Planning (Ch. 10)

PLANNING

Somehow, I don't think you thought your cunning plan all the way through.
Graph Plan

Consider this problem:

Initial: $\text{Sleepy}(me) \land \text{Hungry}(me)$

Goal: $\neg \text{Sleepy}(me) \land \neg \text{Hungry}(me)$

Action( $\text{Eat}(x)$,  
Precondition: $\text{Hungry}(x)$,  
Effect: $\neg \text{Hungry}(x)$)

Action( $\text{Coffee}(x)$,  
Precondition: ,  
Effect: $\neg \text{Sleepy}(x)$)

Action( $\text{Sleep}(x)$,  
Precondition: $\text{Sleepy}(x) \land \neg \text{Hungry}(x)$,  
Effect: $\neg \text{Sleepy}(x) \land \text{Hungry}(x)$)
Mutexes: actions

Mutex Action rules:

1. \( x \in \text{Effect}(A1) \land \neg x \in \text{Effect}(A2) \)
2. \( x \in \text{Pre}(A1) \land \neg x \in \text{Effect}(A2) \)
3. \( x \in \text{Pre}(A1) \land \neg x \in \text{Pre}(A2) \)
There are 2 rules for states, but unlike action-mutexes they can change across levels:

1. Opposite relations are mutexes (x and ¬x)
2. If there are mutexes between all possible actions that “lead” to a pair of states...

Two ways that “leading” can be in mutex:
2.1. Actions are in mutex
2.2. Preconditions of action pair are in mutex
Another way to compute state mutexes:

1. Add mutexes between all pairs in state
2. If any pair of actions can lead to this pair of relationships, un-mutex them

Recap:
If any valid pair of actions = no mutex
All ways of reaching invalid = mutex
1. Opposite relations are mutexes ($x$ and $\neg x$)
2. If there are mutexes between all possible actions that lead to a pair of states
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2. If there are mutexes between all possible actions that lead to a pair of states
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2. If there are mutexes between all possible actions that lead to a pair of states

None... but if we remove coffee...
1. Opposite relations are mutexes (x and \( \neg x \))
2. If there are mutexes between all possible actions that lead to a pair of states

\[ S \leftrightarrow H \]
\[ S \leftrightarrow E \]
\[ S \leftrightarrow \neg E \]

\( Sl \) has mutex with both \( E \) and \( \text{NoOp}(\neg H) \)

This mutex will be gone on the next level (as you can eat again)
1. Opposite relations are mutexes (x and ¬x)
2. If there are mutexes between all possible actions that lead to a pair of states
Mutexes: actions

Consider...
Initial: \( \neg \text{Money} \land \neg \text{Smart} \land \neg \text{Debt} \)
Goal: \( \neg \text{Money} \land \text{Smart} \land \neg \text{Debt} \)

Action( School,
Precondition: ,
Effect: Debt \land Smart )

Action( Job,
Precondition: ,
Effect: Money \land \neg \text{Smart} )

Action( Pay,
Precondition: Money,
Effect: \neg \text{Money} \land \neg \text{Debt} )
Mutexes: actions
Mutexes: actions

Non-trivial mutexes:
(Sc, P),
(J, P),
(Sc, J),
(P, D&M&M&M M),
(Sc, M D&M&M S),
(J, M&M&M)
Mutexes: actions

Non-trivial mutexes:
(SC, P),
(J, P),
(SC, J),
(P, D&M&M M),
(SC, D&D S),
(J, M&M)
Non-trivial mutexes:

- (SC, P),
- (J, P),
- (SC, J),
- (P, D&M&┐M),
- (SC,┐D&S),
- (J,┐M&S)
GraphPlan

GraphPlan can be computed in $O(n(a+l)^2)$, where $n =$ levels before convergence
$a =$ number of actions
$l =$ number of relations/literals/states
(square is due to needing to check all pairs)

The original planning problem is PSPACE, which is known to be harder than NP
Let's consider this problem:

**Initial:** \( Clean \land Garbage \land Quiet \)

**Goal:** \( Food \land \neg Garbage \land Present \)

<table>
<thead>
<tr>
<th>Action</th>
<th>Precondition</th>
<th>Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>((MakeFood, )</td>
<td>(Clean)</td>
<td>(Food)</td>
</tr>
<tr>
<td>((Wrap, )</td>
<td>(Quiet)</td>
<td>(Present)</td>
</tr>
<tr>
<td>((Takeout, )</td>
<td>(Garbage)</td>
<td>(\neg Garbage \land \neg Clean)</td>
</tr>
<tr>
<td>((Dolly, )</td>
<td>(Garbage)</td>
<td>(\neg Garbage \land \neg Quiet)</td>
</tr>
</tbody>
</table>
GraphPlan: states

Take out one more level
Mutexes

Possible state pairs:

F, C  C, Q
F, ┐C  C, ┐Q
F, G  C, P
F, ┐G  ┐C, G
F, Q  ┐C, ┐Q
F, P  ┐C, ┐Q
C, ┐C  ┐C, ┐Q
C, G  ┐C, P
C, ┐G  ... (more)
Mutexes

Make one more level here!
Blue mutexes disappear.

Mutexes

Pink = new mutex
GraphPlan as heuristic

GraphPlan is optimistic, so if any pair of goal states are in mutex, the goal is impossible.

