Percolation Centrality: Introduction and Application

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12/11/2017

Outline

- Motivation
- Introduction on Percolation
- Review on Centrality measure
- Percolation centrality
- Simple example
- Application on Minnesota road network
- Conclusion

Motivation

Network analysis

Physics model

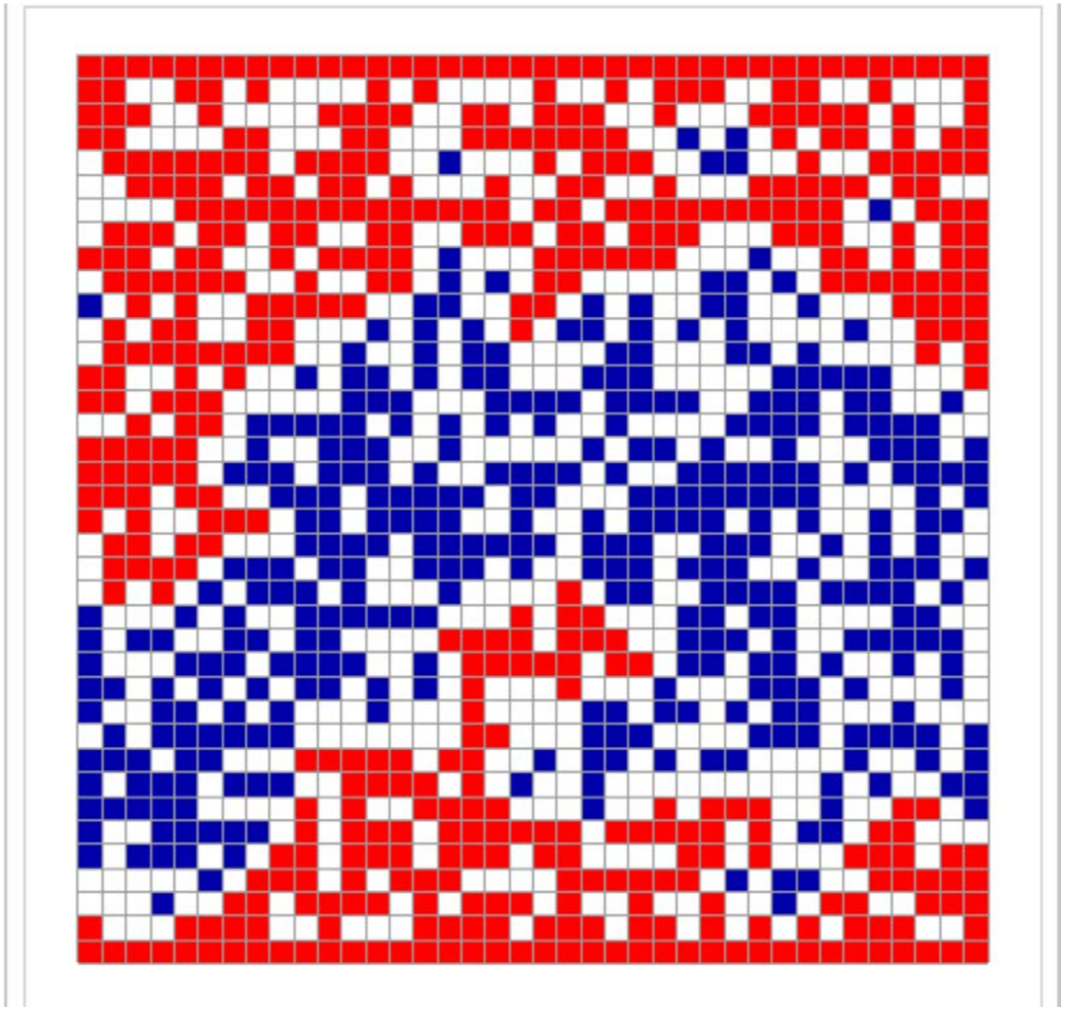
Influence/disease propagation (dynamic network)

- Spreading of computer viruses on computer network
- Transmission of disease over a network of towns
- Rumors or news spread via social networks

