ASSIGNMENT 3:
Assigned: 03/11/19 Due: 03/31/19 at 11:55 PM (submit via Canvas, you may scan or take a picture of your paper answers) Submit only pdf or txt files (for non-code part), separate submission for code files Show as much work as possible for all problems!

Problems 1 to 4 will use the following Bayesian network (specifically, a Hidden Markov Model): (Here X’s represent whether the water table is low/medium/high and E’s represent whether or not your basement gets flooded.)

**Problem 1.** (15 points)
Give the filtering probabilities for the water table’s three values in the following cases:
(1) $P(x_1 | e_1=\text{“not flooded”})$
(2) $P(x_2 | e_1=\text{“not flooded”}, e_2=\text{“not flooded”})$
(3) $P(x_3 | e_1=\text{“not flooded”}, e_2=\text{“not flooded”}, e_3=\text{“flooded”})$

**Problem 2.** (15 points)
Given the same sequence of evidence as problem 1, find the smoothed estimates for $x_1$, $x_2$ and $x_3$. (In other words, the evidence is still: $e_1=\text{“not flooded”}, e_2=\text{“not flooded”}, e_3=\text{“flooded”}$)

Then plot the probabilities both filtering and smoothed estimates on the same graph. (Note: you will need two points/lines for a single probability, so overall you should have four lines.)

**Problem 3.** (20 points)
Assume we are using the same HMM as problems 1 & 2, but we have difference evidence: $e_1=\text{“flooded”}, e_2=\text{“not flooded”}, e_3=\text{“not flooded”}, e_4=\text{“flooded”}, e_5=\text{“not flooded”}$

What is the most likely sequence of water table levels for these five days?

If you found that day 6 was “flooded” (i.e. $e_6=\text{“flooded”}$), what is the most likely sequence now?
**Problem 4.** (25 points)
Use particle filtering to estimate:

\[
P(x_{10} \mid e_1=\text{“not flooded”}, e_2=\text{“not flooded”}, e_3=\text{“flooded”}, e_4=\text{“flooded”}, e_5=\text{“not flooded”}, e_6=\text{“not flooded”}, e_7=\text{“not flooded”}, e_8=\text{“not flooded”}, e_9=\text{“flooded”}, e_{10}=\text{“not flooded”})
\]

(i.e. the days “flooded” are 3, 4 and 9. The rest are “not flooded”.) Give the number of particles used in your sampling, along with the probability for the water table values.

**Problem 5.** (20 points)
Assume we are using the Frisbee example from class, where: \(P(x_0) = N(0,1)\) and \(P(e_t \mid x_t) = N(x_t, 0.75)\).
How accurate do you need to be so that after 10 throws, the variance is not more than 10.

**Problem 6.** (15 points)
Suppose you have the simple HMM 3-node Bayes net shown below. Assume that \(P(x_0)\) is uniformly distributed between [-1, 1]. The transition probability, \(P(x_{t+1} \mid x_t)\), is \(N(x_t, 1)\). The evidence probability is: \(P(e_t \mid x_t) = N(x_t, 0.5)\)

![Diagram of 3-node HMM Bayes net](image)

Use this description to approximate the distribution for \(P(x_1 \mid e_1=0)\) and plot this distribution on a graph. Extend this network to 5 nodes to estimate the distribution for \(P(x_2 \mid e_1=0, e_2=0)\) and plot this distribution on a graph. As \(t \to \infty\), describe what happens to the distribution of the filtering probabilities (assuming the evidence always says zero).