

$$a_{ii} = (Ae_i, e_i)$$

$$a_{ij} = (Ae_j, e_i)$$

$|x|$  k components nonzero  
 $u = \begin{bmatrix} |x| \\ 0 \end{bmatrix}$  followed by zeros.

$$(Au, u) = (A_k x, x)$$

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$(X^T A X v, v) > 0$ ? for all nonzero  $v$ ?

$$(X^T A X v, v) = (A X v, Xv) \equiv (Au, u) \text{ with } u = Xv$$

if  $v$  is nonzero then  $u$  is nonzero then  $(Au, u) > 0$  done  
 because  $X$  is of full rank.

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at kth step :

$$\det(A_k) = \prod_{i=1}^k u(i, i) \quad u(k, k) = \det(A_k) / \det(A_{k-1})$$

$$\implies u(1,1) > 0 \quad u(2,2) > 0 \quad \dots \quad u(i,i) > 0$$

$$A = L D L^T$$

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$$\begin{aligned} (A(x+y), x+y) &= (Ax, x) + (Ay, y) + 2(Ax, y) \\ \xrightarrow{>0} & \xrightarrow{>0} & \xrightarrow{>0} \\ (Ax, y) &= 0 \quad \forall x \quad \forall y \end{aligned}$$

use  $x = e_j \quad y = e_i \implies a_{ij} = 0$  done.

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LU:

$$a(i, j) \leftarrow a(i, j) - \frac{a(i, k) a(k, j)}{a(k, k)}$$