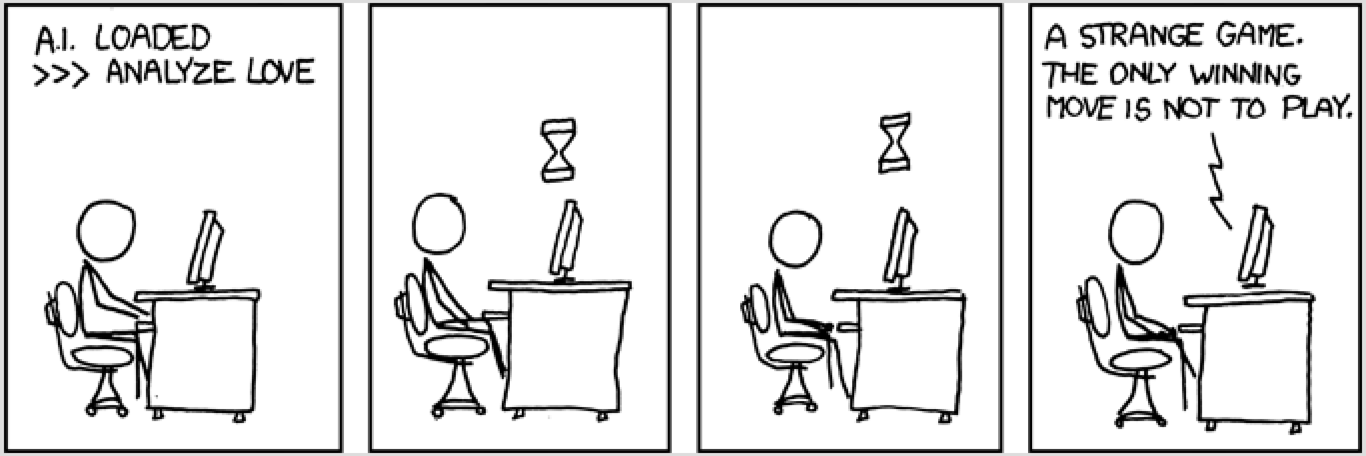
Game theory (Ch. 17.5)



