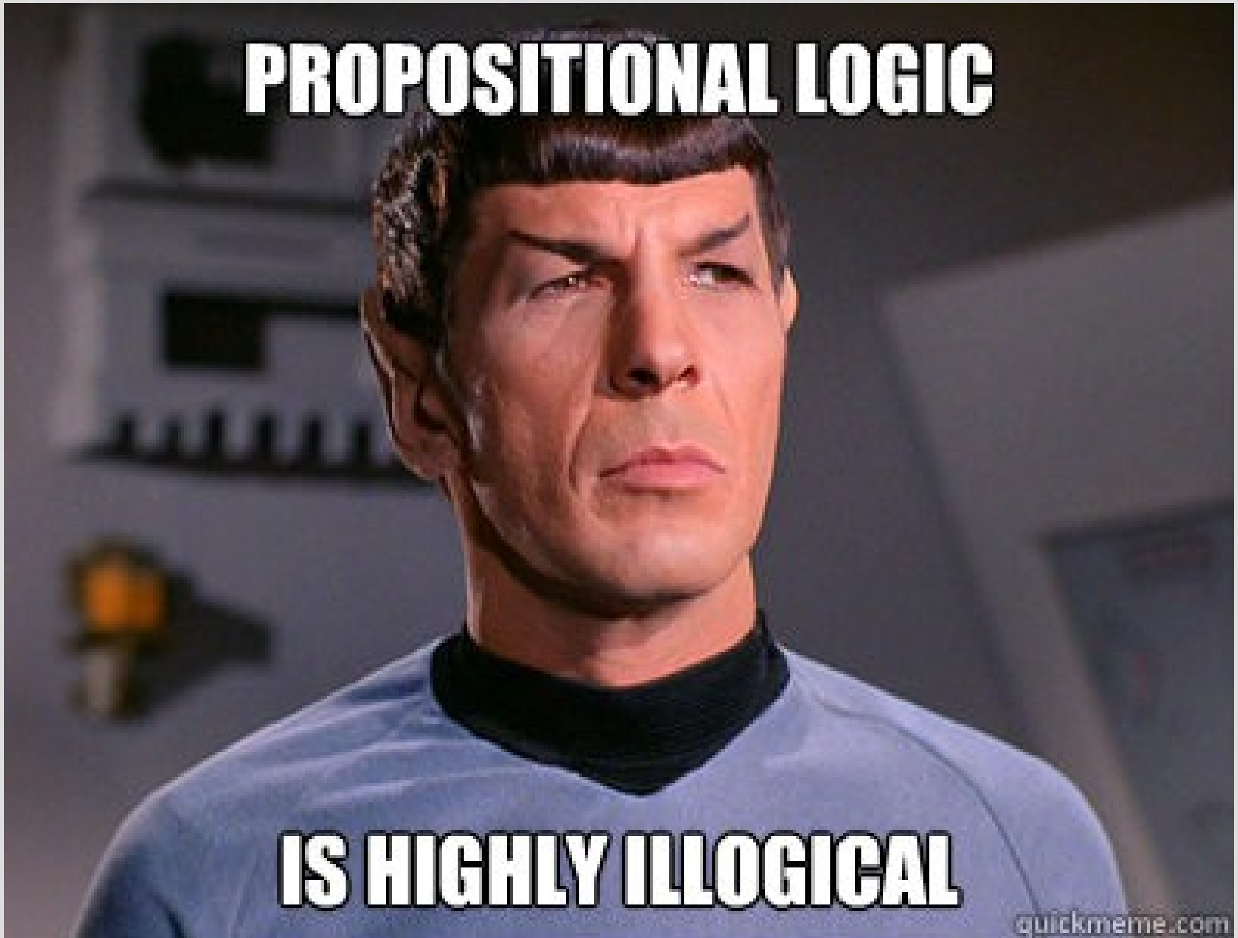
# Propositional logic (Ch. 7)



