1. 20 points
Convert these English sentences to predicate calculus, using predicates such as: Town(x), Dog(x), Postman(x), Apartment(x), Empty(x), Rentable(x), Large(x), LivesIn(x,y) = x lives in y, Bitten(x,y) = x has bitten y.

(a) There is a dog in the town that has bitten every postmen in town.

(b) An apartment is not rentable unless it is empty.

(c) There is no apartment which is rentable.

(d) There is only one large apartment which is rentable.

Note: If you want full credit, do not use $\exists!x$. 
2. 15 points
For each of the following sentences, decide if the logic sentence given is a correct translation of the English sentence or not. If not explain briefly why not and correct it:

(a) Poor people do not own houses that are big.
   \[ \exists x \text{ Poor}(x) \land \exists y \text{ House}(y) \land \neg \text{Owns}(x, y) \Rightarrow \neg \text{Big}(y) \]

(b) Anyone who does not own a house does not own a pool.
   \[ \forall x \forall y \text{ House}(y) \land \text{Pool}(z) \Rightarrow \neg \text{Owns}(x, y) \land \neg \text{Own}(x, z) \]

(c) Some houses have no pool
   \[ \forall x \text{ House}(x) \Rightarrow \neg \text{HasPool}(x) \]
3. 10 points
You are given the following knowledge base in propositional calculus:

(a) Wet ∧ Slippery ⇒ Miserable
(b) Snowy ⇒ Rainy ∧ Cold
(c) Rainy ⇒ Wet
(d) Cold ⇒ Slippery
(e) Snowy

Use resolution with refutation to derive Miserable.
Show each step in the solution, showing the pair of clauses your are resolving, and the resolvent.
4. 15 points
You are given the following knowledge base in predicate calculus:

(a) \( \forall x \ R(x) \land P(x) \Rightarrow \neg Q(x) \lor S(x) \)
(b) \( \exists x \ Q(x) \land P(x) \)
(c) \( \forall x \ P(x) \Rightarrow R(x) \)
(d) \( \forall x \ Q(x) \Rightarrow P(x) \)

Use resolution with refutation to derive that “\( \exists x \ S(x) \)”. Specify the value of \( x \) that was found by resolution.
Show each step in the solution, showing the pair of clauses your are resolving, the results of unification, and the resolvent.
5. 25 points
You are given the following information:
(a) Tom is not older than Karen and Harry
(b) John is older than Karen
(c) There is no one older than Wilt

Represent the knowledge base in predicate calculus using only the predicate Older(x,y) = x is older than y.
Use resolution with refutation to show that “John is older than Tom and Harry is not older than Wilt”.
If there is any additional knowledge you need to find a solution, add it to the knowledge base.
Show each step in the solution, showing the pair of clauses your are resolving, the results of unification, and the resolvent.
(continue question 5 if needed)
6. 15 points
Answer the following questions shortly but precisely. Do not just answer yes or no, you need to explain your answers.

(a) Is it true that in predicate calculus, if a sentence is entailed, it can always be proven using resolution with refutation? Explain.

(b) In predicate calculus, if resolution with refutation fails to produce the empty clause, what can you conclude? Be precise.

(c) Is it true that it is always possible to prove that a sentence in propositional logic is entailed or not entailed by the knowledge base? Explain.
7. 10 points – extra credit

Suppose you use resolution with refutation to prove that $KB \models \alpha$. Does it mean that $KB$ is valid? or is $\alpha$ valid? Explain clearly why or why not.