Consider the matrix

\[ A = \begin{pmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \\ 7 & 8 & -9 \end{pmatrix}. \]

1. Give the 1-norm \( \|A\|_1 \) for the matrix and give a vector \( v \) such that \( \|v\|_1 = 1 \) and \( \|Av\|_1 = \|A\|_1 \). (Hint, try a coordinate unit vector.)

2. Using the matrix in the previous question, we have the identity

\[
\min_{x} \frac{\|Ax\|_1}{\|x\|_1} = \frac{282}{166} = \|A\|_1, \text{ achieved at } x^* = \begin{pmatrix} 93 \\ -6 \\ 67 \end{pmatrix}.
\]

Use this identity to give the condition number of \( A \), using the 1 norm. (hint: what’s \( \|A^{-1}\|_1 \)?) You can leave your answer as a fraction of numbers \( \frac{\text{integer}}{\text{integer}} \).

3. Find a matrix \( B \) with \( \|B\|_1 = \frac{282}{166} \) such that \((A - B)x^* = 0\). Hint: \( B \) has the form

\[
\begin{pmatrix} \pm b & \pm b & \pm b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},
\]

and also note that \( Ax^* = \begin{pmatrix} 282 \\ 0 \\ 0 \end{pmatrix} \) and \( 93 + 6 + 67 = 166 \).