Can be modelled as a specific example of **percolation** in networks

Percolation

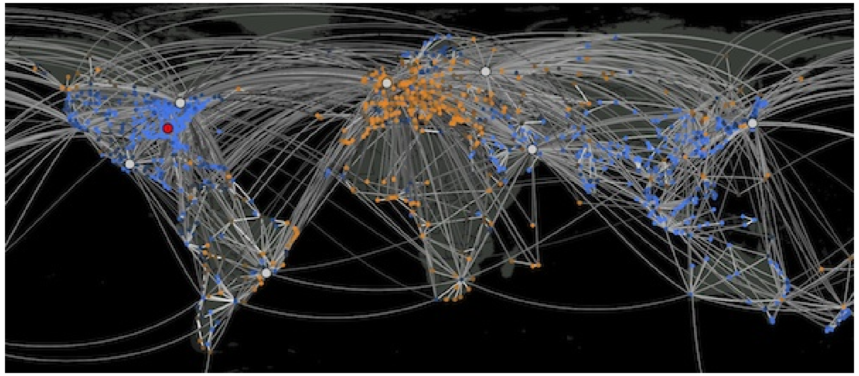
- The movement and filtering of fluids through porous materials
- Second-order Phase transition



Centrality Measure: Determine the relative importance of a node in a complex network

e.g. Disease Outbreak

- Choices for early intervention in the affected network need to be precise
- Need to identify “central” nodes.



Centrality Measure

- Degree centrality

$$DC(v) = \deg(v)$$

- Betweenness centrality

$$BC(v) = \frac{1}{(N-1)(N-2)} \sum_{s \neq v \neq t} \frac{\sigma_{s,t}(v)}{\sigma_{s,t}}$$

- Closeness centrality

$$CC(v) = \frac{1}{\sum_{i \neq v} d_g(v,i)}$$

These measures do not depend on the “state” of the node.

Percolation Centrality

$$PC^T(v) = \frac{1}{(N-2)} \sum_{s \neq v \neq t} \frac{\sigma_{s,t}(v)}{\sigma_{s,t}} \frac{x_s^T}{\left[\sum x_i^T\right] - x_v^T}$$

x_v^T: percolation state

- Binary: received/not received a piece of news
- Discrete: susceptible/infected/recovered
- Continuous: proportion of infected people in a town

$w_{s,v}^T = \frac{x_s^T}{\left[\sum x_i^T\right] - x_v^T}$: the relative contribution of each percolated path originated in the source node "*s*" to the percolation centrality $PC^T(v)$.

