Please review carefully the instructions given for Homework 1. They apply to this assignment, too.

Please hand in your answers to the following problems. Problem numbers, where indicated, are from the seventh edition of the Rosen text:

1. (5 points) In class we discussed how to derive the formula for \( S_n = \sum_{i=1}^{n} i^2 \) using the (known) formula for \( A_n = \sum_{i=1}^{n} i \) and the (unknown) formula for \( C_n = \sum_{i=1}^{n} i^3 \).

   Use a similar approach to derive the formula for \( C_n \) using the (known) formulae for \( S_n \) and \( A_n \) and the (unknown) formula for \( F_n = \sum_{i=1}^{n} i^4 \). Simplify your formula as much as possible.

2. (5 points) p. 168, #16, parts (c) and (f). Your solution to each recurrence should be simplified as much as possible.

   You can use either forward or backward substitution (see pages 158–160) to solve each recurrence.

3. (5 points) Consider the following program fragment:

   ```java
   for i = 1 to n do
     for j = 1 to i do
       for k = 1 to j do
         statement S
       end
     end
   end
   ```

   Write down a summation for the number of times statement S is executed. Derive as simple a formula as possible for the summation.

4. (6 points) p. 176, #10. Justify your answer for each part briefly.

5. (5 points) Let \( S \) be the set of positive integers that are not divisible by 3. Show that \( S \) is countable by giving explicitly a bijective function from \( \mathbb{Z}^+ \) to \( S \). Be sure to show that your function is indeed a bijection.
6. (5 points) Let $A$ and $B$ be infinite sets with the same cardinality. Prove that the powersets $\mathcal{P}(A)$ and $\mathcal{P}(B)$ also have the same cardinality. Do this by giving explicitly a bijective function from $\mathcal{P}(A)$ to $\mathcal{P}(B)$. You must also prove that your function is indeed a bijection.

7. (4 points) Use diagonalization to prove that the set consisting of all infinite-length binary strings is uncountable. Be sure to spell out the steps in your proof carefully.