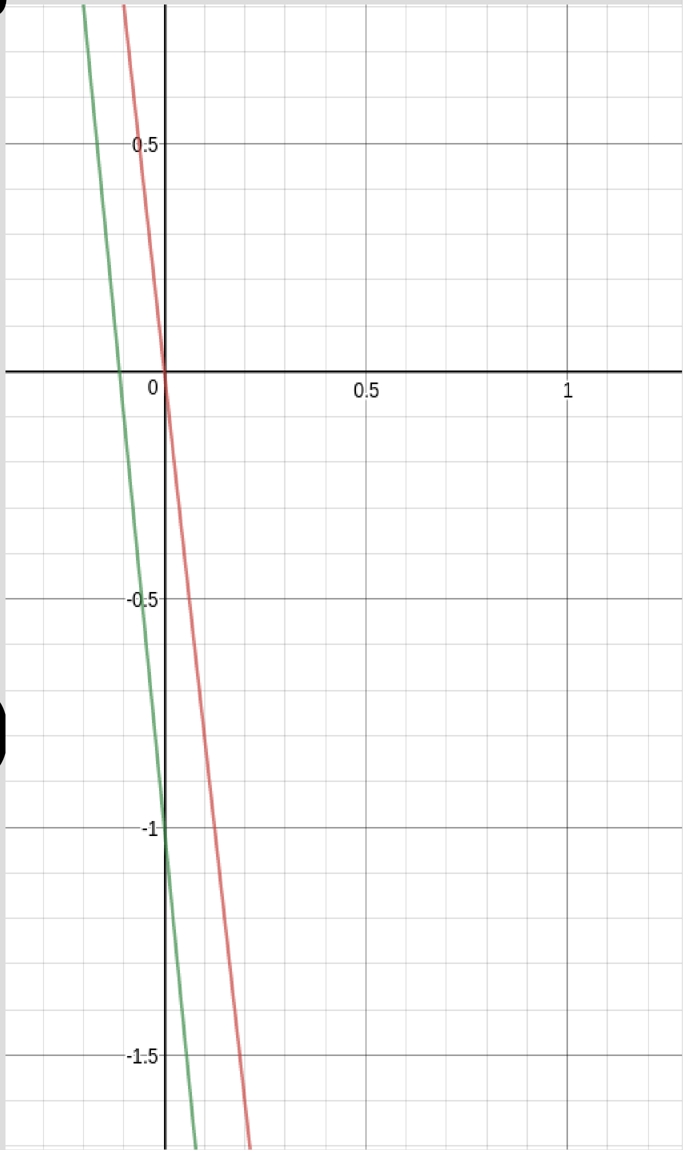
Announcements

Test grades up now

Find best strategy

How does this compare on PD?

	Confess	Lie
Confess	-8 , -8	0 , -10
Lie	-10 , 0	-1 , -1



Player 1: p = prob confess...

P2 Confesses: -8*p + 0*(1-p)

P2 Lies: -10*p + (-1)*(1-p)

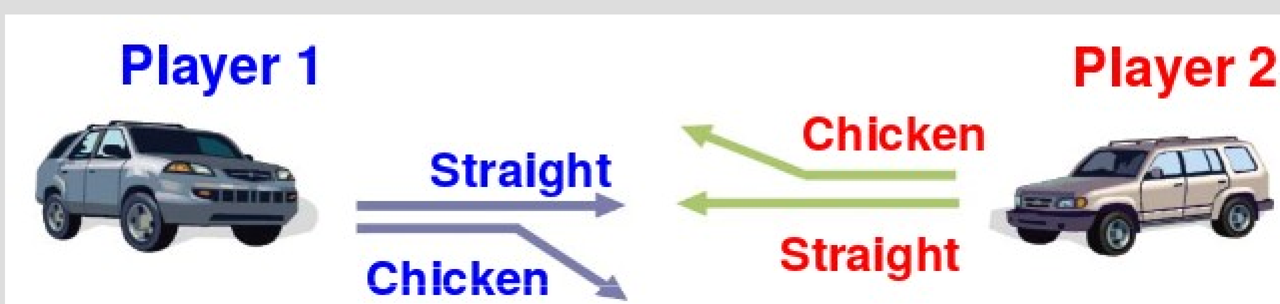
Cross at negative p, but red line is better (confess)

Chicken

What is Nash for this game?
What is Pareto optimum?

	S	C
S	-10, -10	1, -1
C	-1, 1	0, 0

Game of Chicken



Chicken

	S	C
S	-10, -10	1, -1
C	-1, 1	0, 0

To find Nash, assume we (blue) play S probability p, C prob 1-p

Column 1 (red=S): p*(-10) + (1-p)*(1)
Column 2 (red=C): p*(-1) + (1-p)*(0)

Intersection: -11*p + 1 = -p, p = 1/10

Conclusion: should always go straight 1/10 and chicken 9/10 the time

Chicken

	s	c
s	-10, -10	1, -1
c	-1, 1	0, 0

We can see that 10% straight makes the opponent not care what strategy they use:

(Red numbers)

100% straight: (1/10)*(-10) + (9/10)*(1) = -0.1

100% chicken: (1/10)*(-1) + (9/10)*(0) = -0.1

50% straight: (0.5)*[(1/10)*(-10) + (9/10)*(1)]

+ (0.5)*[(1/10)*(-1) + (9/10)*(0)]

=(0.5)*[-0.1] + (0.5)*[-0.1] = -0.1

Chicken

The opponent does
not care about action,
but you still do (never considered our values)

	s	c
s	-10, -10	1, -1
c	-1, 1	0, 0

Your rewards, opponent 100% straight:

(0.1)*(-10) + (0.9)*(-1) = -1.9

Your rewards, opponent 100% curve:

