CSci5512, Fall-2021
ASSIGNMENT 1 :
Assigned: 09/16/21 Due: 09/30/21 at 11:55 PM (submit via Canvas, you may scan or take a picture of your paper answers) Please organize your work before sumbitting.
On all problems you must show work to receive full credit; all answers done individually

Problem 1. (10 points)
In-class we did an example where I said P(a) = 0.2, P(b) = 0.3, and P(a or b) = 0.1. I claimed these probabilities were not consistent with each other (impossible). What property is violated? Prove this property using only these five facts that I gave in-class:

(1) $0 \leq P(\omega) \leq 1$
(2) $\sum_{\omega \in \Omega} P(\omega) = 1$, where Ω is the set of all possible outcomes
(3) $P(a) + P(\neg a) = 1$
(4) $P(a \text{ or } b) = P(a) + P(b) - P(a, b)$
(5) $P(a) = \sum_{b} P(a, b)$

Problem 2 & 3 use these probabilities: (original had a small mistake)

$$p(a, b, c) = 0.096$$
$$p(a, b, \neg c) = 0.06$$
$$p(a, \neg b, c) = 0.384$$
$$p(a, \neg b, \neg c) = 0.06$$
$$p(\neg a, b, c) = 0.024$$
$$p(\neg a, b, \neg c) = 0.14$$
$$p(\neg a, \neg b, c) = 0.096$$
$$p(\neg a, \neg b, \neg c) = 0.14$$

Problem 2. (15 points)
Find the following probabilities using the table above:
(1) $p(a|b, c)$
(2) $p(a, b)$
(3) $p(a|b)$
(4) $p(a)$
(5) $p(\neg a)$

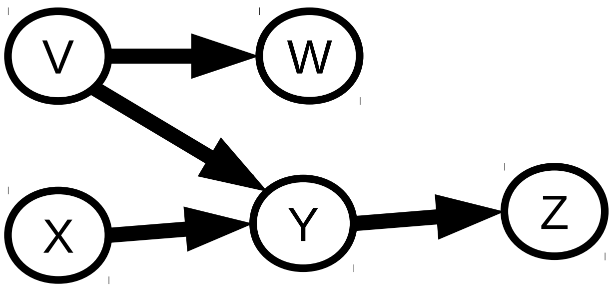
Problem 3. (15 points)
Using the same table as problem 2, are any of the variables independent? Are any of the variables conditionally independent? (Support your claims with work demonstrating why.)

Problem 4. (15 points)
Suppose you had four (true/false valued) variables: a, b, c and d. Variable "a" is independent from all the other variables. "b" and "c" are conditionally independent given "d".

(1) Give the most **efficiently** Bayesian network (least amount of probabilities stored). How many number do you need to store?

(2) Give the most **inefficient** Bayesian network (maximum amount of probabilities to define network without giving the probabilities for opposite events (e.g. can't give both P(a|b) and P(¬a|b))

Problem 5. (15 points)
Suppose you had a Bayesian Network that looks like this:



For the following probabilities, simplify them as much as possible (also write a sentence or two justifying your reasoning):
(1) $p(V, X)$
(2) $p(Z|V, Y)$
(3) $p(Y|V, W)$
(4) $p(V|W, Y, X)$

Problem 6. (10 points)
Suppose you and another family were hosting/taking each other out to dinner occasionally. You host the dinner 40% of the time and the other family hosts 60%. Sometimes the other family are feeling a bit lazy, so the amount of effort/money they put into the dinner varies. 80% of the time they put "4x" hours (x is a variable) of work preparing for the dinner party, the remaining 20% of the time they slack off and only put in "x" hours. You follow suit and 70% of the time you put in 5 hours, and 30% of the time you put in "2x" hours.

(1) Set this word problem up as represented by random variables (hint: plural).
(2) What value of "x" brings the average amount of work to get ready for the dinner party to 6.12 hours?
(3) For the "x" found above, who is putting more effort into these dinner parties? (Short answer expected, but reference a bit of math.)

Problem 7. (15 points)
Suppose you went to a casino to gamble away some money. The machine you decide to stop at lets you win a generous 30% of the time (and lose 70%). Assume you <u>stop playing as soon as you win</u> once. Answer the following questions about probability:
(1) What is the probability you will win within 3 pulls?
(2) What is the probability you will win within 30 pulls? (Please give an exact answer, not an approximation.)

Problem 8. (5 points)
Same setup as problem 7 (30% win, 70% lose), except if you assume it takes $1 to play this machine. On average how many dollars will you need to spend before you win? (Hint: there are multiple ways to solve/setup problem 7... one way makes it easier to solve problem 8 than some others.)