Computational Methods for Intelligent Information Access (Latent Semantic Indexing)

Presented by Yiyi Yin

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Motivation: (Textual) Information Retrieval

- **Information Retrieval** is the "activity of obtaining information resources relevant to an information need from a collection of information resources".
- To goal is to retrieve relevant (in terms of having the same conceptual topics or meaning) textual documents from databases.
- Example:
  Prior art (state of the art) searches for patents
  Searches at google scholar

Latent Semantic Indexing v.s. Lexical Matching

- Lexical Matching is inaccurate because
  i) There are synonymys
  ii) Words have multiple meanings
  so a user's query may literally match irrelevant documents.
- Latent Semantic Indexing (LSI) solved the problem by using conceptual indices rather than individual words.

Example

Consider **car, automobile, driver** and **elephant** where **car** and **automobile** are synonyms.

- In lexical matching, it is the same for query **automobile** to retrieve documents about cars and documents about elephants, if neither included the term **automobile**.
- In latent semantic indexing, the projection space can reflect interrelationships between terms. **Car** and **automobile** are close to each other because they always occur in similar contexts with words (motor, model, vehicle, engine, etc.).

Latent Semantic Indexing

- An indexing and information retrieval method
- Mathematically based on truncated singular value decomposition (SVD)
- Assumes that words used in the same contexts have similar meanings
- "Called latent semantic indexing because of its ability to correlate semantically related terms that are latent in a collection of text"

Singular Value Decomposition (SVD)

WLOG, assume $m \geq n$ and $rank(A) = r \leq min(m, n)$,

$$A_{m\times n} = U_{m\times n}\Sigma_{n\times n}V_{n\times n}^T$$

where $U^T U = V^T V = I$,
$\Sigma = diag(\sigma_1, \ldots, \sigma_n), \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > \sigma_{r+1} = \cdots = \sigma_n = 0$

- Dyadic decomposition:

$$A = \sum_{i=1}^{r} u_i \sigma_i v_i^T$$

where $U = [u_1 u_2 \cdots u_n], V = [v_1 v_2 \cdots v_n]$

Best Rank-k Approximation [Eckart and Young]

Define

$$A_k = \sum_{i=1}^{k} u_i \sigma_i v_i^T$$

then

$$\min_{rank(B)=k} ||A - B||_F^2 = ||A - A_k||_F^2$$

A_k, which is constructed from the k-largest singular triplets of A, is the closest rank-k matrix to A.

Latent Semantic Indexing

- Construct term-document matrix A
- Take SVD decomposition of term-document matrix
- Select k and find A_k, the best rank-k approximation
- Queries
- Updating

Construct Term-Document Matrix

The term-document matrix

$$A = [a_{ij}]$$

where where a_{ij} denotes the frequency that term i occurs in document j. We can write

$$a_{ij} = L(i,j) \times G(i)$$

where $L(i,j)$ is the local weighting for term i in document j, and $G(i)$ is the global weighting for term i.

- By adding local and global weightings, we can change the importance of terms within or among documents.

Example: Database

Label	Medical Topic
M1	study of depressed patients after discharge with regard to age of onset and culture
M2	culture of pleuropneumonia like organisms found in vaginal discharge of patients
M3	study showed oestrogen production is depressed by ovarian irradiation
M4	cortisone rapidly depressed the secondary rise in oestrogen output of patients
M5	boys tend to react to death anxiety by acting out behavior while girls tended to become depressed
M6	changes in children's behavior following hospitalization studied a week after discharge
M7	surgical technique to close ventricular septal defects
M8	chromosomal abnormalities in blood cultures and bone marrow from leukaemic patients
M9	study of christmas disease with respect to generation and culture
M10	insulin not responsible for metabolic abnormalities accompanying a prolonged fast
M11	close relationship between high blood pressure and vascular disease
M12	mouse kidneys show a decline with respect to age in the ability to concentrate the urine during a water fast
M13	fast cell generation in the eye lens epithelium of rats
M14	fast rise of cerebral oxygen pressure in rats

