4511W, Fall-2021
ASSIGNMENT 4:
Assigned: 11/03/21 Due: 11/09/21 at 11:55 PM (submit on gradescope, mark the page(s) associated for each problem when submitting). Show work for full credit.

Problem 1. (20 points)
For the following payoff matrix, consider only **pure strategies** and find:
(1) All Nash equilibrium
(2) All Pareto optimal strategies

P1 \ P2	x	y	z
a	(5,8)	(6,6)	(3,4)
b	(4,9)	(9,4)	(1,3)
c	(7,2)	(8,2)	(5,3)

Problem 2. (20 points)
Suppose you had a 2-by-2 payoff matrix where all payoffs are zero-sum (to the same constant value). Additionally, each payoff value cannot be duplicated. So you cannot make a matrix such as these:

$$\begin{bmatrix} (2,2) & (4,0) \\ (4,0) & (1,3) \end{bmatrix}$$

$$\begin{bmatrix} (0,0) & (0,0) \\ (0,0) & (0,0) \end{bmatrix}$$

For **pure strategies**, what are:
(1) The maximum number of Pareto optimal strategies
(2) The minimum number of Pareto optimal strategies
(3) The maximum number of Nash equilibrium
(4) The minimum number of Nash equilibrium

Problem 3. (20 points)
Consider the following 3-by-3 payoff matrix. For **mixed strategies** find:
(1) At least one Nash equilibrium
(2) All Pareto optimal strategies

(Hint: remember what the definition of a "Nash equilibrium" means, not just the process of how to find it.)

		P2		
		x	y	z
P1	a	(32,14)	(8,22)	(9,12)
	b	(5,3)	(10,1)	(7,9)
	c	(3,8)	(8,8)	(9,12)

Problems 4 and 5 deal with the following constraint satisfaction problem:
w, x, y and z are all single digit integers (0-9)

$w < x < y < z$
$w+x > z$
$5 < y < 8$
$z-y = x-w$

Problem 4. (20 points)
For the problem above, show for k-consistency:
(1) The domains of variables that are 1-consistent
(2) The domains of variables that are 2-consistent (using part (1) as a starting point)
(3) Of the pairs that involve "x", which values are 3-consistent (using part (2) as a starting point) (Note: only need to show pairs with "x" to reduce work, not due to any CSP related reason.)

Problem 5. (20 points)
Suppose we were doing backtracking search.
(1) If we are following the four recommendations for backtracking search, what variable should be picked first?
(2) From your answer in part 1, what value should be picked for this variable?
(3) Ignoring parts (1) and (2), assume you have assigned {z=7, y=6}. What are the domains of the remaining variables that are 2-consistent?

Programming:

We are still using the AIMA code, though we have finally graduated from search (mostly). Instead both these problems deal with constraint satisfaction problems, so we will be looking in:
`csp.py`
`tests/test_csp.py`

Problem 6. (10 points)
We will again test the N-queens problem (framed as a constraint satisfaction problem this time).

(1) Report the run-time for board sizes = {11, 20, 28} when solving the N-Queens CSP problem with backtracking search.

(2) Report the run-time for board sizes = {11, 20, 28, 40} when solving the N-Queens CSP problem with min-conflicts.

(3) Find what board size takes approximately 10 seconds to solve with min-conflicts (make sure it is not ending early).

Problem 7. (15 points)
The “Zebra” problem is a classic constraint satisfaction problem defined as a set of 5: houses, people (of nationality), pets, cigars, drinks and colors. House 1 is on the far left with house 5 on the far right (house 3 is in the middle). Each house (numbered) has a single person (nationality), pet, cigar, drink and color associated with the house. No two houses share the same of any property (for example, each house has its own unique pet). The rules are:

- There are five houses.
- The Englishman lives in the red house.
- The Spaniard owns the dog.
- Coffee is drunk in the green house.
- The Ukrainian drinks tea.
- The green house is immediately to the right of the ivory house.
- The Old Gold smoker owns snails.
- Kools are smoked in the yellow house.
- Milk is drunk in the middle house.
- The Norwegian lives in the first house.
- The man who smokes Chesterfields lives in the house next to the man with the fox.
- Kools are smoked in the house next to the house where the horse is kept.
- The Lucky Strike smoker drinks orange juice.
- The Japanese smokes Parliaments.
- The Norwegian lives next to the blue house.

The question is then: Who owns the Zebra? And who drinks water?

(5/15 points) This classic problem is what is already put in as the Zebra problem. Run the backtracking_search() on this problem and report the answer to the two questions above. (It is fine to use the default parameters for backtracking_search().)

(10/15 points) Modify the problem to match the Zebra problem below (with names, sports, transportation and lawns) and report: Who has flowers in their yard? Who likes to watch Starcraft2?

- There are five houses.
- Albert likes baseball.
- Dietfried has a lawn with cleanly cut grass.
- Virgilijus enjoys watching rugby.
- Gallchobhar gets to work by walking.
- Bricius lives next to Virgilijus.

- The owner of the house with an Astroturf lawn likes baseball.
- The house with flowers in the lawn is between (directly next to) the house with trees and the house with rocks in the lawns.
- The person who likes baseball goes to work on a motorcycle.
- The 2nd house owners like to watch debates.
- The owner of the 4th house goes to work on a bicycle.
- The person who likes soccer takes a bus to work.
- The house with trees on the lawn to the right of the house with an owner who drives a car to work.
- Gallchobhar lives to the right of the person who likes Rugby.