

1 [Matlab]

(a) Use Matlab to generate the array X of size 5×3 with entries $x_{ij} = i + j - 1$. Then generate a tridiagonal matrix T whose sub-diagonal is $X(1 : n - 1, 1)$, diagonal is $X(1 : n, 2)$, and whose super-diagonal is $X(2 : n, 3)$.

(b) Convert T to sparse format by using

(i) $S = \text{sparse}(T)$;

(ii) The command `spdiags`;

(iii) The command `spconvert` or `sparse` but using arrays obtained from X ..

2 This exercise is about computational graphs and ‘back-propagation’. Consider the simple expression:

$$c(x, y, z) = z * (x + y) + 2 * y + z$$

[a] Show a computational graph that computes $c(x, y, z)$ where each node performs an atomic operation comprising an add a multiply (at most) [$a = x + y$, $b = 2 * y + z$ and finally (at top) $c = z * a + b$, and x, y, z are ‘leafs’]. Is the resulting graph a tree? Represent the dependencies by directed edges. Build the graph from left (‘leaves’) to right.

[b] Forward loop: Show how calculation proceeds for case $x = y = 1, z = 2$.

[c] Show how to calculate $\partial c / \partial y$ in the same forward manner.

[d] Now start from c at the top, and see how you can calculate ∇c with a chain-rule in situation where a, b, c have already been computed.