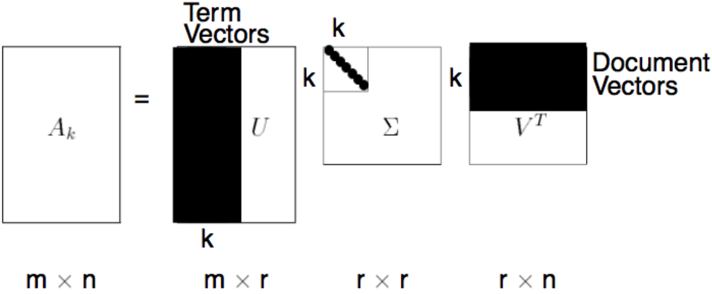
Example: Term-Document Matrix

Terms	Documents													
	M 1	M 2	M 3	M 4	M 5	M 6	M 7	M 8	M 9	M 10	M 11	M 12	M 13	M 14
abnormalities	0	0	0	0	0	0	0	1	0	1	0	0	0	0
age	1	0	0	0	0	0	0	0	0	0	0	1	0	0
behavior	0	0	0	0	1	1	0	0	0	0	0	0	0	0
blood	0	0	0	0	0	0	0	1	0	0	1	0	0	0
close	0	0	0	0	0	0	1	0	0	0	1	0	0	0
culture	1	1	0	0	0	0	0	1	1	0	0	0	0	0
depressed	1	0	1	1	1	0	0	0	0	0	0	0	0	0
discharge	1	1	0	0	0	1	0	0	0	0	0	0	0	0
disease	0	0	0	0	0	0	0	0	1	0	1	0	0	0
fast	0	0	0	0	0	0	0	0	0	1	0	1	1	1
generation	0	0	0	0	0	0	0	0	1	0	0	0	1	0
oestrogen	0	0	1	1	0	0	0	0	0	0	0	0	0	0
patients	1	1	0	1	0	0	0	1	0	0	0	0	0	0
pressure	0	0	0	0	0	0	0	0	0	0	1	0	0	1
rats	0	0	0	0	0	0	0	0	0	0	0	0	1	1
respect	0	0	0	0	0	0	0	1	0	0	0	1	0	0
rise	0	0	0	1	0	0	0	0	0	0	0	0	0	1
study	1	0	1	0	0	0	0	0	1	0	0	0	0	0

SVD Decomposition and Best Rank-k Approximation

Take SVD decomposition of term-document matrix $A = U\Sigma V^T$, approximated by $A_k = U_k \Sigma_k V_k^T = \sum_{i=1}^{k} u_i \sigma_i v_i^T$.

A_k	= Best rank-k approximation to A		
		m	= Number of terms
U	= Term vectors	n	= Number of documents
Σ	= Singular values	k	= Number of factors
V	= Document vectors	r	= Rank of A



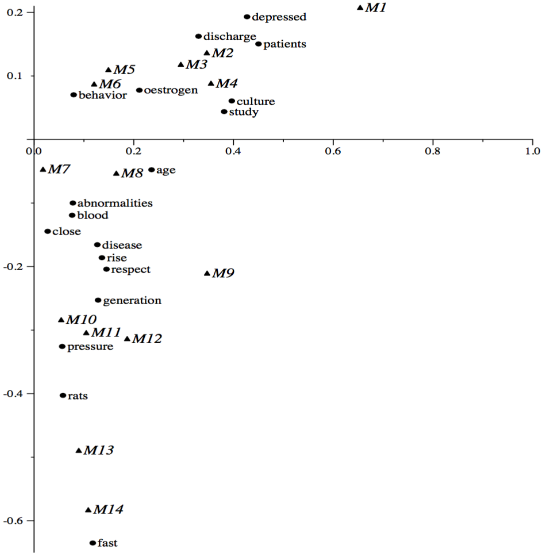
Projection Space

- k-dimensional Term Projection Space: $(U_k \Sigma_k)_{m \times k}$
- k-dimensional Document Projection Space: $(V_k \Sigma_k)_{n \times k}$

For example, when $k = 2$, $A_2 = U_2 \Sigma_2 V_2$

- 2-dimensional Term Projection Space: $(u_1 \sigma_1, u_2 \sigma_2)$
- 2-dimensional Document Projection Space: $(v_1 \sigma_1, v_2 \sigma_2)$

Example: Projection Space



Queries

- A query is a phrase
- In order to retrieve information, query must be projected in the projection space and compared to all existing documents.