# Announcements

Writing 2 graded

- 2 weeks from today to resubmit

# Complete-state CSP

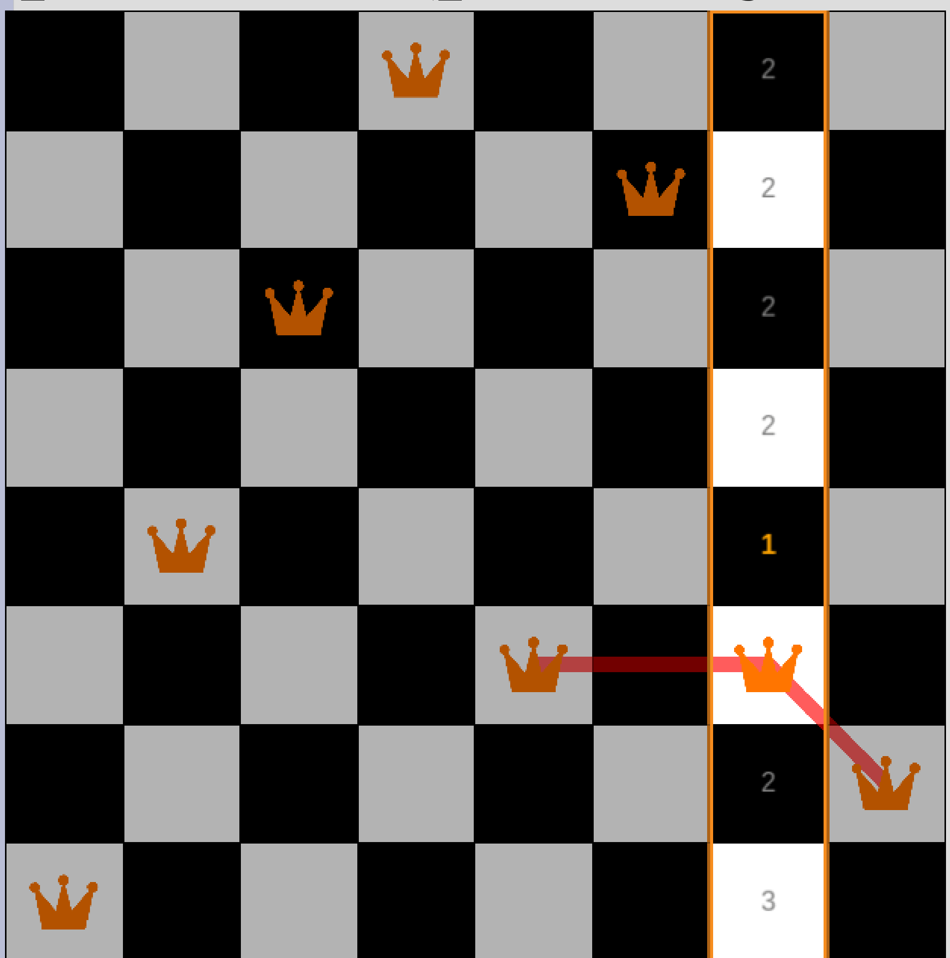
So far we have been looking at incremental search (adding one value at a time)

