Pr. Ex. 10

Part a.

$\| A - B Q \|_F$ A: m x p B: m x p Q: pxp $Q^T Q = I$

$\| A - B Q \|_F^2 = Tr\ (A-BQ)\ (A-BQ)^T = Tr\ [A\ A^T - A\ Q^T\ B^T - BQ\ A^T + B\ B^T]$

$= Tr\ (A\ A^T) + Tr\ (B\ B^T) - Tr(A\ Q^T\ B^T) - Tr(\ B\ Q\ A^T)$

[Note: $(A\ Q^T\ B^T)^T = B\ Q\ A^T$]

$= Tr\ (A\ A^T) + Tr\ (B\ B^T) - 2\ Tr(BQ\ A^T)$

===> Need to maximize $Tr\ (B\ Q\ A^T) = Tr((A^T\ B)\ Q)$ mxm matrix

$C = A^T B = U\ \Sigma\ V^T$

Need to maximize:

$Tr((A^T\ B)\ Q) = Tr\ (\ U\ \Sigma\ V^T\ Q) = Tr\ (\ \Sigma\ V^T\ Q\ U)$

Optimal solution is reached when $V^T\ Q\ U = I$!

This give $Q = V\ U^T$ ...

Notes:
$V^T\ Q\ U$ is unitary $V^T\ Q\ U\ U^T\ Q^T\ V = I$

!! $Tr(\ X\ Y) = Tr\ (Y\ X)$

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