Percolation Centrality:  
Introduction and Application

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Outline

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Motivation

Influence/disease propagation (dynamic network)

- Spreading of computer viruses on computer network
- Transmission of disease over a network of towns
- Rumors or news spread via social networks

Can be modelled as a specific example of *percolation* in networks
Percolation

- The movement and filtering of fluids through porous materials
- Second-order Phase transition
Centrality Measure: Determine the relative importance of a node in a complex network

e.g. Disease Outbreak

• Choices for early intervention in the affected network need to be precise

• Need to identify “central” nodes.
Centrality Measure

• Degree centrality

\[ DC(v) = \text{deg}(v) \]

• Betweenness centrality

\[ BC(v) = \frac{1}{(N - 1)(N - 2)} \sum_{s \neq v \neq t} \frac{\sigma_{s,t}(v)}{\sigma_{s,t}} \]

• Closeness centrality

\[ CC(v) = \frac{1}{\sum_{i \neq v} d_g(v, i)} \]

These measures do not depend on the “state” of the node.
Percolation Centrality

\[ PC^T(v) = \frac{1}{(N-2)} \sum_{s \neq v \neq t} \frac{\sigma_{s,t}(v)}{\sigma_{s,t}} \frac{x_s^T}{\left[ \sum x_i^T \right] - x_v^T} \]

\( x_v^T \): percolation state
- Binary: received/not received a piece of news
- Discrete: susceptible/infected/recovered
- Continuous: proportion of infected people in a town

\[ w_{s,v}^T = \frac{x_s^T}{\left[ \sum x_i^T \right] - x_v^T} : \text{the relative contribution of each percolated path originated in the source node “s” to the percolation centrality } PC^T(v).\]
Percolation Centrality

- $x_s^T = 0$, zero percolated nodes, $PC^T(v) = 0$

- Once a node is percolated, this node will affect the $PC$ of multiple nodes, resulting in the average $PC$ of these nodes being significantly higher than the average betweenness of these nodes.

- If all nodes are fully percolated, $w_{s,v}^T = \frac{1}{N-1}$

$$PC^T(v) = \frac{1}{(N-1)(N-2)} \sum_{s \neq v \neq t} \frac{\sigma_{s,t}(v)}{\sigma_{s,t}} = BC(v)$$
Simple example

Betweenness centrality
Simple example

Percolation centrality
Minnesota Road Network

Application on Minnesota Road Network
Percolation
Betweenness Centrality
Conclusion

• Percolation centrality can be used for analyzing importance of nodes during percolation in networks
• When a network is fully percolated, the percolation centrality reduces to betweenness centrality

References