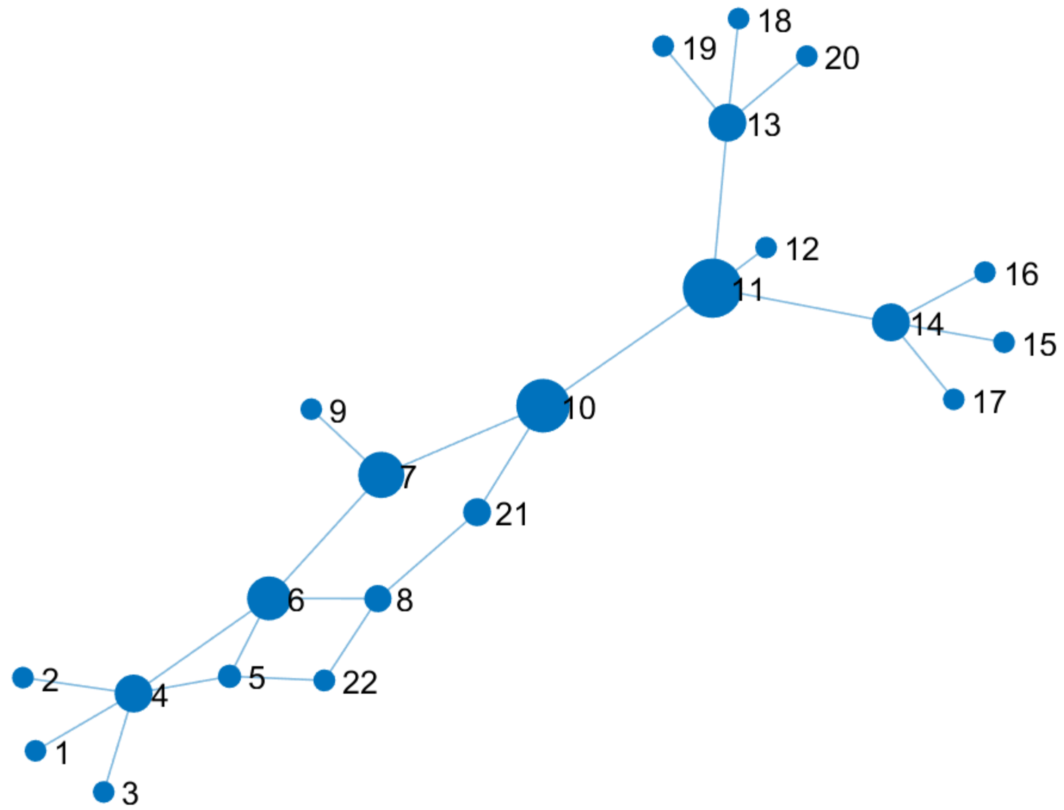
Percolation Centrality

- $x_s^T = 0$, zero percolated nodes, $PC^T(v) = 0$
- Once a node is percolated, this node will affect the PC of multiple nodes, resulting in the average PC of these nodes being significantly higher than the average betweenness of these nodes.
- If all nodes are fully percolated, $w_{s,v}^T = \frac{1}{\mathrm{N}-1}$

$$PC^T(v) = \frac{1}{(N-1)(N-2)} \sum_{s \neq v \neq t} \frac{\sigma_{s,t}(v)}{\sigma_{s,t}} = BC(v)$$