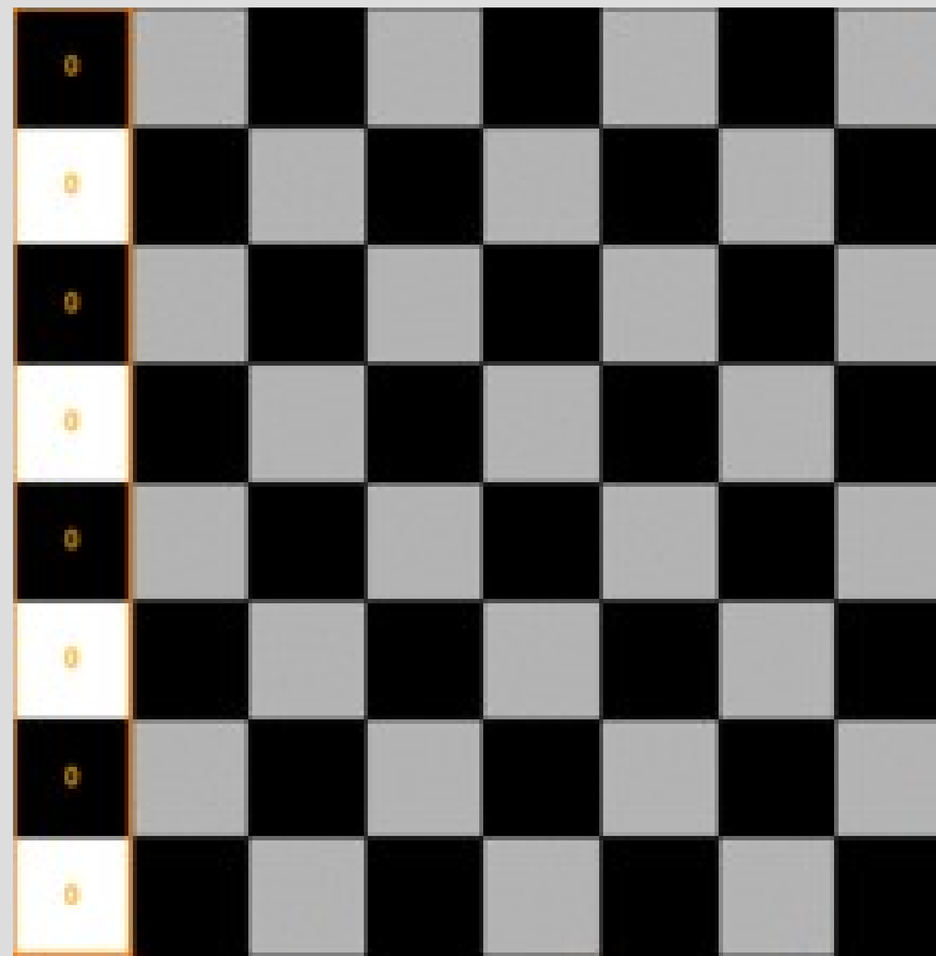
Complete-state searches are also possible in CSPs and can be quite effective

A popular method is to find the min-conflict, where you pick a random variable and update the choice to be one that creates the least number of conflicts

# Complete-state CSP

This works incredibly well for the n-queens problem (partially due to dense solutions)



# Complete-state CSP

As with most local searches (hill-climbing), this method has issues with plateaus

This can be mitigated by avoiding recently assigned variables (forces more exploration)

You can also apply weights to constraints and update them based on how often they are violated (to estimate which constraints are more restrictive than others)

# Complete-state CSP

Local search does not have "locally optimal" solution our general search does

As we have a CSP, the "local optimal" may occur, but if it is not 0 then we know we are not satisfied (unless we searched the whole space and find no goal)

This is almost as if we had an almost perfect heuristic built in to the problem!

# Representing knowledge

So far we have looked at algorithms to find goals via search, where we are provided with all the knowledge and possibly a heuristic

With CSP we saw how to apply inference to rules to find the goal

Now we will expand more on that and fully represent a knowledge base that will store the rules/constraints and what we see/deduce

# Logic

Minesweep?


http://minesweeperonline.com/

Write down any “deductions/rules” you find!

# Logic

One example of a simple rule:

The 1 in corner marks flag as a mine

Another rule:

The two can mark the two outer mines if flanked by ones

# Logic

The goal is to simply tell the computer about the rules of the game

Then based on what it sees as it plays, it will automatically realize these “safe plays”

This type of reasoning is important in partially observable environments as the agent must often reason on new/unseen information

# Logic: definitions

A symbol represents a part of the environment (e.g. a minesweep symbol might be if a cell has a mine or not), like math variables

Each single piece of the knowledge base is a sentence involving at least one symbol

A model is a valid assignment of symbols, a “possible outcome” of the environment

# Logic: definitions

Side note:

A model is just any assignment of true/false to the variables

The models of a sentence are all possible true assignments (i.e. the set of all models)

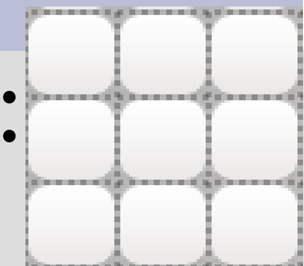
# Logic: definitions

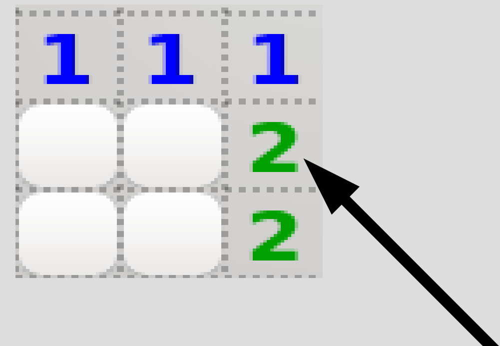
In propositional logic, a <u>symbol</u> is either true or false (as it represents a proposal of a “variable”)

