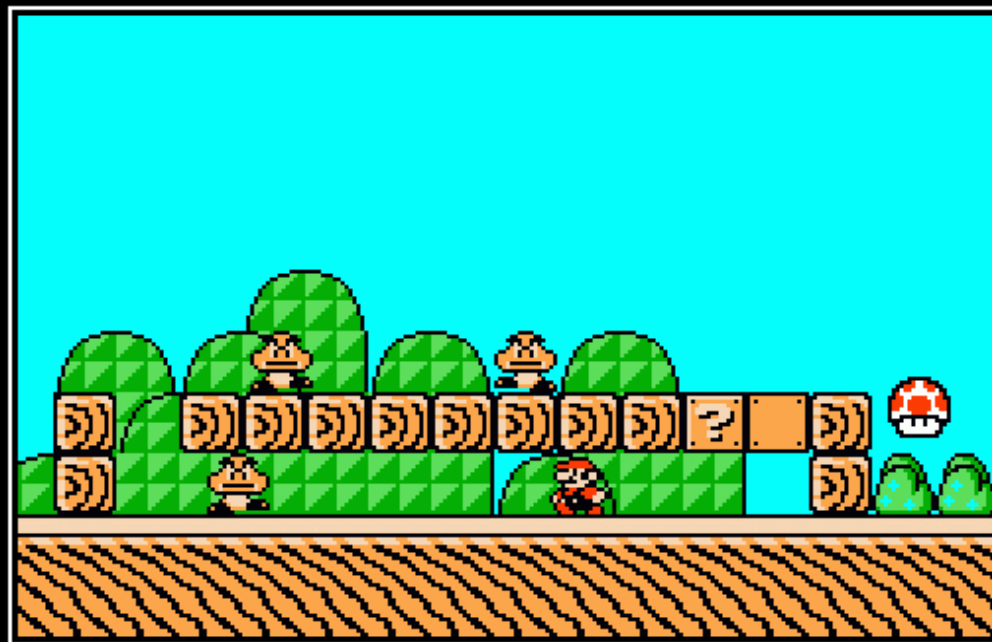


# Planning (Ch. 10)



## PLANNING

Somehow, I don't think you thought your cunning plan all the way through.

# Graph Plan

Consider this problem:

Initial:  $Sleepy(me) \wedge Hungry(me)$

Goal:  $\neg Sleepy(me) \wedge \neg Hungry(me)$

Action(  $Eat(x)$ ,

Precondition:  $Hungry(x)$ ,

Effect:  $\neg Hungry(x)$ )

Action(  $Coffee(x)$ ,

Precondition: ,

Effect:  $\neg Sleepy(x)$ )

Action(  $Sleep(x)$ ,

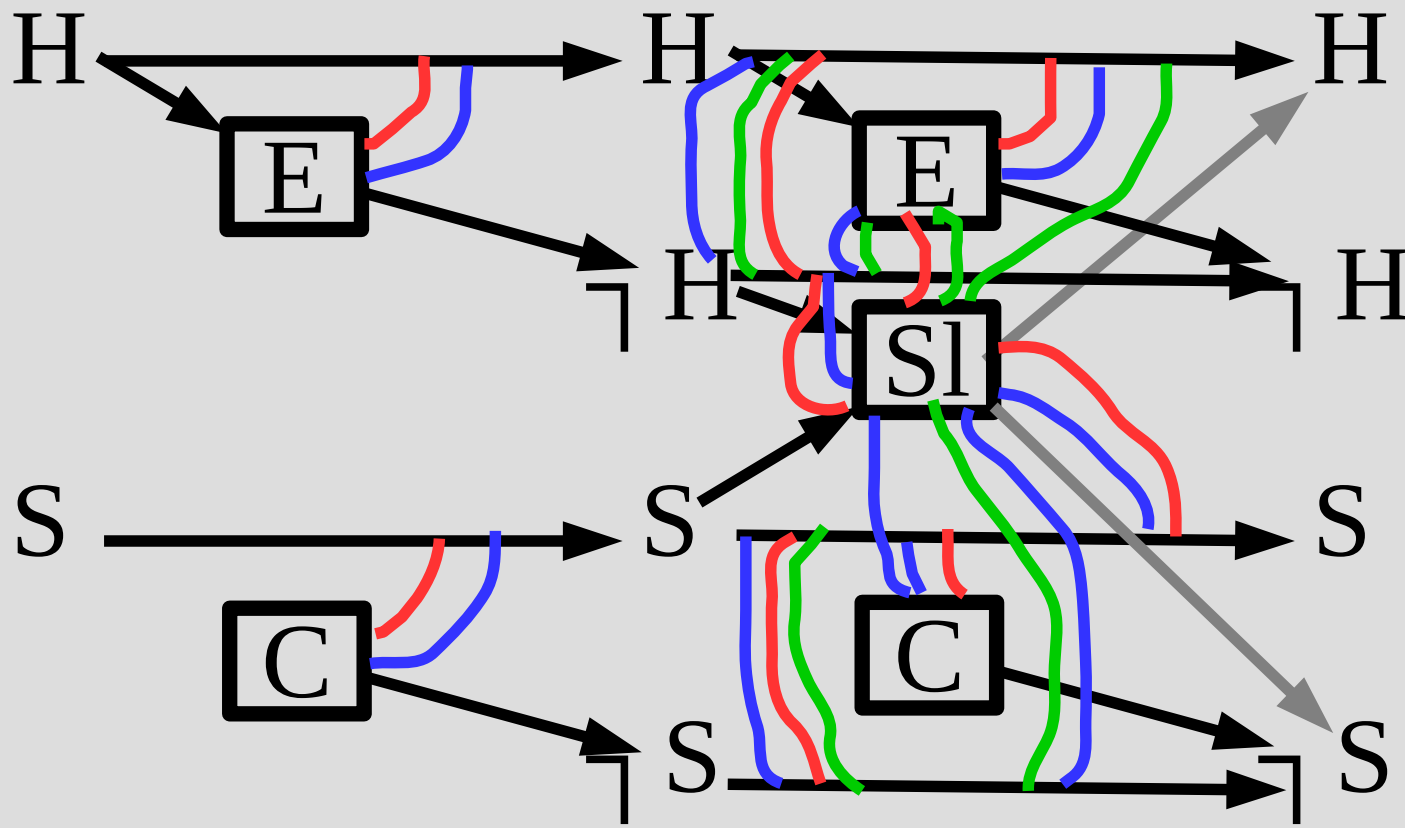
Precondition:  $Sleepy(x) \wedge \neg Hungry(x)$ ,

Effect:  $\neg Sleepy(x) \wedge Hungry(x)$ )

# Mutexes: actions

Mutex Action rules:

1.  $x \in Effect(A1) \wedge \neg x \in Effect(A2)$
2.  $x \in Pre(A1) \wedge \neg x \in Effect(A2)$
3.  $x \in Pre(A1) \wedge \neg x \in Pre(A2)$



# Mutexes: states

There are 2 rules for states, but unlike action-mutexes they can change across levels

1. Opposite relations are mutexes ( $x$  and  $\neg x$ )
2. If there are mutexes between all possible actions that “lead” to a pair of states...

Two ways that “leading” can be in mutex:

- 2.1. Actions are in mutex
- 2.2. Preconditions of action pair are in mutex

# Mutexes: states

Another way to compute state mutexes:

- (1) Add mutexes between all pairs in state
- (2) If any pair of actions can lead to this pair of relationships, un-mutex them

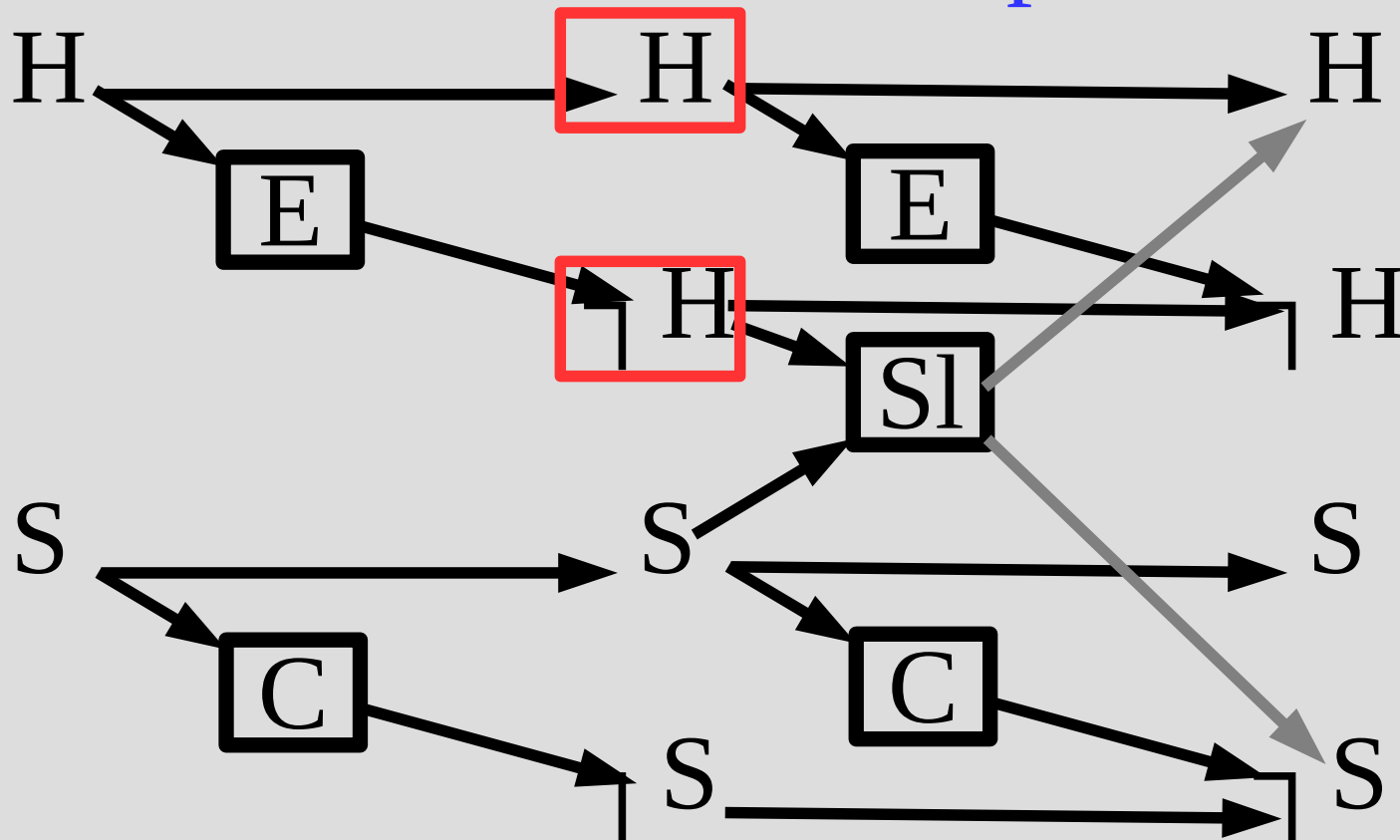
Recap:

If any valid pair of actions = no mutex

All ways of reaching invalid = mutex

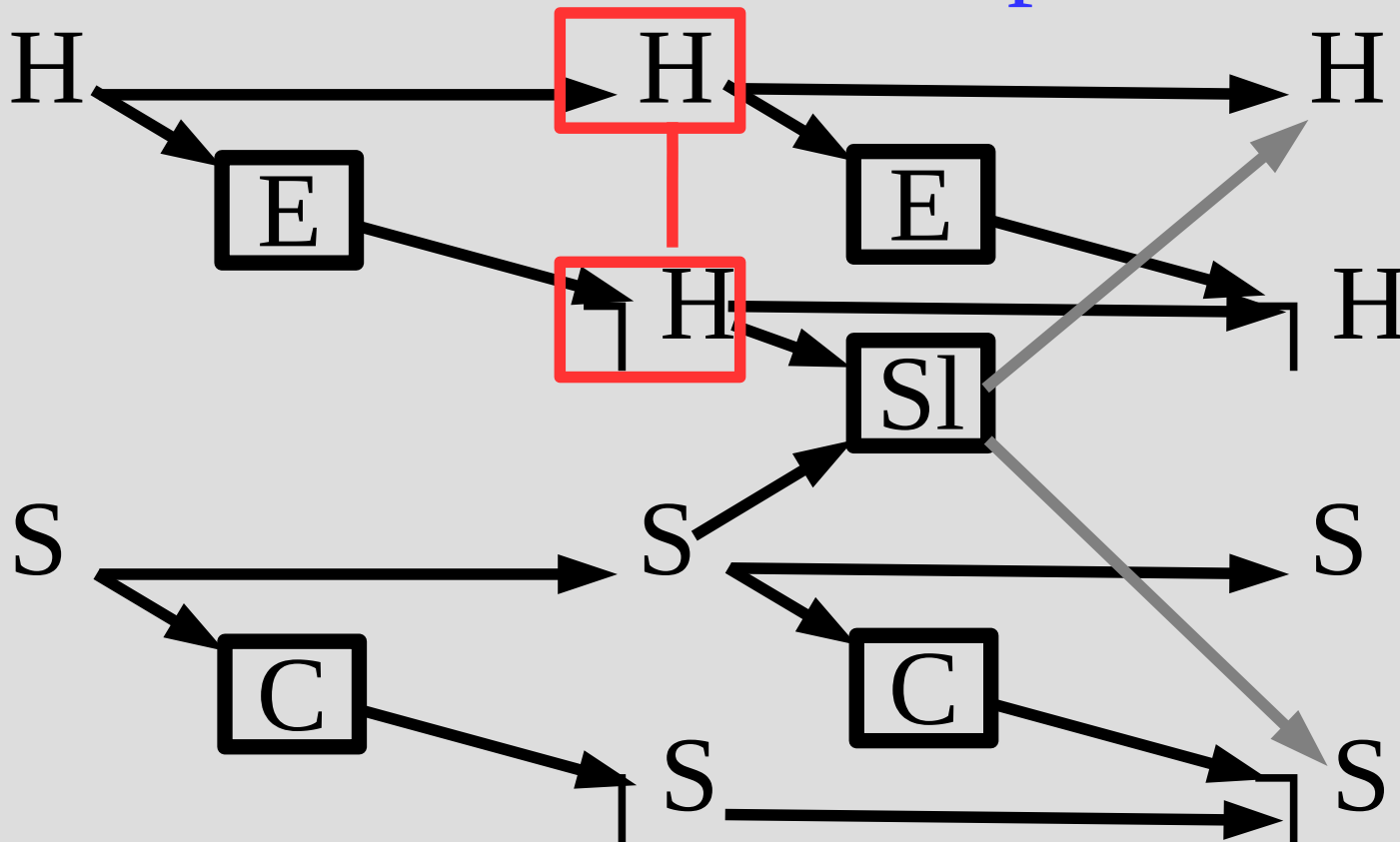
# Mutexes: states

1. Opposite relations are mutexes ( $x$  and  $\neg x$ )
2. If there are mutexes between all possible actions that lead to a pair of states



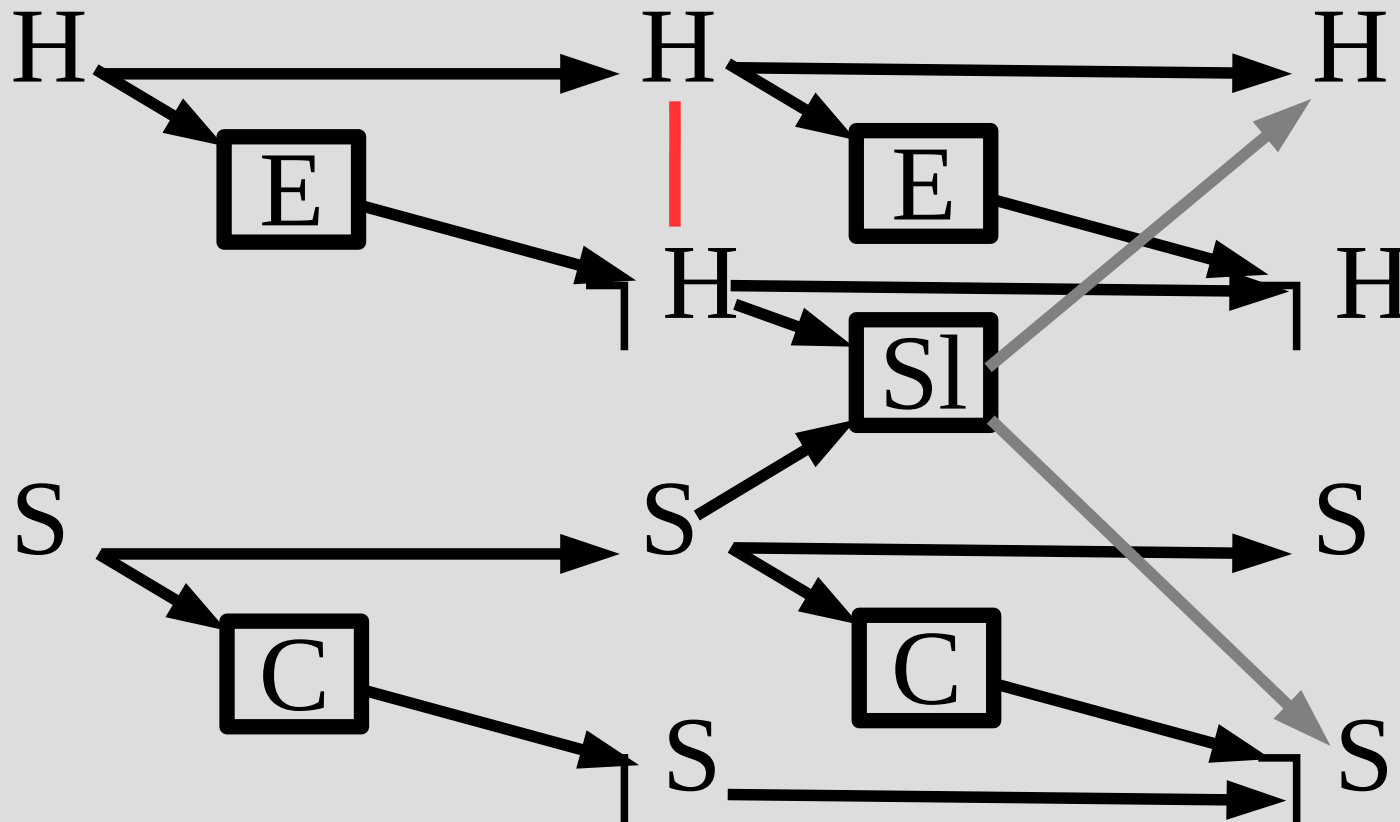
# Mutexes: states

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# Mutexes: states

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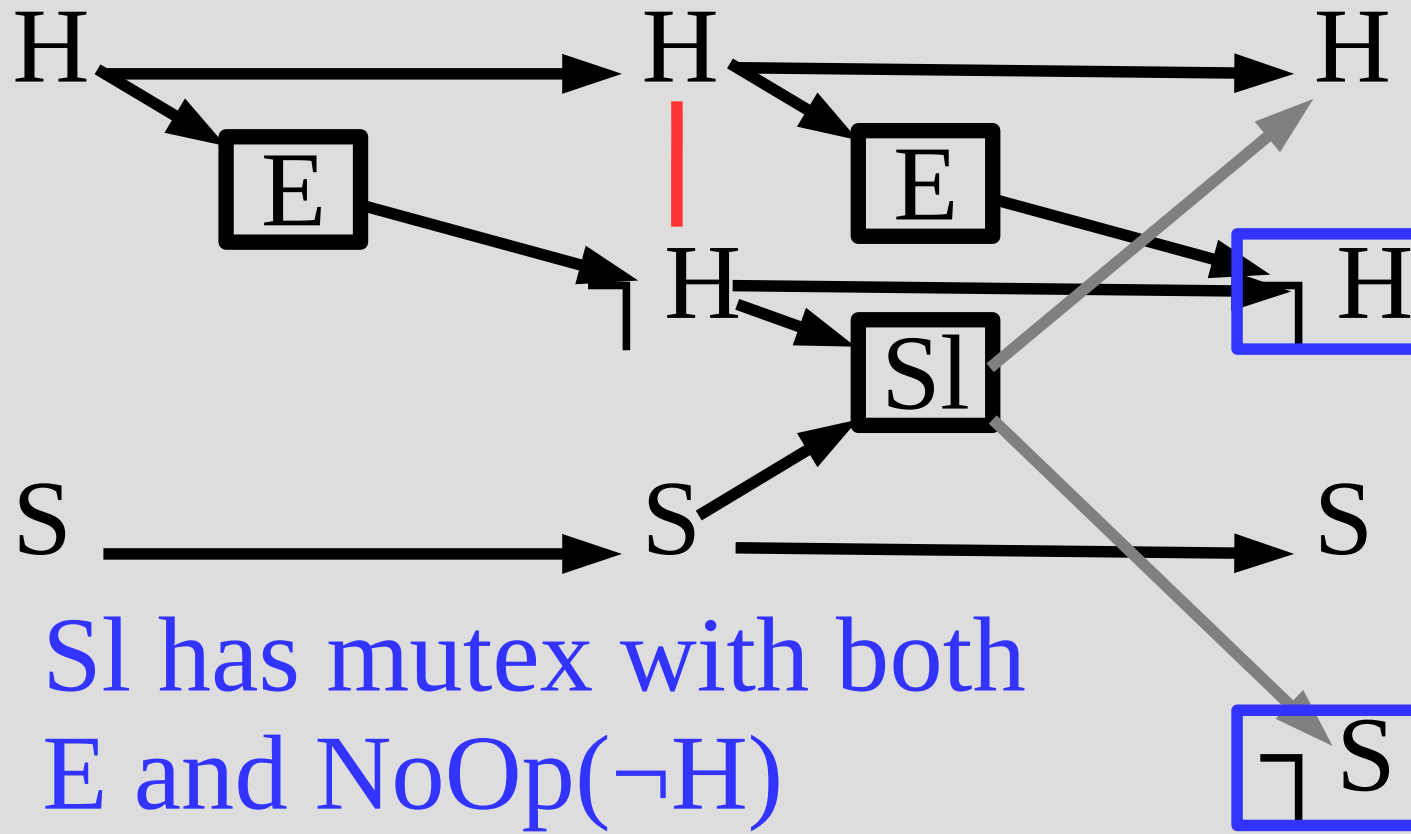


None...  
but if we  
remove  
coffee...



