Rayleigh – Ritz:

$\tilde{u} = Q\, y$ approximate eigenvector

$(A - \mu I)\, \tilde{u} \perp X \rightarrow Q^H (A - \mu I)\, \tilde{u} = 0 \rightarrow Q^H (A - \mu I)\, Q\, y = 0$

$\rightarrow Q^H A\, Q\, y = \mu\, y$

mxm eigenvalue problem

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Projection method: case of invariant subspaces:

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By definition:

X invariant if $A X \subseteq X$

Assume X invariant and let Q = orthonormal basis of X

$\mu$, $\tilde{u}$ approximate eigenpair obtained from proj. method.

$\mathbb{R}^n = X \oplus X\perp$

Then:
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$$(A - \mu I)\, \tilde{u} = \underset{\in X}{A \tilde{u}} - \underset{\in X}{\mu \tilde{u}} \in X \rightarrow (A - \mu I)\, \tilde{u} = Q\, z$$

But recall that $Q^H (A - \mu I)\, \tilde{u} = 0 \rightarrow Q^H Q\, z = z = 0 \rightarrow$

$(A - \mu I)\, \tilde{u} = 0 \rightarrow \mu, \tilde{u}$ exact eigenpair □

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$w := Av_j$

$$w := Av_j - \sum_{i=1}^{j} h_{ij}\, v_i \rightarrow A\, v_j = \sum_{i=1}^{j+1} h_{ij}\, v_j$$

$h_{j+1,j} = ||w||$

$v_{j+1} = w / h_{j+1,j}$

$A\, V_m = V_{m+1}\, \bar{H}_m$

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