The projected query is

$$\hat{q} = q^T U_k \Sigma_k^{-1}$$

where q is the vector of words in the query

- In terms of similarity, a common measure is the cosine between query and document.
- Generally, all documents exceeding some cosine threshold are returned.

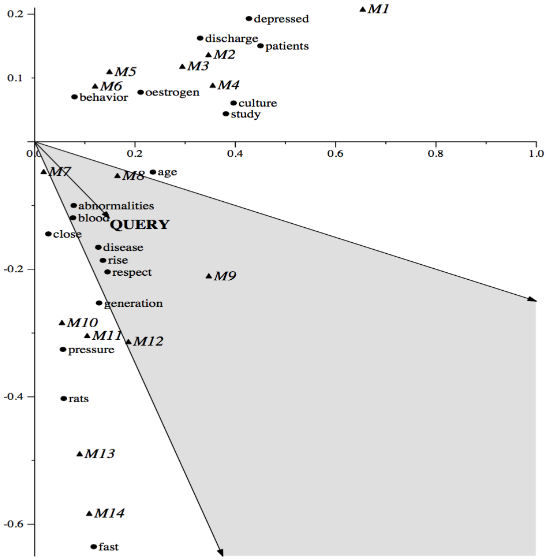
Example: Queries

- We want to identify documents that contain information related to the **age of children with blood abnormalities**.

$$
\begin{pmatrix} 0.1491 & -0.1199 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T \begin{pmatrix} 0.1623 & -0.1372 \\ 0.2068 & -0.0488 \\ 0.0597 & 0.0614 \\ 0.1663 & -0.1313 \\ 0.0258 & -0.1246 \\ 0.4534 & 0.0386 \\ 0.3579 & 0.1710 \\ 0.2931 & 0.1426 \\ 0.0690 & -0.1576 \\ 0.0940 & -0.6535 \\ 0.0599 & -0.2378 \\ 0.1560 & 0.0661 \\ 0.4948 & 0.1091 \\ 0.0460 & -0.3393 \\ 0.0369 & -0.4196 \\ 0.1797 & -0.1456 \\ 0.1087 & -0.2126 \\ 0.3814 & 0.0941 \end{pmatrix} \begin{pmatrix} 3.5919 & 0 \\ 0 & 2.6471 \end{pmatrix}^{-1}
$$

- All documents with cosine greater than 0.85 is considered relevant.

Example: Queries



Example: Queries

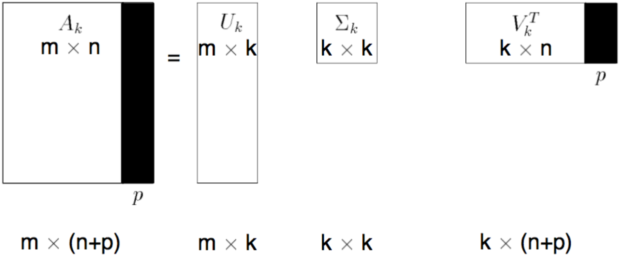
- LSI returned relevant topics **M8**, **M9** and **M12**
- Lexical-matching returned **M1**, **M8**, **M10**, **M11** and **M12**, but **M1** and **M10** are not relevant, **M9** would be missed.
- In LSI, the most relevant topic **M9** is returned.
  Query: age of children with blood abnormalities
  M9: study of christmas disease with respect to generation and culture
  (christmas disease is the name of hemophilia in young children)

Updating

Suppose an LSI model is already built based on the database, more terms and documents are added. How are we going to update the model fitting?