# Mutexes: states

1. Opposite relations are mutexes ( $x$  and  $\neg x$ )
2. If there are mutexes between all possible actions that lead to a pair of states

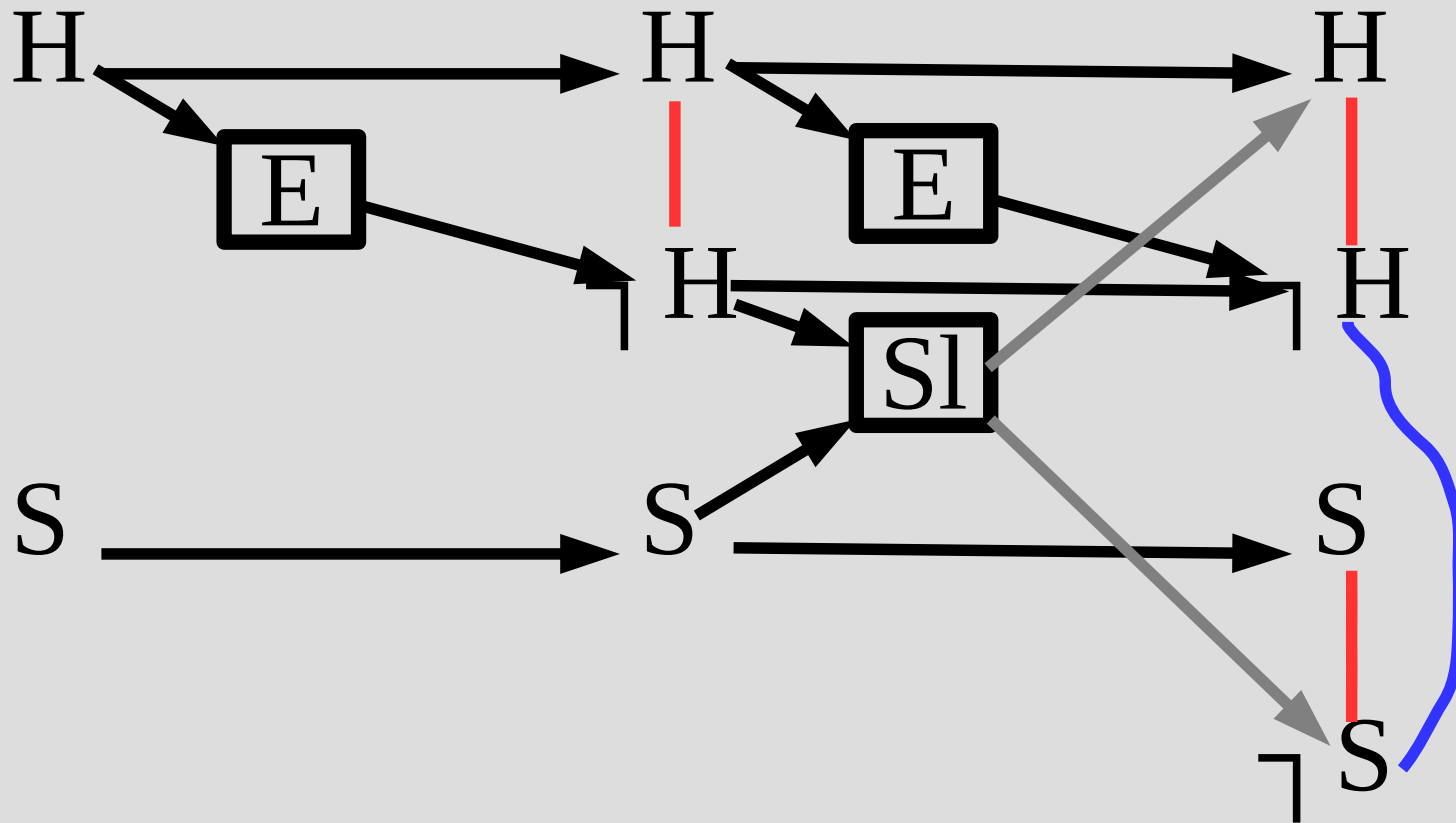


This mutex will be gone on the next level (as you can eat again)

SI has mutex with both E and NoOp( $\neg H$ )

# Mutexes: states

1. Opposite relations are mutexes ( $x$  and  $\neg x$ )
2. If there are mutexes between all possible actions that lead to a pair of states



# Mutexes: actions

Consider...

Initial:  $\neg Money \wedge \neg Smart \wedge \neg Debt$

Goal:  $\neg Money \wedge Smart \wedge \neg Debt$

Action( *School*,

Precondition: ,

Effect:  $Debt \wedge Smart$ )

Action( *Job*,

Precondition: ,

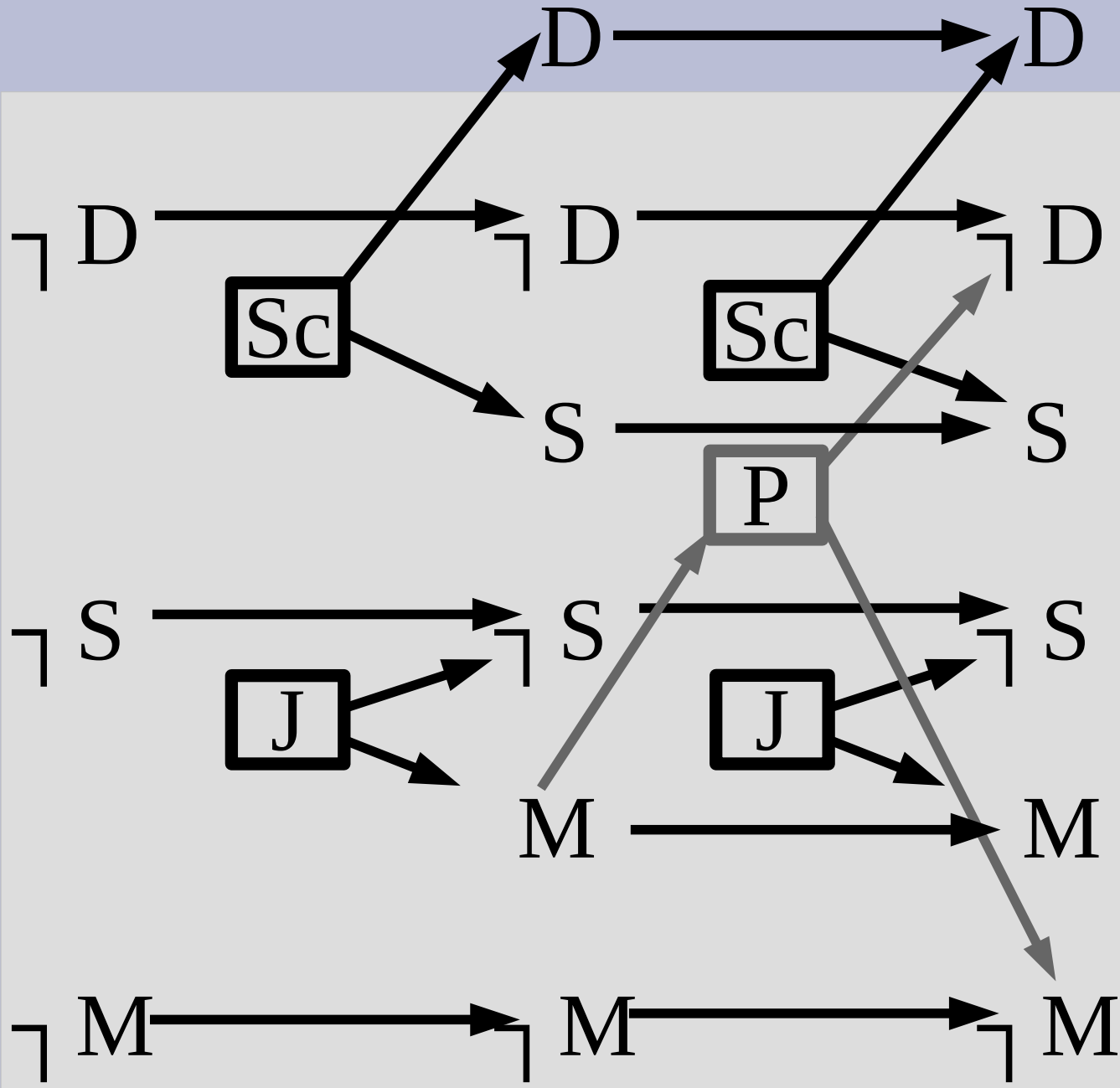
Effect:  $Money \wedge \neg Smart$ )

Action( *Pay*,

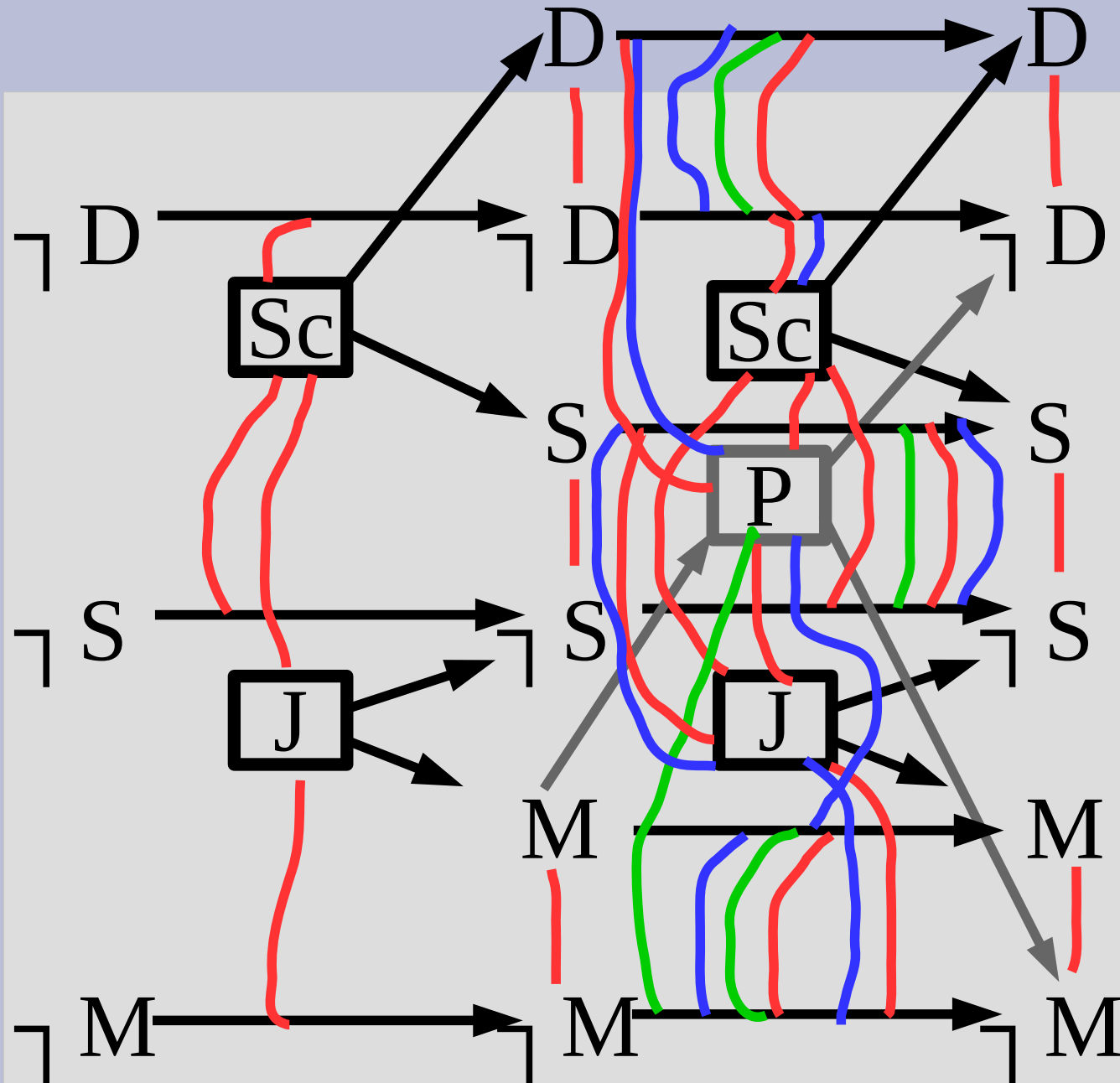
Precondition: *Money*,

Effect:  $\neg Money \wedge \neg Debt$ )

