System to solve

$$\begin{pmatrix} 2 & 1 \\ 0 & \varepsilon \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$x_2 = 1/\varepsilon$ ===>

$\hat{x}_2 = (1/\varepsilon)(1+ \delta_1)$ $\quad |\delta_1| < \bar{u}$

$\hat{x}_2 = (1+ \delta_1)/\varepsilon = 1 / [\varepsilon /(1+\delta_1)] \equiv 1 / [\varepsilon(1+\eta_1)]$

NOTATION: $1+\delta = 1/(1+\eta)$ so $\eta = 1/(1+\delta) -1 = -\delta /(1+\delta)$
note $| \eta | \le \bar{u} / (1-\bar{u}) \equiv \gamma_1$

$\hat{x}_2 = (1/\varepsilon)(1+ \delta_1) = 1 /(\varepsilon (1+\eta_1))$
==> replace ε by $\varepsilon(1+\eta_1)$ in matrix

===

$$\begin{pmatrix} 2 & 1 \\ 0 & \varepsilon \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

get x_1 ==> $x_1 = (3 - x_2)/2$ ==> In inexact arithmetic:

$$\hat{x}_1 = [(3 - \hat{x}_2)(1+\delta_2)/2.0] (1+\delta_3)$$

$$= \frac{3 - \hat{x}_2}{2.0(1+\eta_2)(1+\eta_3)}$$

$$2.0 (1+\eta_2)(1+\eta_3) \equiv 2 (1+\theta_2) = 2 + 2 \theta_2$$
$$\theta_2 = \eta_2+ \eta_3+\eta_2\eta_3$$

we can return to the $\delta_i$ls

$(A+E)\hat{x} = b$ where :

$$E = \begin{pmatrix} 2\theta_2 & 0 \\ 0 & \varepsilon \eta_1 \end{pmatrix} \qquad \begin{matrix} |\theta_2 | \le \ldots \\ |\eta_1 | \le \ldots \end{matrix}$$

Note: all the backward error in A not b..

$$\begin{pmatrix} I & -Z \\ 0 & I \end{pmatrix}$$

$\| A \| = \sqrt{2 n + \| Z \|^2}$
$\mathrm{cond}(A,F) = 2 n + \| Z \|^2$

$\| A A^{-1} \| = 1 \le \| A \| \| A^{-1} \|$

$3 - \hat{x}_2$

-UUU:**--F1 tmp1   Top L8   (Fundamental) --------------------------

$\hat{A} = A + \varepsilon E \quad E = \text{randn}(n,n) \;.\!*\; A$

$\hat{a}_{ij} = a_{ij}(1+\varepsilon) \quad E = \{ \varepsilon_{ij} a_{ij} \}$

$r = b - Ay$
$A y = b - r = b + \Delta b \quad \Delta b = -r$

$\| \Delta b \| / \| b \| = \| r \| / \|b\|$

$r = b - Ay$ ==> $A y + r = b$ find E s.t. $Ey = r$

$(A + t \; r y^T) y = b$

$t = 1/(y^T y)$

$|| u v^T ||_2 = || u ||_2 \; || v||_2$

$|| A v || = \tau$

$|| A^{-1} A v || =1$ ==> $|| A^{-1}|| \; || A v || \geq 1$

$\frac{(Au,u)}{(u,u)} \geq \alpha$??

$v = u / \| u \|_2$

$(Av,v) > 0$

$\min (Av,v) = (A v_0, v_0) ==$ some number > 0

$(Av,v) \geq \alpha$

$\| v \| =1$