- Folding-in: essentially the same with query representation, quick and simple
- Recomputing the SVD: repeat the whole procedure, requires more computation time
- SVD-updating: computationally efficient

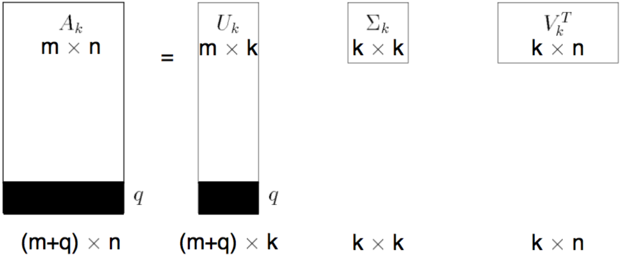
Folding-in: Document



- p new documents
- folding-in a new document vector, d, into the existing LSI projection model, a projection, $\hat{d}$ is computed by

$$\hat{d} = d^T U_k \Sigma_k^{-1}$$

Folding-in: Term



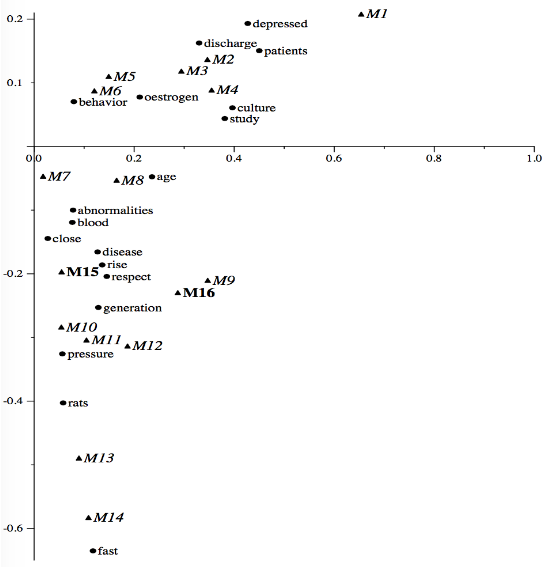
- q new terms
- folding-in a new term vector, t, into the existing LSI projection model, a projection, $\hat{t}$ is computed by

$$\hat{t} = t^T V_k \Sigma_k^{-1}$$

Example: Updating

Label	Medical Topic
M15	behavior of rats after detected rise in oestrogen
M16	depressed patients who feel the pressure to fast

Example: Folding-in



Example: Recomputing the SVD

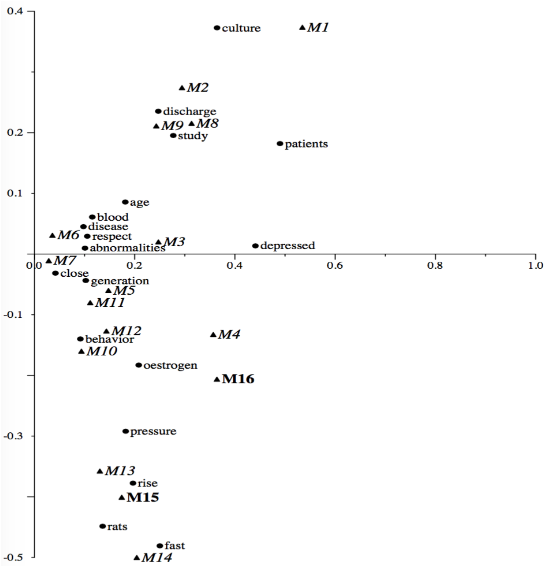
Simply compute the SVD of a reconstructed term-document matrix, say $\tilde{A}$. Now the new term-document matrix $\tilde{A}$ is 18×16.
Recomputing the SVD

$$\tilde{A} = \tilde{U}\tilde{\Sigma}\tilde{V}^T$$

Construct the rank-2 approximation to $\tilde{A}$ by

$$\tilde{A}_2 = \tilde{U}_2\tilde{\Sigma}_2\tilde{V}_2^T$$

Example: Recomputing the SVD



Example: Difference between Folding-in and Recomputing the SVD

- Folding-in didn't reconstruct the semantic representation in the new database
- The existing LSI didn't take the association between **behavior** and **rats** into consideration. Thus it fails to form the cluster **M13**, **M14** and **M15**