# Mutexes: actions



# Mutexes: actions



Non-trivial  
mutexes:

$(SC, P),$

$(J, P),$

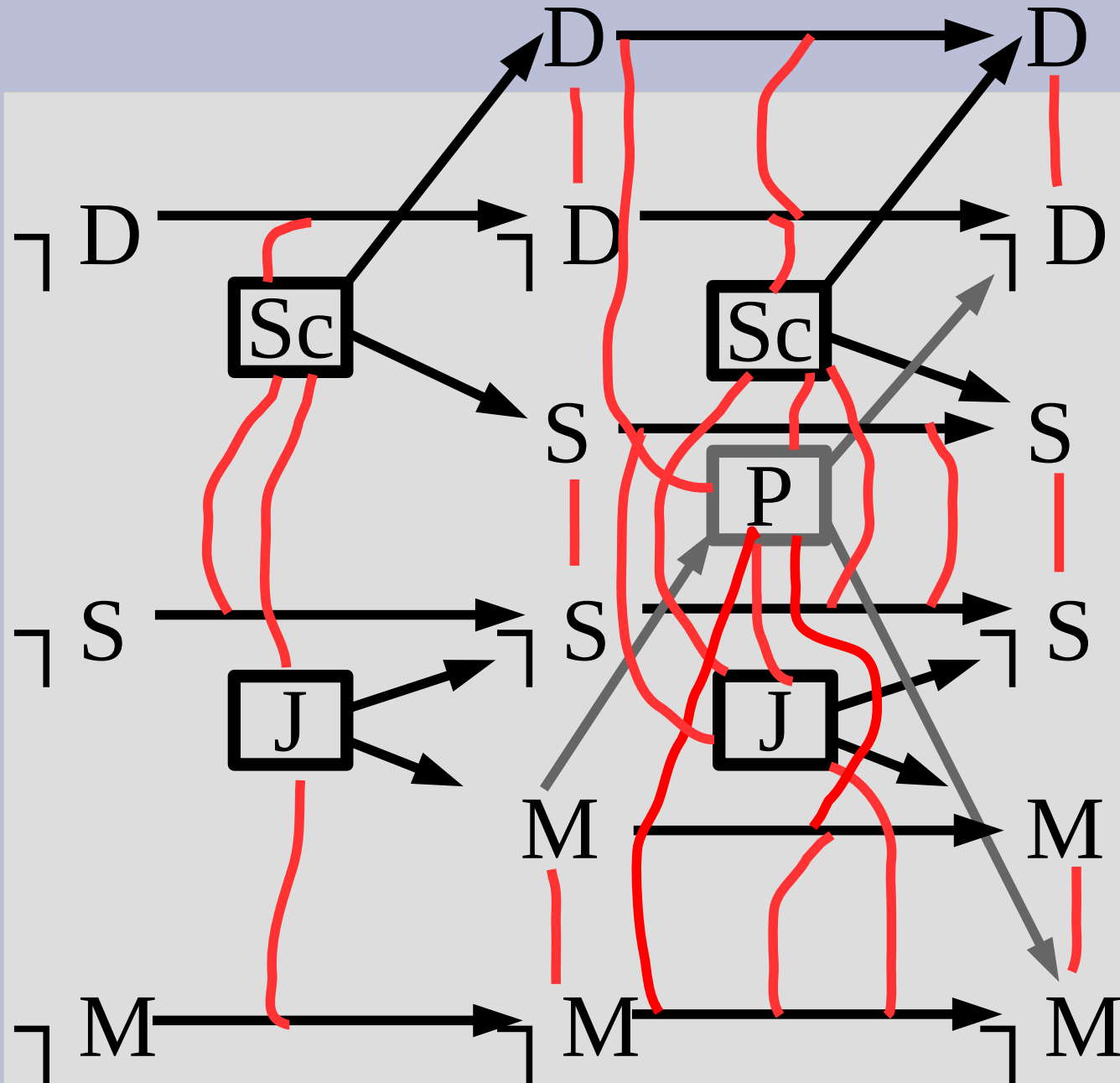
$(SC, J),$

$(P, D \& M \& \neg M),$

$(SC, \neg D \& \neg S),$

$(J, \neg M \& S)$

# Mutexes: actions



Non-trivial  
mutexes:

(SC, P),

(J, P),

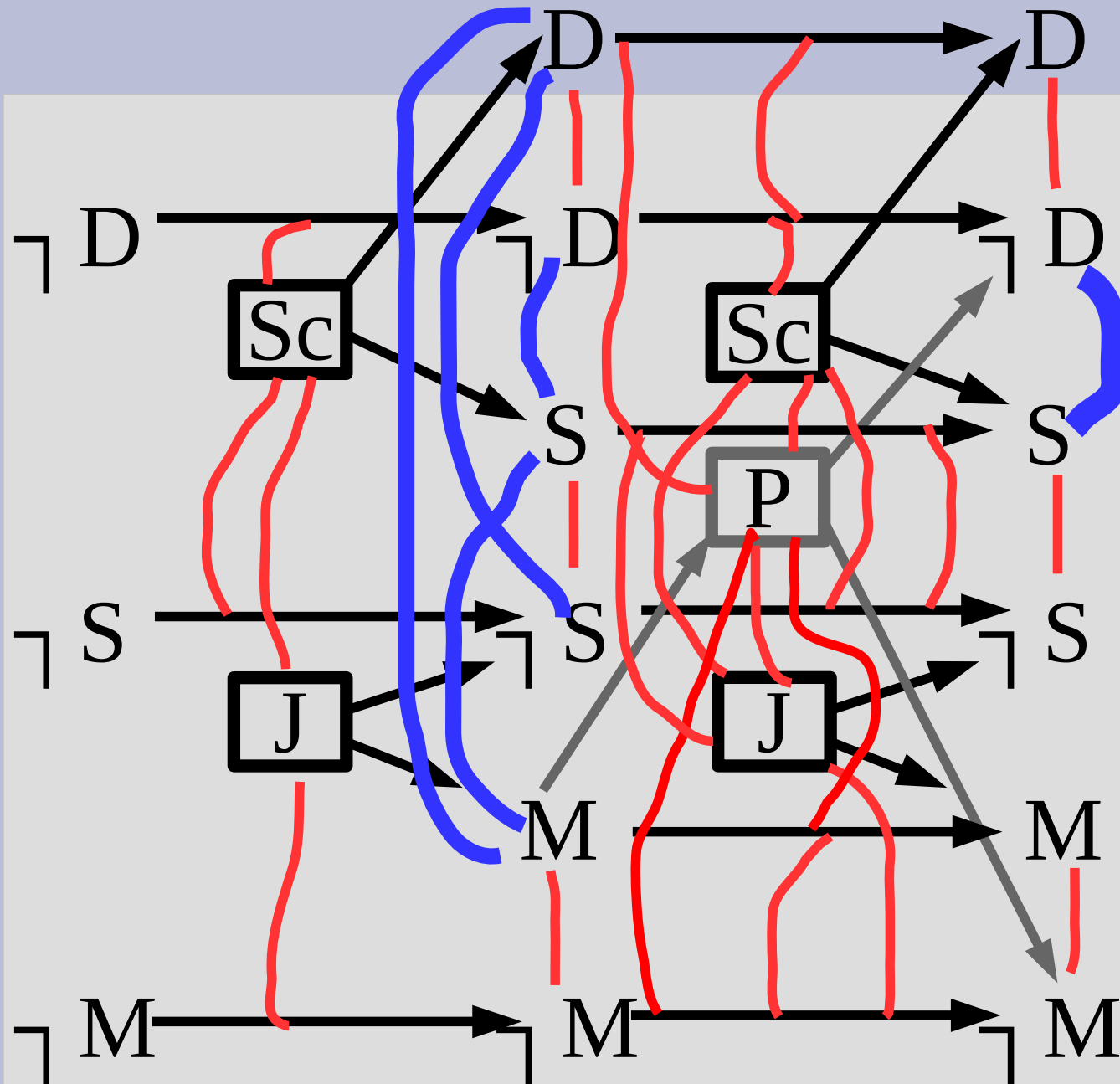
(SC, J),

(P,  $D \& M \& \neg M$ ),

(SC,  $\neg D \& \neg S$ ),

(J,  $\neg M \& S$ )

# Mutexes: actions



Non-trivial  
mutexes:

(SC, P),

(J, P),

(SC, J),

(P,  $D \& M \& \neg M$ ),

(SC,  $\neg D \& \neg S$ ),

(J,  $\neg M \& S$ )

# GraphPlan

GraphPlan can be computed in  $O(n(a+1)^2)$ ,  
where  $n$  = levels before convergence  
 $a$  = number of actions  
 $l$  = number of relations/literals/states  
(square is due to needing to check all pairs)

The original planning problem is PSPACE,  
which is known to be harder than NP



# GraphPlan: states

Let's consider this problem:

Initial:  $Clean \wedge Garbage \wedge Quiet$

Goal:  $Food \wedge \neg Garbage \wedge Present$

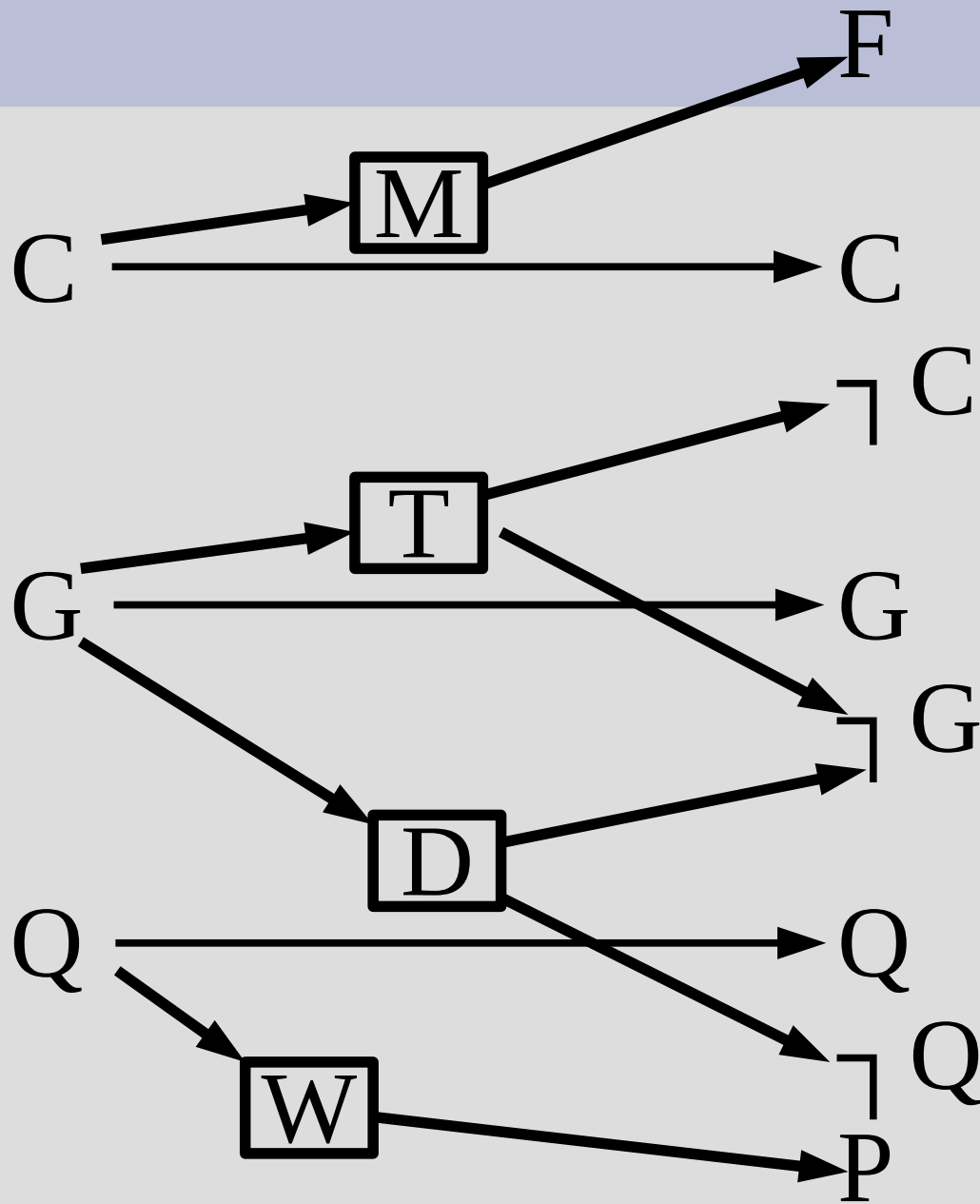
Action: ( *MakeFood*,  
Precondition: *Clean*,  
Effects: *Food*)

Action: ( *Takeout*,  
Precondition: *Garbage*,  
Effects:  $\neg Garbage \wedge \neg Clean$ )

Action: ( *Wrap*,  
Precondition: *Quiet*,  
Effects: *Present*)

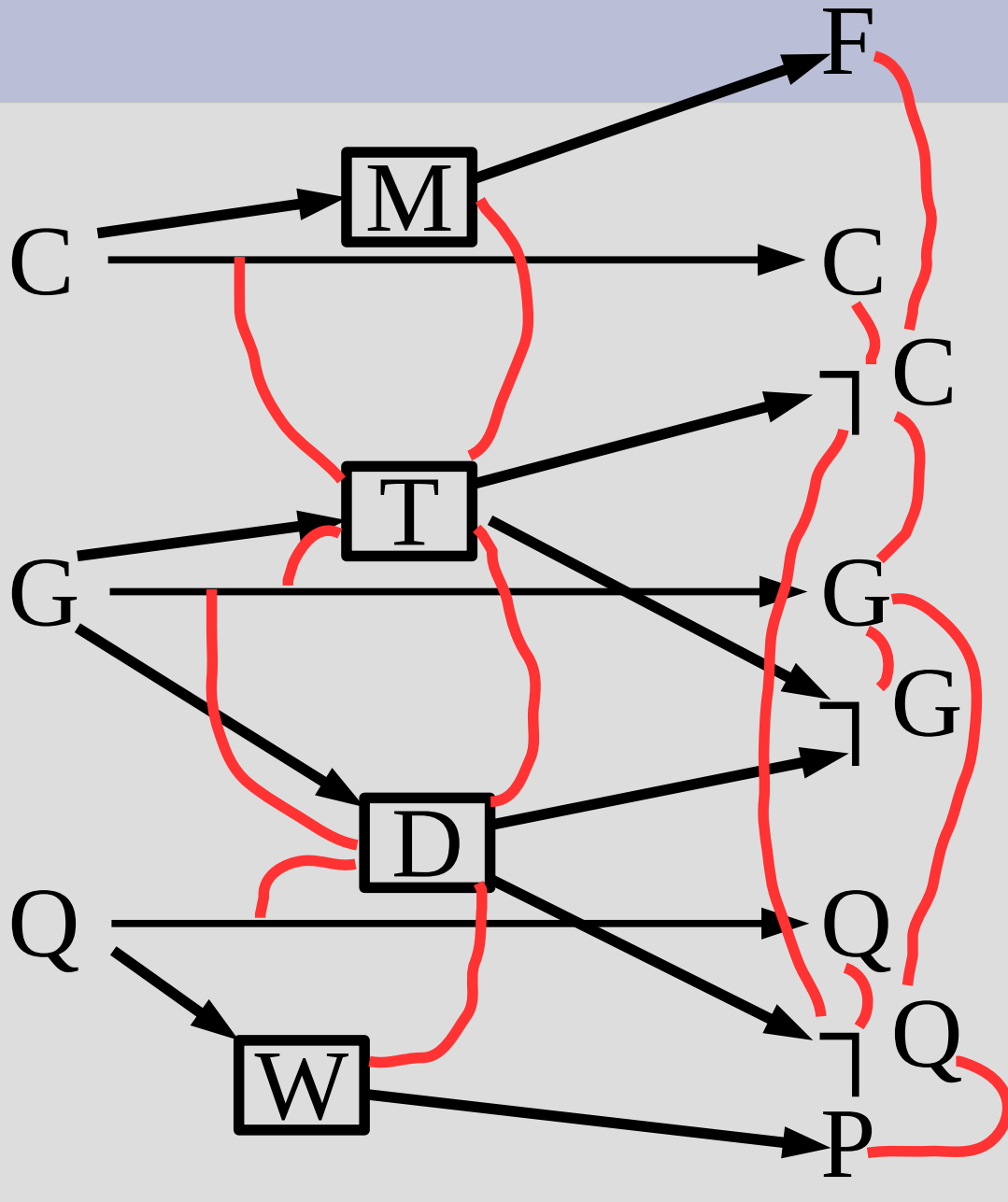
Action: ( *Dolly*,  
Precondition: *Garbage*,  
Effects:  $\neg Garbage \wedge \neg Quiet$ )

# GraphPlan: states



Take out one more level

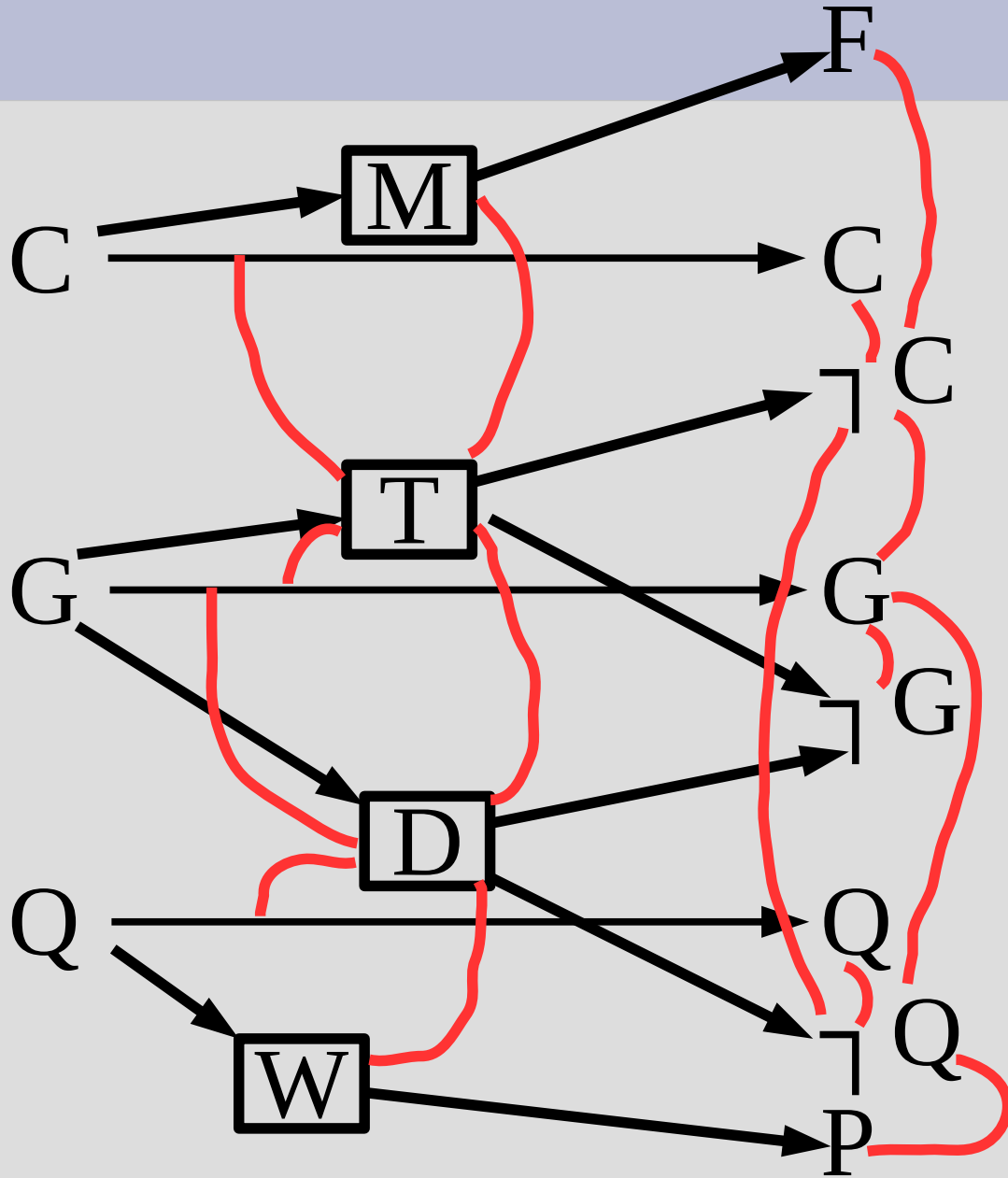
# Mutexes



Possible state pairs:

F, C	C, Q
<del>F, <math>\neg</math>C</del>	C, $\neg$ Q
F, G	C, P
F, $\neg$ G	<del><math>\neg</math>C, G</del>
F, Q	$\neg$ C, $\neg$ G
F, $\neg$ Q	$\neg$ C, Q
F, P	<del><math>\neg</math>C, <math>\neg</math>Q</del>
<del>C, <math>\neg</math>C</del>	$\neg$ C, P
C, G	... (more)
C, $\neg$ G	