If “m” is a model and is “α” a sentence, m <u>satisfies</u> α means α is true in m (also said as “m models α”)

Let M(α) be all models of α

# Logic: example

For example, consider a 3x3 minesweep: 

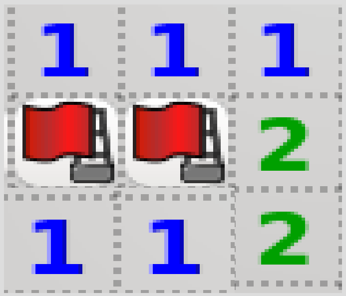
After the first play we have: 


Let us define P2,3,2 as the proposition that row 2, column 3 cell has value 2 (i.e. α=P2,3,2)

After playing the first move, we add to the knowledge base that this proposition is true (this representation has 10^9 states)

# Logic: example

Here is one possible assignment:




This does **not** satisfy our proposition P2,3,2 as there are only two mines adjacent to row 2, column 3 cell

So the assignment does not represent our knowledge base (i.e. the picture not in M(α))

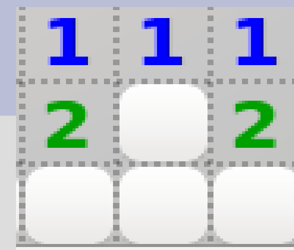
# Logic: entailment

We say $\alpha$ entails $\beta$ ($\alpha \models \beta$) if and only if every model with $\alpha$ true, $\beta$ is also true

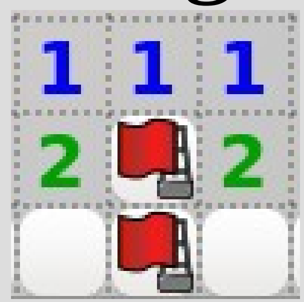
Another definition (mathy):
$\alpha \models \beta$ if and only if $M(\alpha)$ subset $M(\beta)$

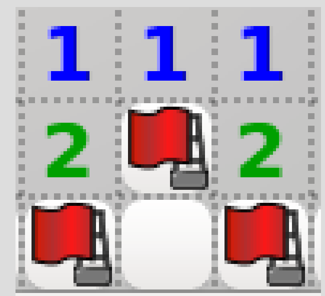
This means there are fewer models true with proposition $\alpha$ than $\beta$

# Logic: entailment

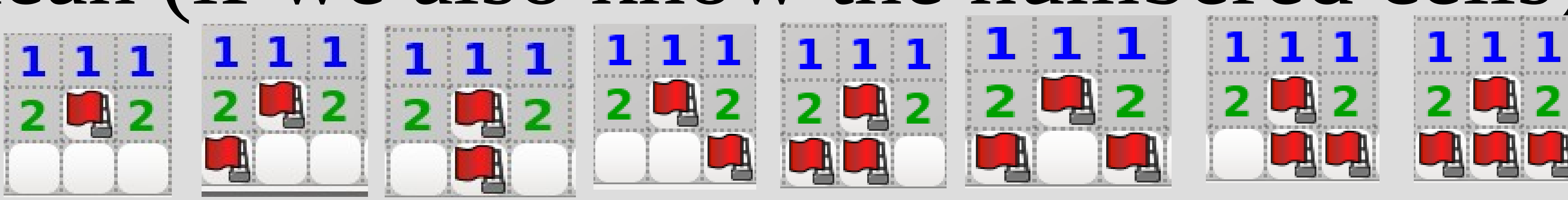
Consider this example: 


There are two valid configurations based on our knowledge base:



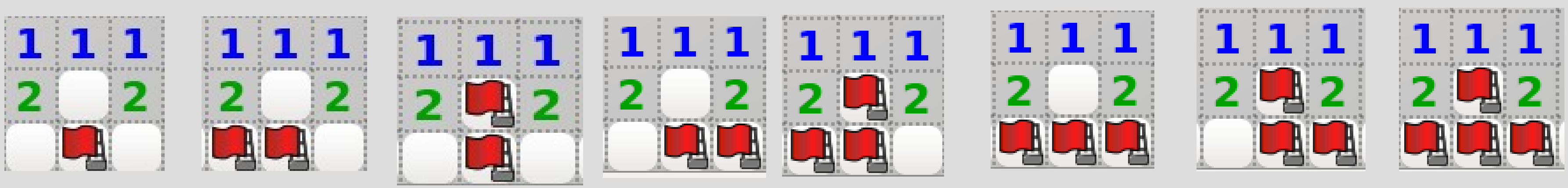

If we let α = {mine at (2,2)}, then this can mean (if we also know the numbered cells):



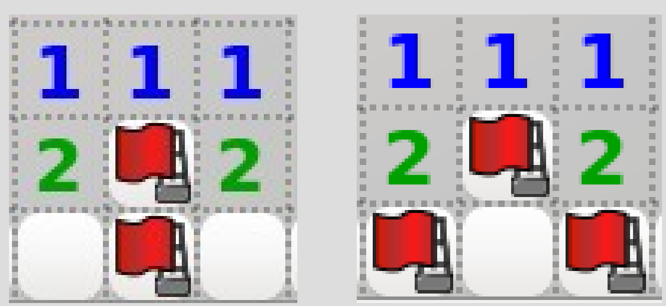
We can see that M(above) subset M(α(below))

# Logic: entailment

However, if we let β = mine at (3,2), we get:




M(knowledge base (KB)) is (again):




This is not entailment, as this is not in M(β), thus KB $\not\models \beta$ (in other words “from the KB, you cannot conclude (3,2) is a mine”)

# Logic: model checking

Entailment can generate new sentences for our knowledge base(i.e. can add “mine at (2,2)”)

Model checking is when we write out all the actual models (as I did in the last example) then directly check entailment

This is exponential, and unfortunately this is very typical (although some are much worse exponential than others)

# Logic: model checking

Model checking...

1. Preserves truth through inference
2. Is complete, meaning it can derive any sentence that is entailed (and in finite time)

The "complete" is important as some environments have an infinite number of possible sentences

# Logic syntax

In our (current) logic, we allow 5 operations:

$\neg$ = logical negation (i.e. "not" T = F)

$\wedge$ = AND operation

$\vee$ = OR operation (Note: not XOR)

$\Rightarrow$ = "implies" operation

$\Longleftrightarrow$ = "if and only if" operation (iff)

The order of operations (without parenthesis) is top to bottom

# Logic syntax

We mentioned a symbol is P1,3,2 but a <u>literal</u> is either P1,3,2 or ┐ P1,3,2

Two notes:

OR is not XOR (exclusive or), which is not the English "or" (e.g. ordering food)

"implies" only provides information if left hand side is true (e.g. F = cats can fly, B = cats are birds: F implies B is true...)

# Logic syntax

Here are the truth tables:

| p | q | $p \wedge q$ | $p \vee q$ | $p \rightarrow q$ | $p \leftrightarrow q$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | T | F | F |
| F | T | F | T | T | F |
| F | F | F | F | T | T |

And equivalent laws:

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$ commutativity of $\wedge$
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$ commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$ associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$ associativity of $\vee$
$\neg(\neg\alpha) \equiv \alpha$ double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$ contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$ implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$ biconditional elimination
$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$ de Morgan
$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$ de Morgan
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$ distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$ distributivity of $\vee$ over $\wedge$

# Check model

We can make use model checking to make an inference algorithm, much the same way we modified DFS to do backtracking search

1. Enumerate possibilities on a symbol (repeat)
2. Once all symbols are assigned, check if consistent, if not return false (all the way up tree due to recursive call)

# Check model

Example: suppose our KB is “P implies Q”
We want to check α = “not P”
Enumerate P: {P = true}, {P = false}
Enumerate Q: {P=T,Q=T}, {P=T,Q=F}, {P=F,Q=T}, {P=F,Q=F}

Consistent?

| P | Q | not P | P → Q |
|---|---|---|---|
| T | T | F | T |
| T | F | F | F |
| F | T | T | T |
| F | F | T | T |

No! (top row)
“not P” is false when “P implies Q” is true