(0.1)*(1) + (0.9)*(0) = 0.1

The opponent also needs to play at your value intersection to achieve Nash

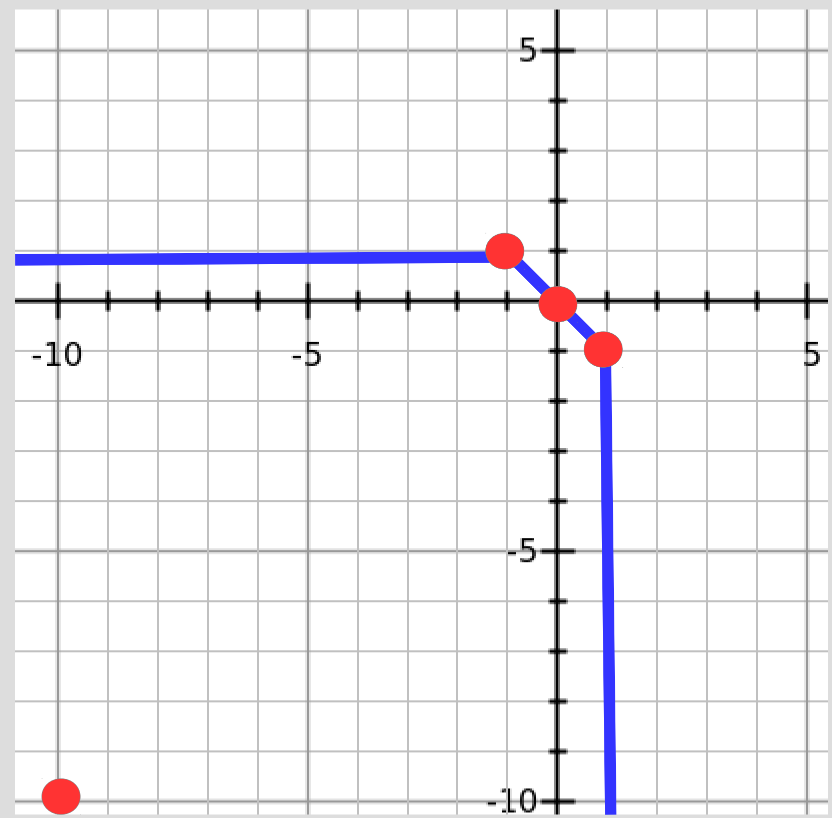
Chicken

Pareto optimum?
All points except (-10,10)

Can think about this
as taking a string from the
top right and bringing the
it down & left

Stop when string going
straight left and down

	s	c
s	-10, -10	1, -1
c	-1, 1	0, 0



Find best strategy

We have two actions, so one parameter (p) and thus we look for the intersections of lines

If we had 3 actions (rock-paper-scissors), we would have 2 parameters and look for the intersection of 3 planes (2D)

This can generalize to any number of actions (but not a lot of fun)

		Player 2		
		Stone	Paper	Scissors
Player 1	Stone	$(0, 0)$	$(-1, 1)$	$(1, -1)$
	Paper	$(1, -1)$	$(0, 0)$	$(-1, 1)$
	Scissors	$(-1, 1)$	$(1, -1)$	$(0, 0)$

Repeated games

In repeated games, things are complicated

For example, in the basic PD, there is no benefit to “lying”

		PRISONER 2	
		Confess	Lie
PRISONER 1	Confess	-8 , -8	0 , -10
	Lie	-10 , 0	-1 , -1

However, if you play this game multiple times, it would be beneficial to try and cooperate and stay in the [lie, lie] strategy

Repeated games

One way to do this is the <u>tit-for-tat</u> strategy:

1. Play a cooperative move first turn
2. Play the type of move the opponent last played every turn after (i.e. answer competitive moves with a competitive one)

This ensure that no strategy can "take advantage" of this and it is able to reach cooperative outcomes