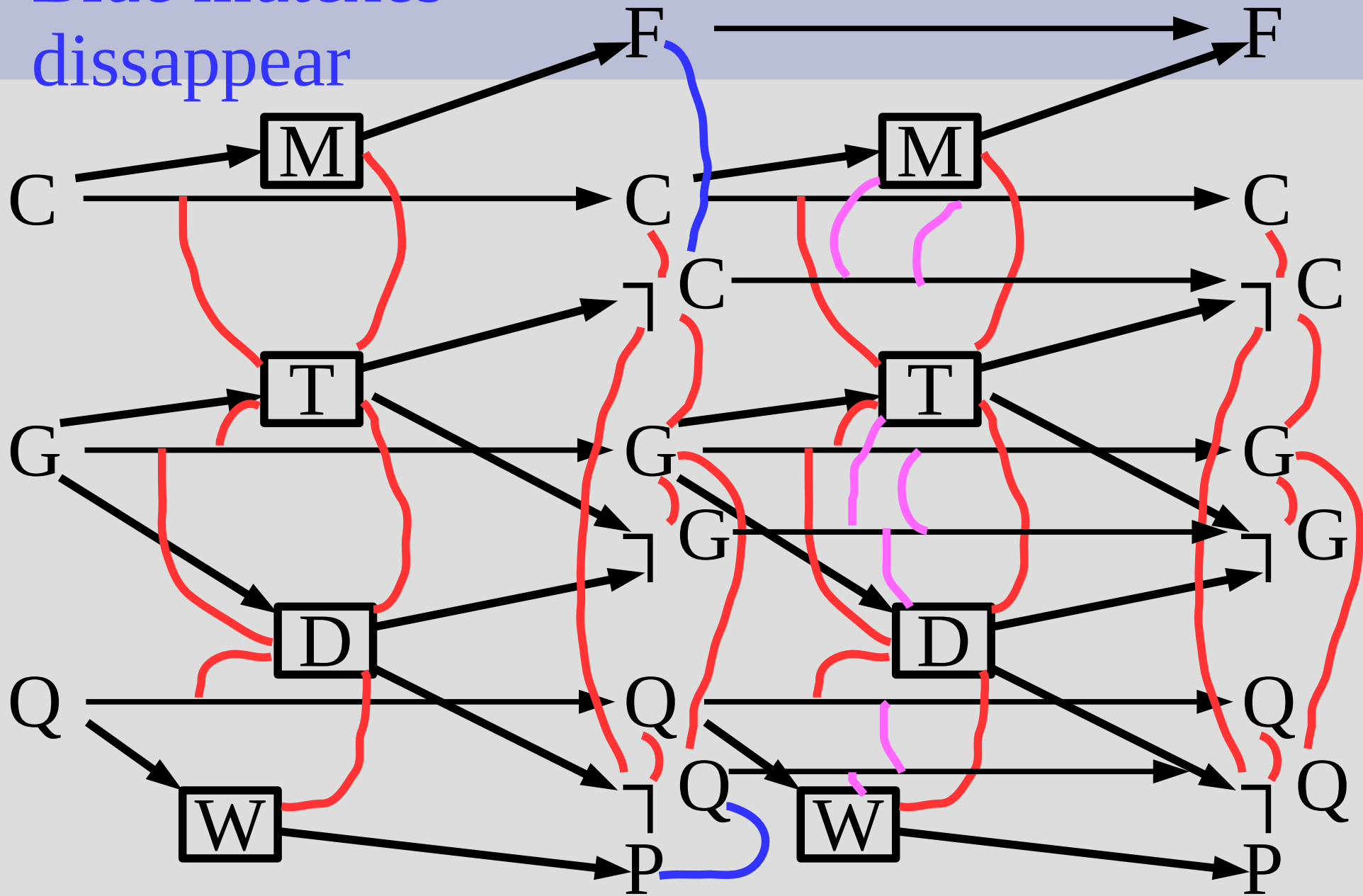
# Mutexes



Make  
one  
more  
level  
here!

# Mutexes

Blue mutexes disappear  
Pink = new mutex



# GraphPlan as heuristic

GraphPlan is optimistic, so if any pair of goal states are in mutex, the goal is impossible

3 basic ways to use GraphPlan as heuristic:

- (1) Maximum level of all goals
- (2) Sum of level of all goals (not admissible)
- (3) Level where no pair of goals is in mutex

(1) and (2) do not require any mutexes, but are less accurate (quick 'n' dirty)

# GraphPlan as heuristic

For heuristics (1) and (2), we relax as such:

1. Multiple actions per step, so can only take fewer steps to reach same result
2. Never remove any states, so the number of possible states only increases

This is a valid simplification of the problem, but it is often too simplistic directly

# GraphPlan as heuristic

Heuristic (1) directly uses this relaxation and finds the first time when all 3 goals appear at a state level

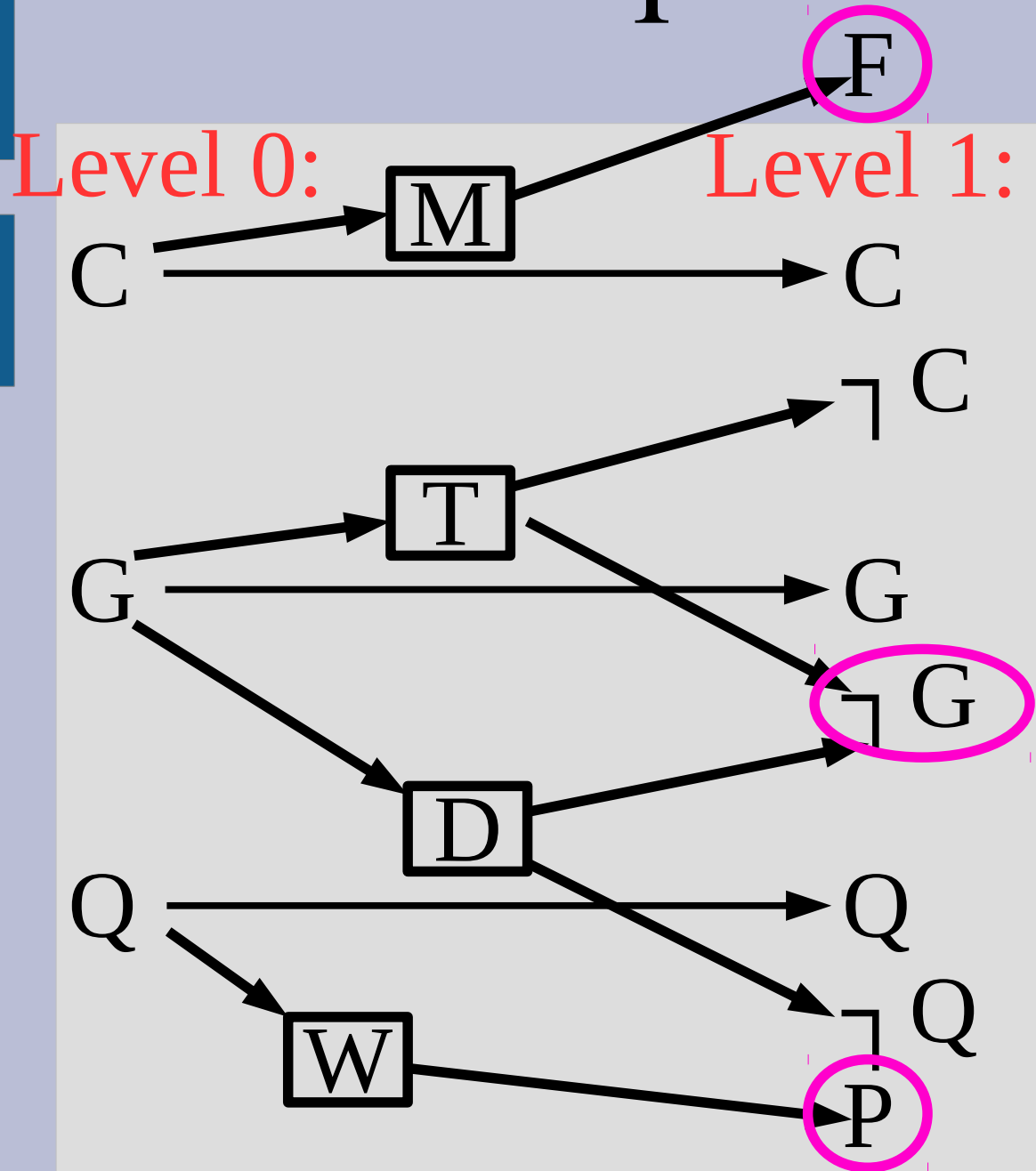
(2) tries to sum the levels of each individual first appearance, which is not admissible (but works well if they are independent parts)

Our problem: goal={Food,  $\neg$  Garbage, Present}

First appearance: F=1,  $\neg$  G=1, P=1



# GraphPlan: states



Heuristic (1):  
 $\text{Max}(1,1,1) = 1$

Heuristic (2):  
 $1+1+1=3$

# GraphPlan as heuristic

Often the problem is too trivial with just those two simplifications

So we add in mutexes to keep track of invalid pairs of states/actions

This is still a simplification, as only impossible state/action pairs in the original problem are in mutex in the relaxation

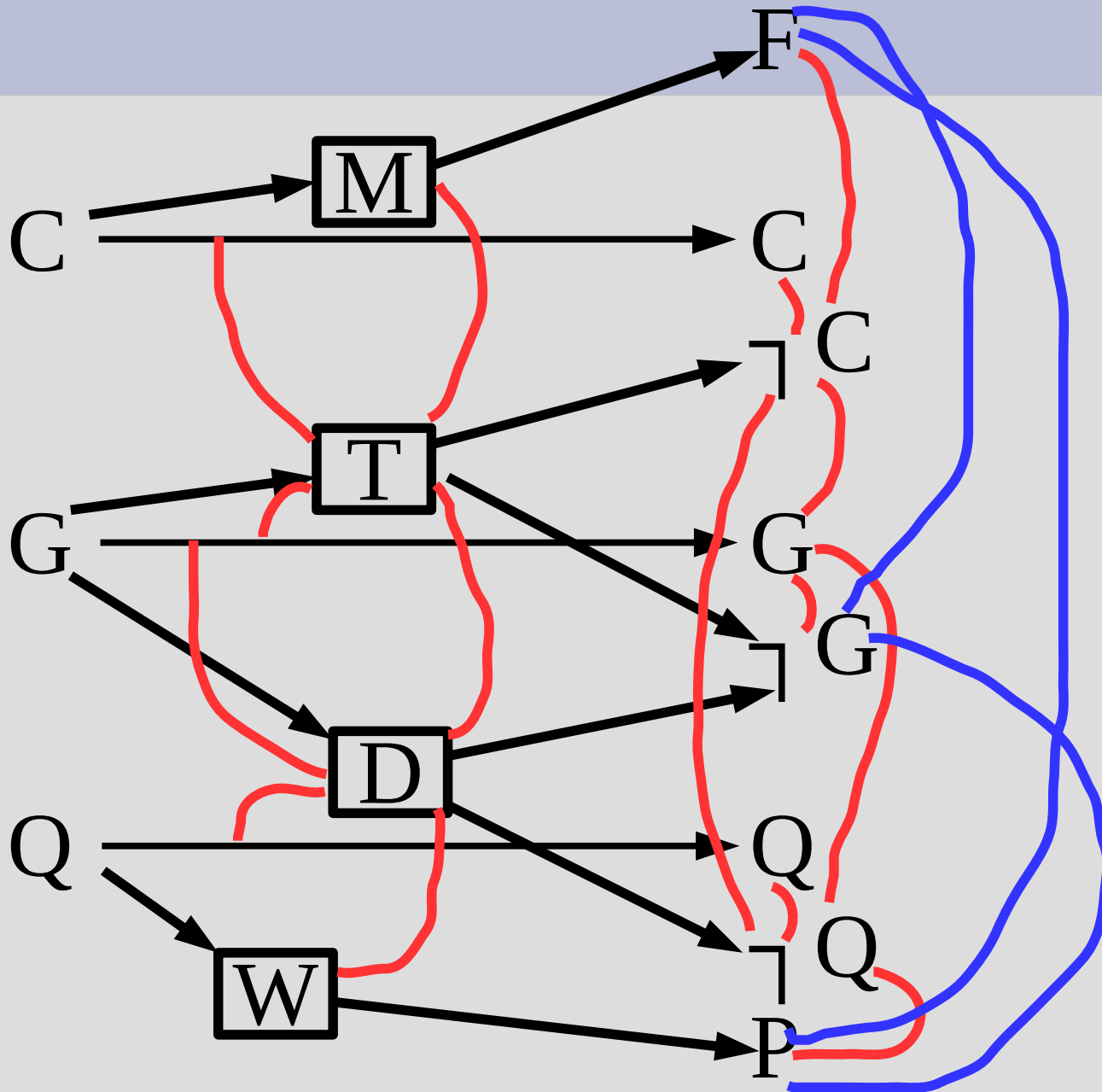
# GraphPlan as heuristic

Heuristic (3) looks to find the first time none of the goal pairs are in mutex

For our problem, the goal states are:  
(Food,  $\neg$  Garbage, Present)