SVD-updating: Documents

Let $D_{m \times p}$ denote new document vectors, $B = (A_k | D)$, define $SVD(B) = U_B \Sigma_B V_B^T$. Then

$$U_k^T B \begin{pmatrix} V_k & 0 \\ 0 & I_p \end{pmatrix} = (\Sigma_k | U_k^T D)$$

since $A_k = U_k \Sigma_k V_k^T$. If $F = (\Sigma_k | U_k^T D)$ and $SVD(F) = U_F \Sigma_F V_F^T$, then

$$U_B = U_k U_F, V_B = \begin{pmatrix} V_k & 0 \\ 0 & I_p \end{pmatrix} V_F, \Sigma_F = \Sigma_B$$

SVD-updating: Terms

Let $T_{q \times n}$ denote new term vectors, $C = \begin{pmatrix} A_k \\ T \end{pmatrix}$, define $SVD(C) = U_C \Sigma_C V_C^T$. Then

$$\begin{pmatrix} U_k^T & 0 \\ 0 & I_q \end{pmatrix} C V_k = \begin{pmatrix} \Sigma_k \\ T V_k \end{pmatrix}$$

If $H = \begin{pmatrix} \Sigma_k \\ T V_k \end{pmatrix}$ and $SVD(H) = U_H \Sigma_H V_H^T$, then

$$U_C = \begin{pmatrix} U_k & 0 \\ 0 & I_q \end{pmatrix} U_H, V_C = V_k V_H, \Sigma_H = \Sigma_C$$

SVD-updating: Term Weight Correction

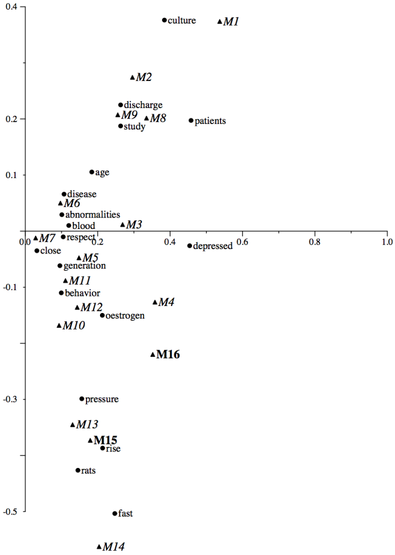
For a change of weightings in j terms, let Y_j be an $m \times j$ matrix with rows of zeros or rows of j-th order identity matrix I_j, let Z_j be $n \times j$ matrix whose columns specify the actual differences between old and new weights for each of the j terms. The correction step is actually to compute the SVD decomposition of $W = A_k + Y_j Z_j^T$. Define $SVD(W) = U_W \Sigma_W V_W^T$. Then

$$U_k^T W V_k = (\Sigma_k + U_k^T Y_j Z_j^T V_k)$$

If $Q = (\Sigma_k + U_k^T Y_j Z_j^T V_k)$ and $SVD(Q) = U_Q \Sigma_Q V_Q^T$, then

$$U_W = U_k U_Q, V_W = V_k V_Q$$

Example: SVD-updating



Orthogonality

One important difference between the folding-in and the SVD-updating is the guarantee of orthogonality.

- The folding-in process corrupts the orthogonality by appending non-orthogonal submatrices.
- The SVD-updating can guarantee the orthogonality.

Compare Updating Methods

	Computational Complexity	Orthogonality
Folding-in	*	No
Recomputing the SVD	***	Yes
SVD-updating	**	Yes

References

Berry, Michael W., Susan T. Dumais, and Todd A. Letsche., 1995
Computational methods for intelligent information access.
In Supercomputing, 1995. Proceedings of the IEEE/ACM SC95 Conference (pp. 20-20). IEEE.

Thanks!