3 basic ways to use GraphPlan as heuristic:
(1) Maximum level of all goals
(2) Sum of level of all goals (not admissible)
(3) Level where no pair of goals is in mutex

(1) and (2) do not require any mutexes, but are less accurate (quick 'n' dirty)
For heuristics (1) and (2), we relax as such:

1. Multiple actions per step, so can only take fewer steps to reach same result
2. Never remove any states, so the number of possible states only increases

This is a valid simplification of the problem, but it is often too simplistic directly
GraphPlan as heuristic

Heuristic (1) directly uses this relaxation and finds the first time when all 3 goals appear at a state level

(2) tries to sum the levels of each individual first appearance, which is not admissible (but works well if they are independent parts)

Our problem: goal={Food, Garbage, Present}
First appearance: F=1, G=1, P=1
GraphPlan: states

Level 0:
- C
- G
- Q
- M

Level 1:
- F
- C
- T
- G
- D
- W
- P

Heuristic (1):
\[ \text{Max}(1,1,1) = 1 \]

Heuristic (2):
\[ 1+1+1=3 \]
GraphPlan as heuristic

Often the problem is too trivial with just those two simplifications.

So we add in mutexes to keep track of invalid pairs of states/actions.

This is still a simplification, as only impossible state/action pairs in the original problem are in mutex in the relaxation.
GraphPlan as heuristic

Heuristic (3) looks to find the first time none of the goal pairs are in mutex

For our problem, the goal states are: (Food, ¬ Garbage, Present)

So all pairs that need to have no mutex: (F, ¬ G), (F, P), (¬ G, P)
None of the pairs are in mutex at level 1

This is our heuristic estimate
Finding a solution

GraphPlan can also be used to find a solution:
(1) Converting to a Constraint Sat. Problem
(2) Backwards search

Both of these ways can be run once GraphPlan has all goal pairs not in mutex (or converges)

Additionally, you might need to extend it out a few more levels further to find a solution (as GraphPlan underestimates)
GraphPlan as CSP

Variables = states, Domains = actions to there
Constraints = mutexes & preconditions

Variables: \( G_1, \ldots, G_4, P_1 \ldots P_6 \)

Domains: 
- \( G_1: \{ A_1 \} \)
- \( G_2: \{ A_2 \} \)
- \( G_3: \{ A_3 \} \)
- \( G_4: \{ A_4 \} \)
- \( P_1: \{ A_5 \} \)
- \( P_2: \{ A_6, A_{11} \} \)
- \( P_3: \{ A_7 \} \)
- \( P_4: \{ A_8, A_9 \} \)
- \( P_5: \{ A_{10} \} \)
- \( P_6: \{ A_{10} \} \)

Constraints (normal):
- \( P_1 = A_5 \Rightarrow P_4 \neq A_9 \)
- \( P_2 = A_6 \Rightarrow P_4 \neq A_8 \)
- \( P_2 = A_{11} \Rightarrow P_3 \neq A_7 \)

Constraints (Activity):
- \( G_1 = A_1 \Rightarrow Active\{ P_1, P_2, P_3 \} \)
- \( G_2 = A_2 \Rightarrow Active\{ P_4 \} \)
- \( G_3 = A_3 \Rightarrow Active\{ P_5 \} \)
- \( G_4 = A_4 \Rightarrow Active\{ P_1, P_6 \} \)

Init State: \( Active\{ G_1, G_2, G_3, G_4 \} \)

(a) Planning Graph

(b) DCSP from Do & Kambhampati
Finding a solution

For backward search, attempt to find arrows back to the initial state (without conflict/mutex).

Start by finding actions that satisfy all goal conditions, then recursively try to satisfy all of the selected actions’ preconditions.

If this fails to find a solution, mark this level and all the goals not satisfied as: (level, goals). (level, goals) stops changing, no solution.
Graph Plan

Remember this...
Initial: $\neg Money \land \neg Smart \land \neg Debt$
Goal: $\neg Money \land Smart \land \neg Debt$

Action(\textit{School},
Precondition: ,
Effect: $Debt \land Smart$)

Action(\textit{Job},
Precondition: ,
Effect: $Money \land \neg Smart$)

Action(\textit{Pay},
Precondition: $Money$,
Effect: $\neg Money \land \neg Debt$)
Graph Plan

Ask:  
\( \neg D \land \neg S \land \neg M \)

Find first no mutex...
Ask: \( D^S \land M \)

... then back search

Error! States of 1&4 in mutex

1.

2.

3.
Ask:

\[ D^S M \]

try different back path...

Graph Plan

Error, actions 3&4 in mutex
Ask: \[ D^S^M \] found solution!
Finding a solution

Formally, the algorithm is:

\[
\text{graph} = \text{initial} \\
\text{noGoods} = \text{empty table (hash)} \\
\text{for level} = 0 \text{ to infinity} \\
\quad \text{if all goal pairs not in mutex} \\
\quad \quad \text{solution} = \text{recursive search with noGoods} \\
\quad \quad \text{if success, return paths} \\
\quad \text{if graph & noGoods converged, return fail} \\
\text{graph} = \text{expand graph}
\]
Initial: $Clean \land Garbage \land Quiet$

Goal: $Food \land \neg Garbage \land Present$

You try it!