So all pairs that need to have no mutex:  
(F,  $\neg$  G), (F, P), ( $\neg$  G, P)

# Mutexes



None of the pairs are in mutex at level 1

This is our heuristic estimate

# Finding a solution

GraphPlan can also be used to find a solution:

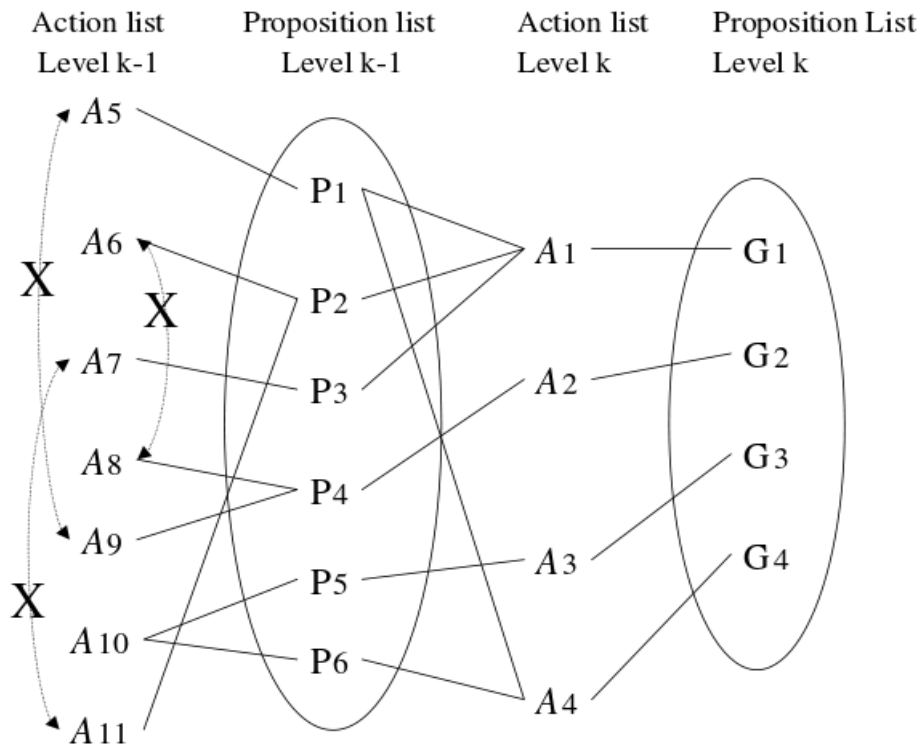
- (1) Converting to a Constraint Sat. Problem
- (2) Backwards search

Both of these ways can be run once GraphPlan has all goal pairs not in mutex (or converges)

Additionally, you might need to extend it out a few more levels further to find a solution (as GraphPlan underestimates)

# GraphPlan as CSP

Variables = states, Domains = actions to there  
 Constraints = mutexes & preconditions



(a) Planning Graph

Variables:  $G_1, \dots, G_4, P_1 \dots P_6$

Domains:  $G_1: \{A_1\}, G_2: \{A_2\}, G_3: \{A_3\}, G_4: \{A_4\}$   
 $P_1: \{A_5\}, P_2: \{A_6, A_{11}\}, P_3: \{A_7\}, P_4: \{A_8, A_9\}$   
 $P_5: \{A_{10}\}, P_6: \{A_{10}\}$

Constraints (normal):  $P_1 = A_5 \Rightarrow P_4 \neq A_9$   
 $P_2 = A_6 \Rightarrow P_4 \neq A_8$   
 $P_2 = A_{11} \Rightarrow P_3 \neq A_7$

Constraints (Activity):  $G_1 = A_1 \Rightarrow \text{Active}\{P_1, P_2, P_3\}$   
 $G_2 = A_2 \Rightarrow \text{Active}\{P_4\}$   
 $G_3 = A_3 \Rightarrow \text{Active}\{P_5\}$   
 $G_4 = A_4 \Rightarrow \text{Active}\{P_1, P_6\}$

Init State:  $\text{Active}\{G_1, G_2, G_3, G_4\}$

(b) DCSP  
 from Do & Kambhampati

# Finding a solution

For backward search, attempt to find arrows back to the initial state (without conflict/mutex)

Start by finding actions that satisfy all goal conditions, then recursively try to satisfy all of the selected actions' preconditions

If this fails to find a solution, mark this level and all the goals not satisfied as: (level, goals)  
(level, goals) stops changing, no solution

# Graph Plan

Remember this...

Initial:  $\neg Money \wedge \neg Smart \wedge \neg Debt$

Goal:  $\neg Money \wedge Smart \wedge \neg Debt$

Action( *School*,

Precondition: ,

Effect:  $Debt \wedge Smart$ )

Action( *Job*,

Precondition: ,

Effect:  $Money \wedge \neg Smart$ )

Action( *Pay*,

Precondition: *Money*,

Effect:  $\neg Money \wedge \neg Debt$ )



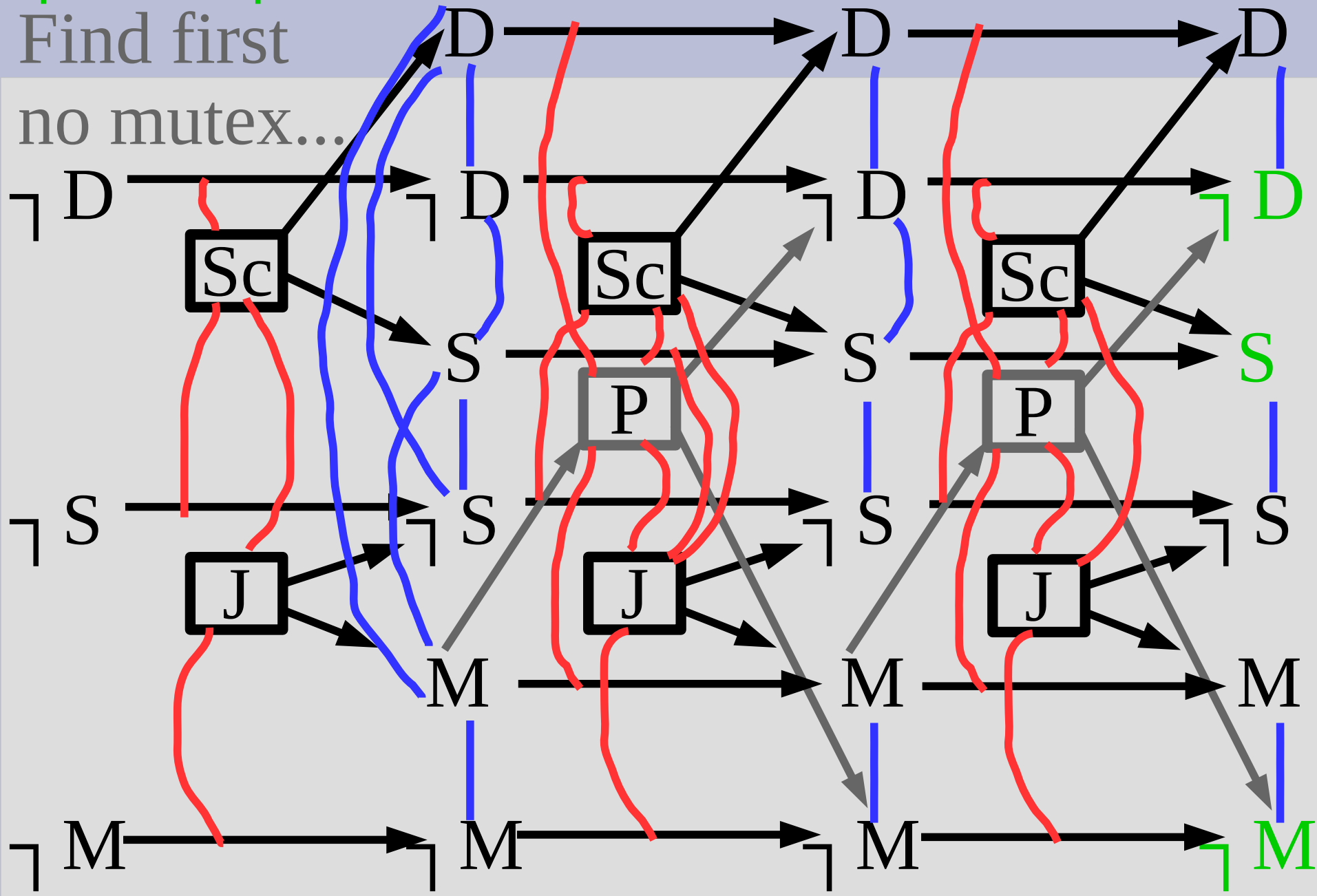
Ask:

$\neg D \wedge S \wedge \neg M$

Find first

no mutex...

