✍1 Exact solution of system

$$\begin{pmatrix} 2 & 4 & 4 \\ 1 & 5 & 6 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 8 \end{pmatrix}$$

Solution: You will find $\boldsymbol{x} = [\mathbf{1}, \mathbf{3}, -\mathbf{2}]^T$. □

✍3 Exact operation count for GE.

Solution:

$$
\begin{aligned}
T &= \sum_{k=1}^{n-1} \sum_{i=k+1}^{n} (2(n-k)+3) && (1)\\
&= \sum_{k=1}^{n-1} (2(n-k)+3)(n-k) && (2)\\
&= 2\sum_{k=1}^{n-1}(n-k)^2 + 3\sum_{k=1}^{n-1} (n-k) && (3)\\
&= 2\sum_{j=1}^{n-1} j^2 + 3\sum_{j=1}^{n-1} j && (4)\\
&= 2\frac{(n-1)(n)(2n-1)}{6} + 4 \times \frac{n(n-1)}{2} && (5)\\
&= .... && (6)\\
&= n(n-1)\left(\frac{2n}{3} + \frac{7}{6}\right) && (7)
\end{aligned}
$$

In (7) we used the fact that $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$ and $\sum_{k=1}^{n} k = n(n+1)/2$. From (3) to (4) we made a change of variables $j = n - k$. Finally observe the remarkable fact that the final expression (7) is always an integer (it has to be) no matter what (integer) value n takes. □