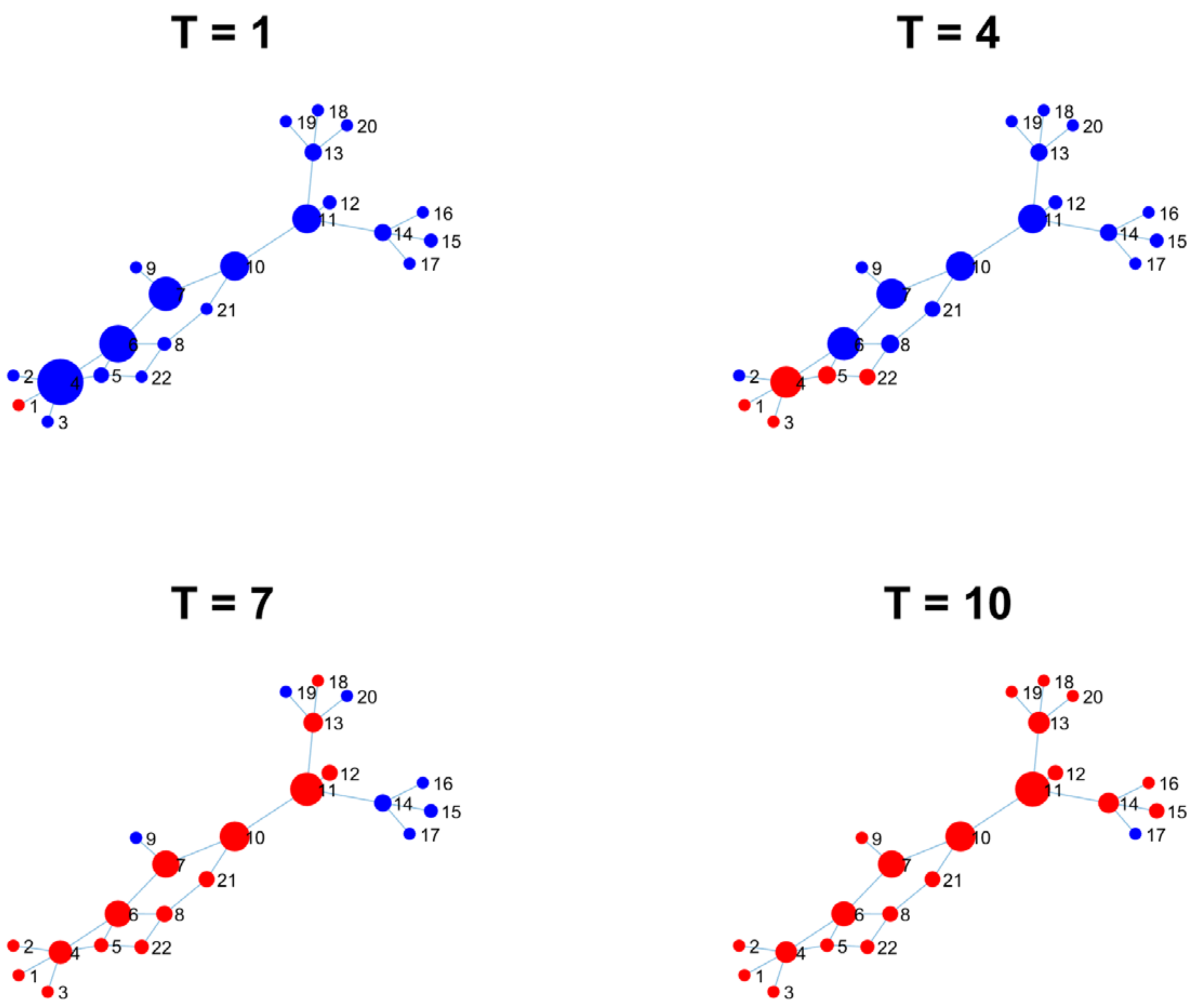
Simple example

Betweenness centrality



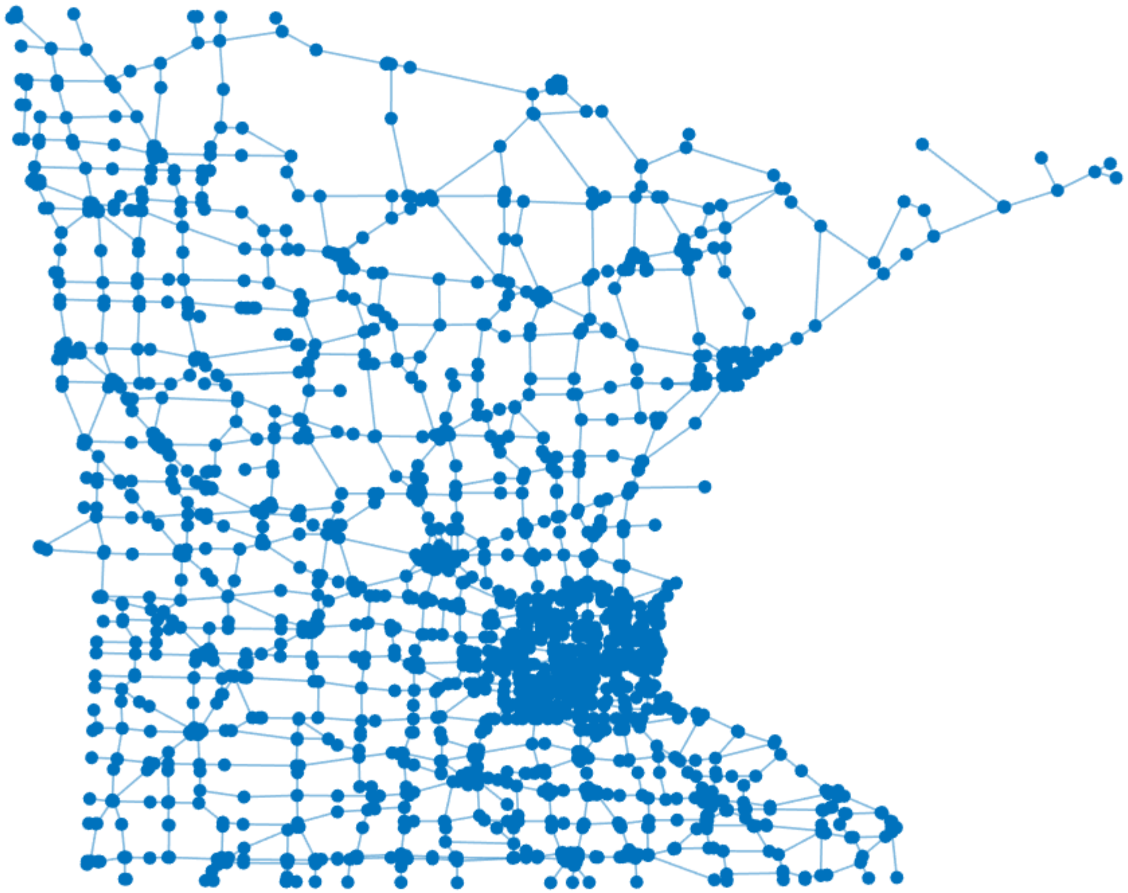
Simple example

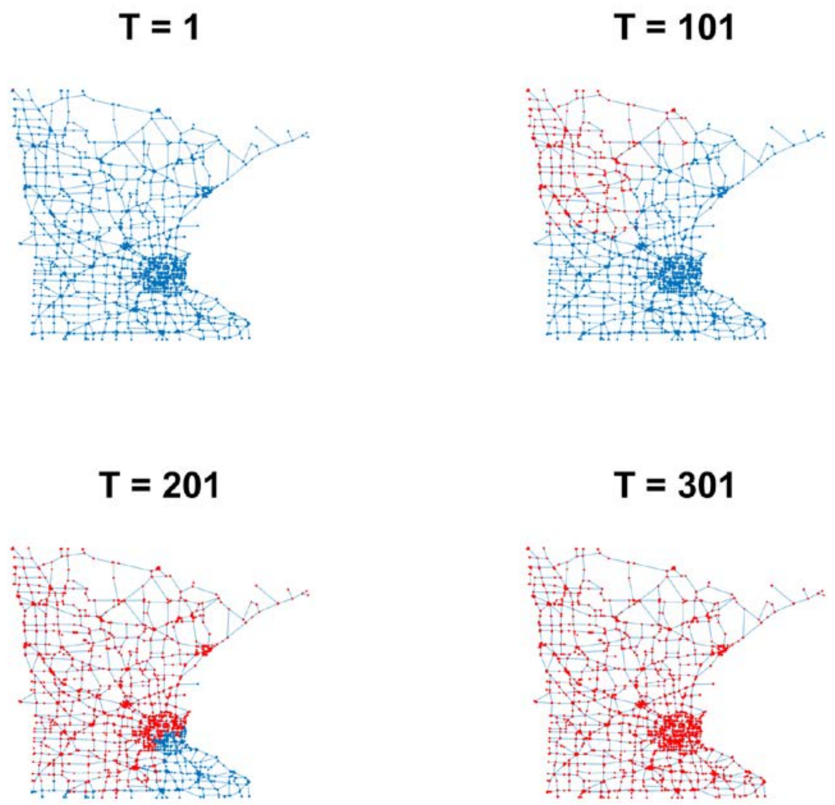
Percolation centrality

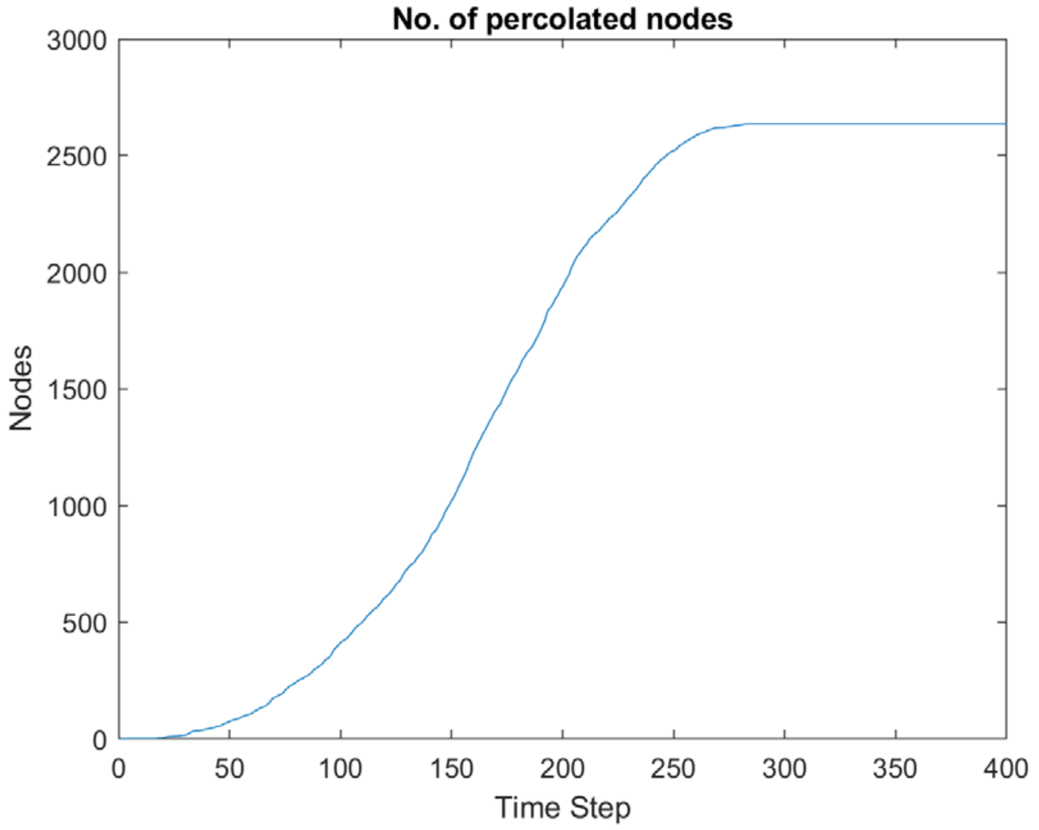


Application on Minnesota Road Network

Minnesota Road Network

