

$y_1(t)$
 $y_2(t)$
 \vdots
 $y_n(t)$

—
 y'

$y'_1(t)$
 $y'_2(t)$
 $y'_n(t)$

—
 $y' = A \cdot y \quad A : n \times n$

$y(0)$ known Then Solution:

$$y(t) = \exp(tA) \cdot y(0)$$

$$\exp(A) = I + A + A^2/2! + \dots + A^k/(k!) + \dots$$

$$y' = F(y)$$

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Total Population: N (e.g., $N = 100000$)

$$S + I + R = N \quad (*)$$

- Removed population varies with I :

$$R' = I/t \quad \rightarrow t = 1/\mu$$

$$R' = \mu I \quad ; \quad 1/\mu = \text{infection period [e.g., 5 days or 10 days]}$$

"rate of variation of R is proportional to I "

- Susceptible population :

$$S' = c (S/N) \times I$$

$$S' = -\beta S I$$

β written in the form $\beta = \mu R_0 / N \implies R_0 = \beta N/\mu$
reproduction number

Because of (*) we have $S' + I' + R' = 0 \implies$

$$I' = -S' - R' = \beta S I - \mu I = [\beta S - \mu] I$$

$$y = \begin{matrix} S \\ I \\ R \end{matrix} \implies y' = \begin{matrix} -\beta I & 0 & 0 \\ 0 & \beta S - \mu & 0 \\ 0 & \mu & 0 \end{matrix} \cdot \begin{matrix} S \\ I \\ R \end{matrix}$$

of the form: $y' = A(y) \cdot y$

Forward Euler: y_0, y_1, \dots, y_k at times $0, 1, \dots, k$
 $(y(t+\delta t) - y(t)) / \delta t \sim y'(t)$

$$y_{\{k+1\}} = y_k + \delta t A(y_k) y_k$$