# Graph Plan



Ask:

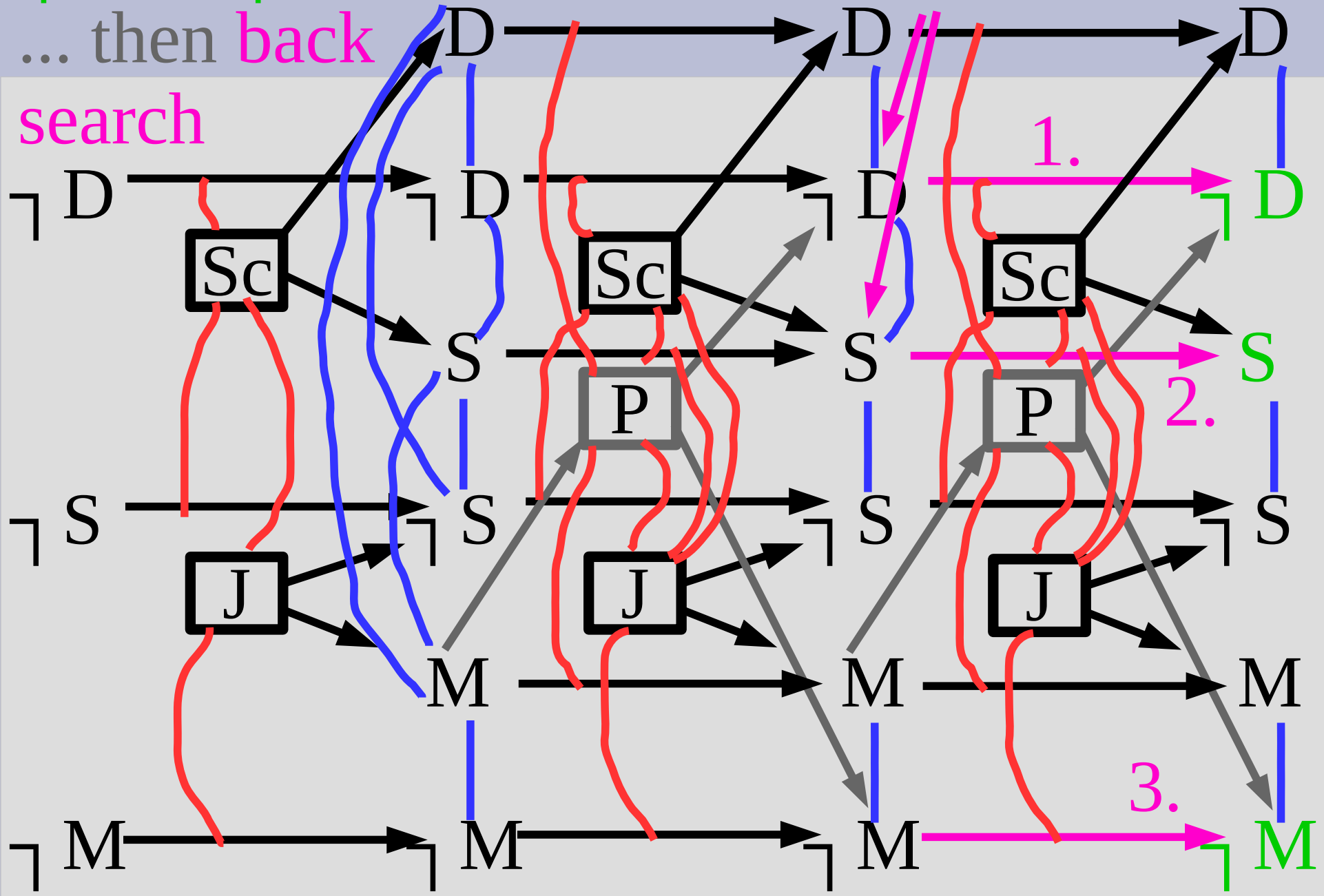
$\neg D \wedge S \wedge \neg M$

... then back

search

# Graph Plan

Error! States of 1&4 in mutex



Ask:

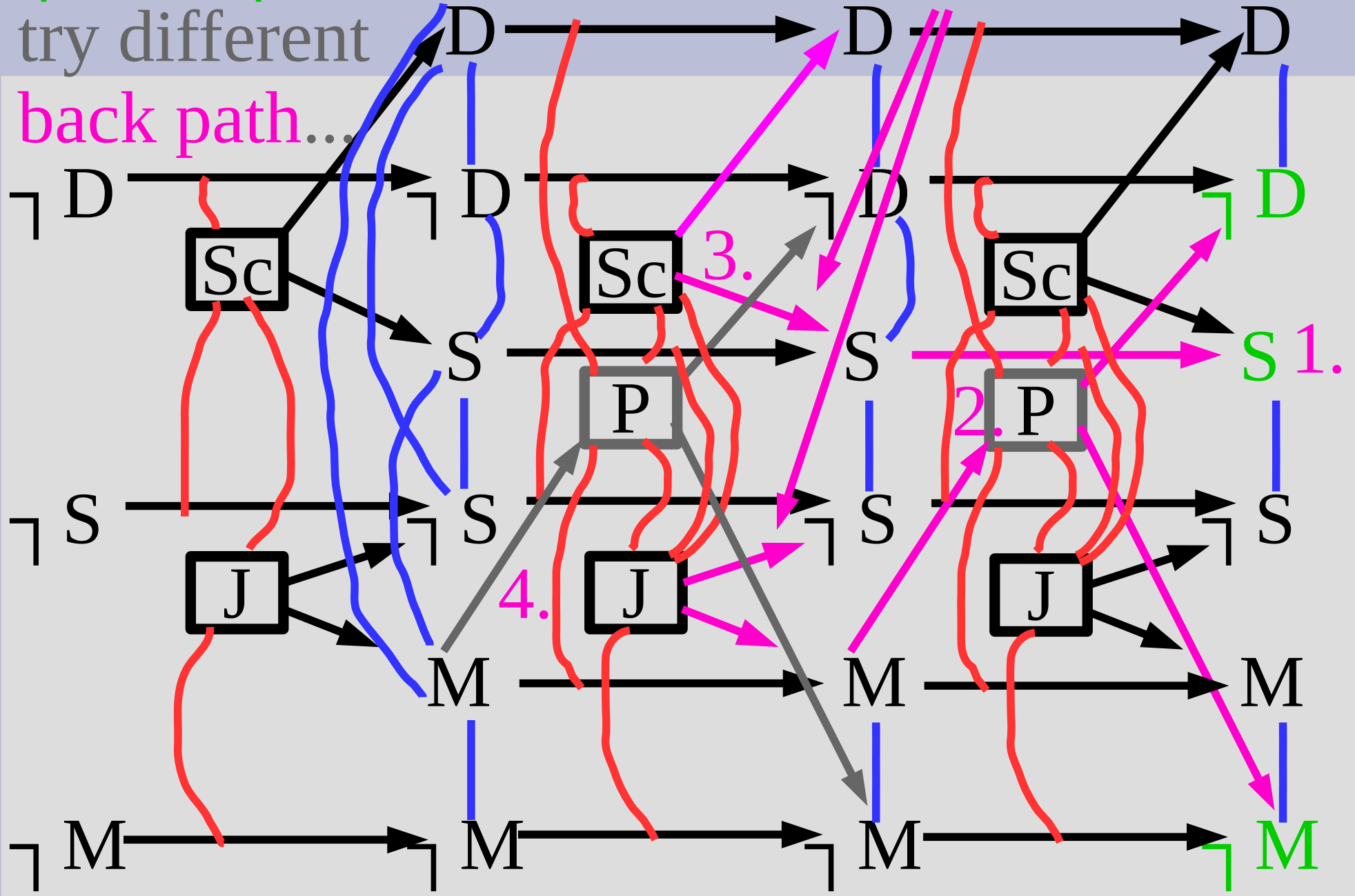
$\neg D \wedge S \wedge \neg M$

try different

back path...

# Graph Plan

Error, actions  
3&4 in mutex



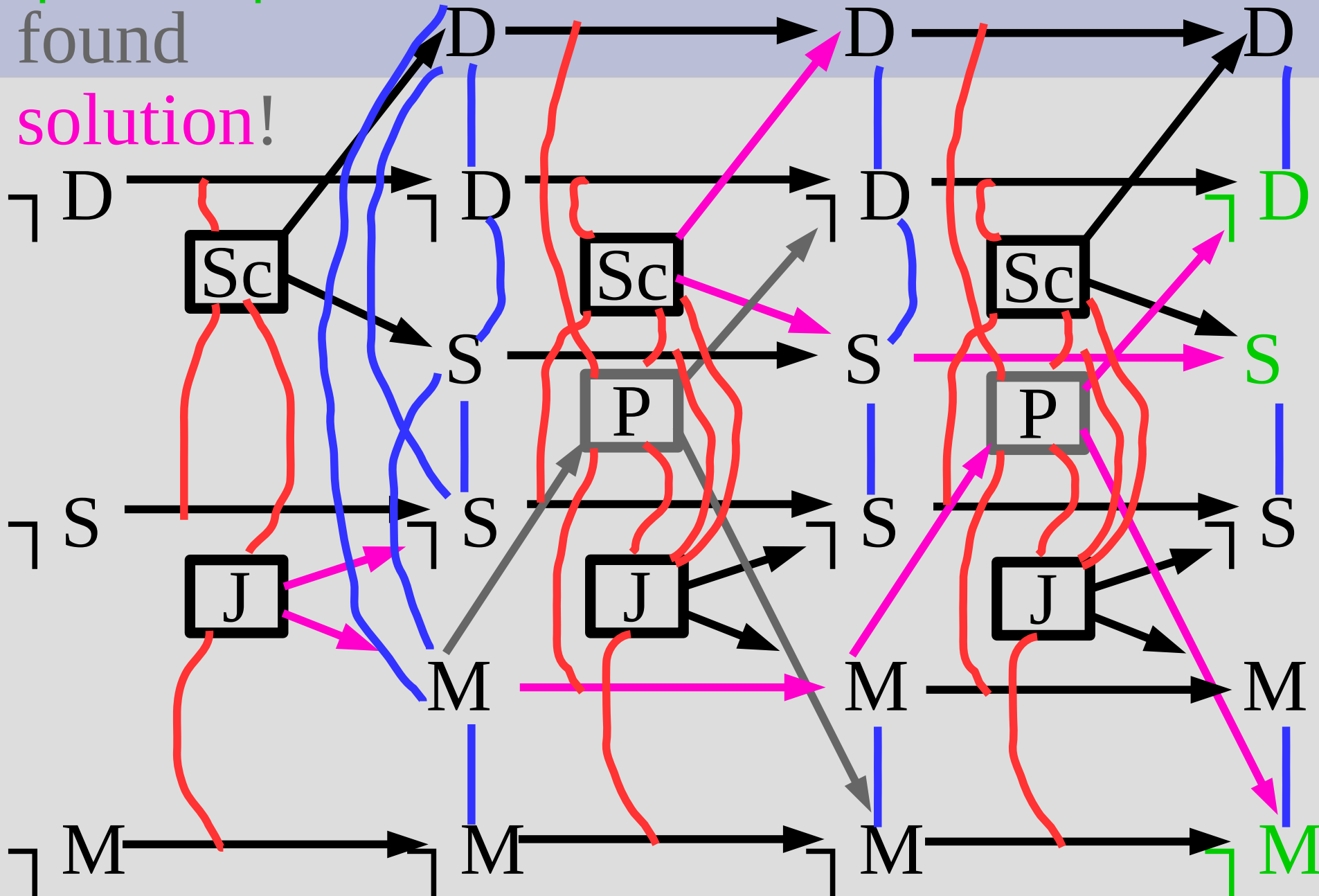
Ask:

$\neg D \wedge S \wedge \neg M$

found

solution!

# Graph Plan



# Finding a solution

Formally, the algorithm is:

graph = initial

noGoods = empty table (hash)

for level = 0 to infinity

  if all goal pairs not in mutex

    solution = recursive search with noGoods

    if success, return paths

  if graph & noGoods converged, return fail

  graph = expand graph

Initial:  $Clean \wedge Garbage \wedge Quiet$

Goal:  $Food \wedge \neg Garbage \wedge Present$

You try it!

