1. In this homework you implement a basic recursive spectral bisection partitioning in matlab. Two-way bisection partitioning was demonstrated in class and the matlab diaries are available online from the class web-site. You will need to implement a recursive version of this technique which will keep partitioning the largest available partition at a given step, until a desired number of parts is reached. The interface to the function must be:

   function [Sets, labl] = RSB(A, maxsets) .

On input, \( A \) is a sparse matrix graph Laplacian, and \( \text{maxsets} \) is the desired number of partitions. On return, \( \text{Sets} \) is a cell array with \( \text{Sets}\{i\} \) containing the list of nodes (indices) of partition \( i \), and \( \text{labl} \) is an array of length \( n \) containing the labels (partition number) of the nodes. (The information provided by \( \text{labl} \) is redundant but it is returned because it will be available from the script \( \text{RSB} \)).

The algorithm starts with a Laplacean \( A \) and a list of all the nodes in some array, e.g., \( \text{list} = \{1:n\} \). The goal is to build an array of cells called \( \text{Sets} \) that will contain the partitions. Initially we have one set, so \( \text{Sets}\{1\} = \text{list} \). At any given step we are partitioning a set, whose number is, say, \( \text{oldset} \) in two subsets. Initially, \( \text{oldset} = 1 \). A matrix \( B \) is associated with the current list, starting with \( B = A \). We compute the lowest 2 eigenvalues of \( B \) (using \texttt{eigs}) , and get the Fiedler vector \( v \). We partition \( \text{list} \) into two sets \( \text{listp} \) and \( \text{listm} \) according to the sign of \( v - \text{median}(v) \). Thus, two sets are created. One of these sets is new, and is added to \( \text{Sets} \): \( \text{Sets}\{\text{newset}\} = \text{listm} \). The other one will just replace the current set that is being partitioned in two: \( \text{Sets}\{\text{oldset}\} = \text{listm} \).
The process is illustrated in the above Figure where are already 4 partitions and the algorithm is in the process of partitioning set \texttt{oldset}=2. Note that at each step one partition is created and another one replaced.

You will be given a data set containing a finite element mesh to partition. In the matlab web-page you will find scripts for reading the mesh and for plotting it (\texttt{mygplot}) and a couple of other utilities.

For this homework I will ask that you upload the following on Canvas:

(a) Your script \texttt{RSB}

(b) ... and any comments you may have [optional]

I will download your driver and RSB script and execute them - so make sure there is nothing missing. It is important to supply only those 2 files your script \texttt{RSB.m} mentioned above and to not make any changes to the other ones provided to you.