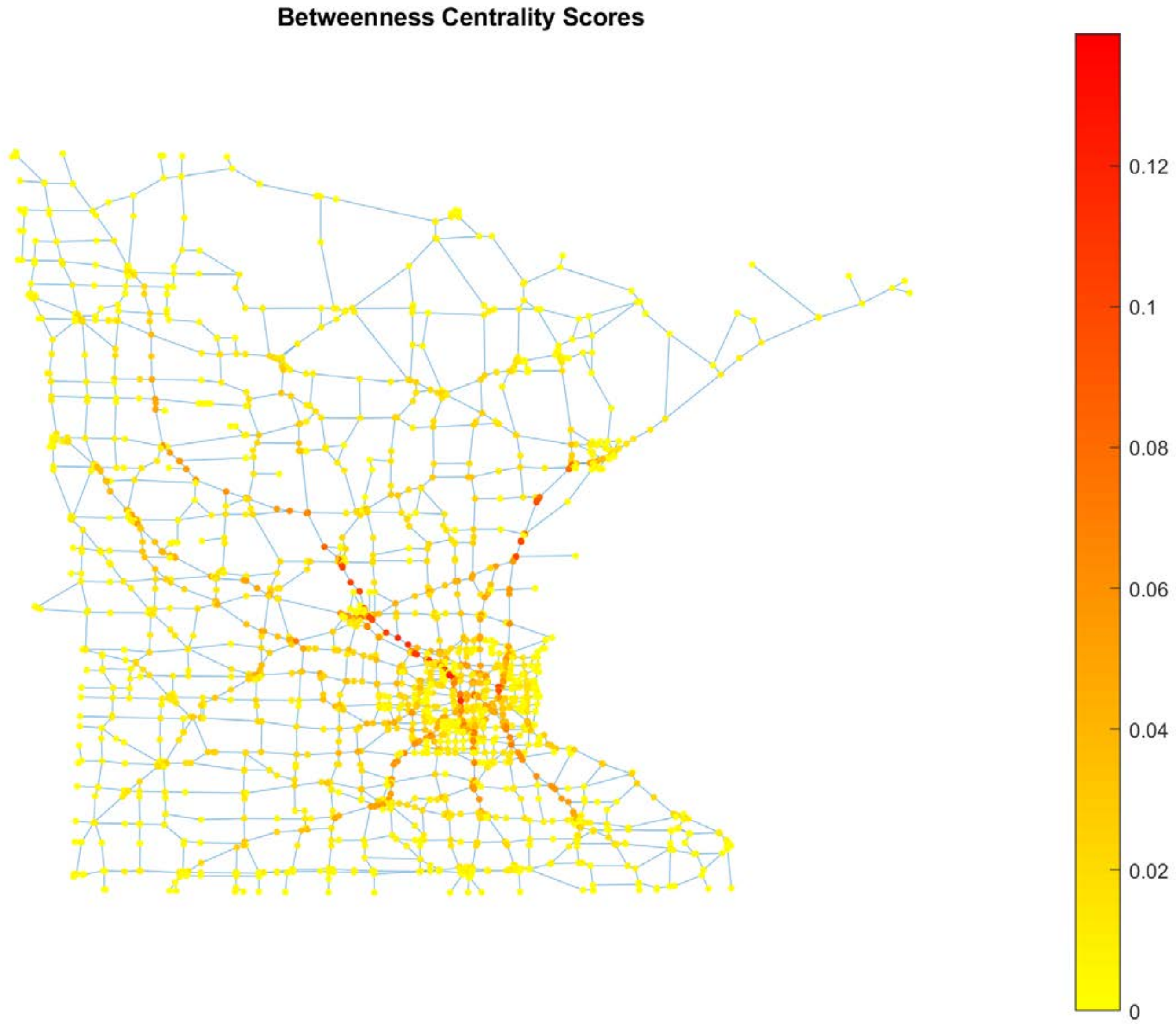






Percolation

Betweenness Centrality



Conclusion

- Percolation centrality can be used for analyzing importance of nodes during percolation in networks
- When a network is fully percolated, the percolation centrality reduces to betweenness centrality

References

- Piraveenan M, Prokopenko M, Hossain L. Percolation centrality: Quantifying graph-theoretic impact of nodes during percolation in networks[J]. PloS one, 2013, 8(1): e53095.
- Brandes U. A faster algorithm for betweenness centrality[J]. Journal of mathematical sociology, 2001, 25(2): 163-177.