution $x_c$?

3. How many decimal digits of accuracy is in the solution $x_c$?

4. Find a nearby polynomial $\tilde{p}$ by making a small modification to the coefficients such that $x_c$ is an exact root of $\tilde{p}$. Hint: apply the change to the constant term.

5. What are the absolute and relative \textit{backward} errors in the solution $x_c$?