CSci 8314 Practice Exercises Set #5 04-17-2019

To solve the linear system $Ax = b$, where A is nonsingular, we consider a projection method which uses a two-dimensional space at each step. At a given step, we take $\mathbf{K} = \text{Span}\{r, Ar\}$, where $r = b - Ax$ is the current residual, and $\mathbf{L} = A\mathbf{K}$.

1. For a basis of $\mathbf{K}$ we use the vectors $p_1 = \frac{r}{\|Ar\|_2}$ and the vector $p_2 = Ap_1 - \gamma p_1$ such that Ap_2 is orthogonal to Ap_1. Give the formula for computing p_2.
2. Write the algorithm for performing the projection method described above.
3. To which other method is this algorithm mathematically equivalent? Analyze its convergence.