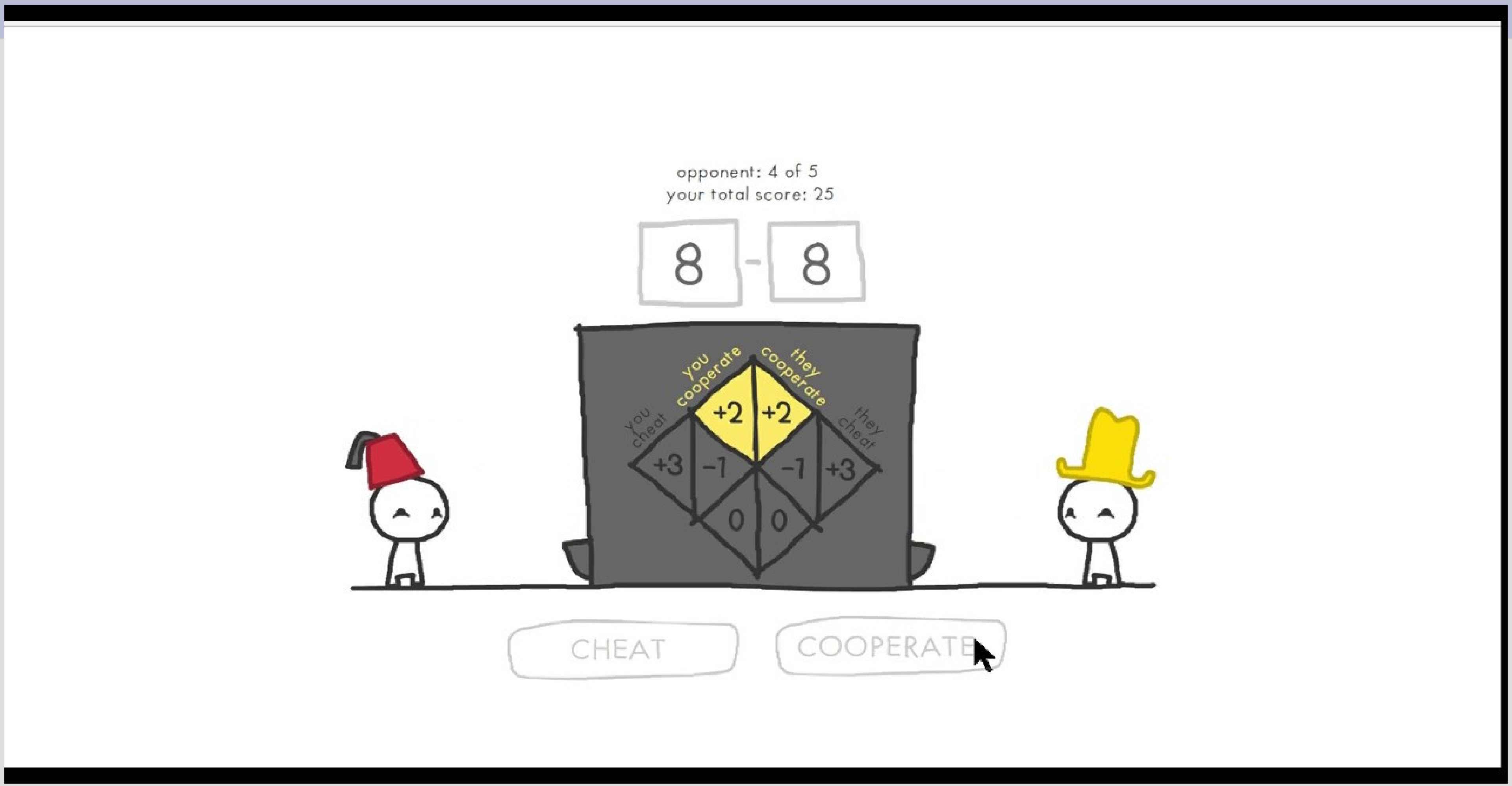
Repeated games

Two “hard” topics (if you are interested) are:

1. We have been talking about how to find best responses, but it is very hard to take advantage if an opponent is playing a sub-optimal strategy

2. How to “learn” or “convince” the opponent to play cooperatively if there is an option that benefits both (yet dominated)

Repeated games



http://ncase.me/trust/